4.2a: Null spaces MA322-001 Feb 26 Worksheet

Define Nul(A) to be the solution set $\{\vec{x} : A\vec{x} = \vec{0}\}$ to the homogeneous equation $A\vec{x} = 0$. It turns out that Nul(A) is always a subspace, no matter which matrix A is.

Verify the three part test to be a subspace here:

It is not to hard to test if a vector $\vec{v}$ is in Nul(A): just multiply it by A and check that you get $\vec{0}$. On the other hand, how do you find lots of vectors in Nul(A)? RREF!

Explain how to get all vectors in Nul(A) for a matrix A that is row equivalent to this matrix B in row echelon form:

$$A \xrightarrow{\text{Row ops}} B = \left[\begin{array}{cccccccc|c} 1 & 2 & 0 & 3 & 4 & 0 & 5 & 6 & 0 \\ 0 & 0 & 1 & 7 & 8 & 0 & 9 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 11 & 12 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

How is column dependence related?

4.2b: Column spaces MA322-001 Feb 26 Worksheet

Define Col(A) to be the span of the columns of A. It turns out this is always a subspace, no matter which matrix A is.

Verify the three part test to be a subspace here:

It is easy to find some elements of Col(A) (the columns of A, any linear combination of them). However, given a vector $\vec{b}$, it can be hard to decide if $\vec{b}$ is in Col(A). How do we figure it out? RREF!

Explain how to test if a vector $\vec{b}$ all vectors in Col(A) for a matrix A that is row equivalent to this matrix B in row echelon form:

$$A \xrightarrow[R_3+R_1]{R_2-3R_1} \cdots \xrightarrow[R_1-R_4]{R_4-R3} B = \left[\begin{array}{cccccccc|c} 1 & 2 & 0 & 3 & 4 & 0 & 5 & 6 & 0 \\ 0 & 0 & 1 & 7 & 8 & 0 & 9 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 11 & 12 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

How is this related to row dependence?

4.2c: Linear transformations MA322-001 Feb 26 Worksheet

Matrices are convenient for computers and spreadsheets, but sometimes there are much clearer ways of describing a linear transformation.

A **linear transformation** is a function $T : V \to W$ between two vector spaces V and W (so $T(\vec{v}) = \vec{w}$) satisfying the following two axioms:

(Additive) $T(\vec{v_1} + \vec{v_2}) = T(\vec{v_1}) + T(\vec{v_2})$, and
(Multiple) $T(c\vec{v}) = cT(\vec{v})$

Let V be the vector space of polynomials and let T be the derivative. Is T a linear transformation? From where to where? Show the two part test:

The **null space** of T is all $\vec{v}$ so that $T(\vec{v}) = \vec{0}$. If T is given by a matrix A, then the null space of T is just Nul(A). What is the null space of T when T is the derivative operator?

The **image** of T is all $\vec{w}$ so that there is some $\vec{v}$ with $T(\vec{v}) = \vec{w}$. If T is given by a matrix A, the image of T is just Col(A). What is the image of T when T is the derivative operator?

MA322-001 Feb 26 Quiz Name:________________________

4.1 (HW4.1#2) V is the vector space of all vectors $\begin{bmatrix} x \\ y \end{bmatrix}$, but W is only those vectors with $xy \geq 0$. For each of the three tests, check W:

(a) Is $\vec{0}$ in W?

(b) If $\vec{v}$ in W and c is a number, is $c\vec{v}$ in W?

(c) if $\vec{v}$ and $\vec{u}$ are both in W, is $\vec{v} + \vec{u}$ in W?

4.2 (HW4.1#6) V is the vector space of all polynomials, $p(t)$. W is only those vectors of the form $p(t) = a + t^2$ for numbers a. Repeat the last question (parts a,b,c).

4.3 (HW4.1#19) V is the vector space of all rel valued functions, $f(t)$. W is only those that can be written as $f(t) = c_1 \cos(\pi t) + c_2 \sin(\pi t)$ for numbers c_1 and c_2. Repeat the last question (parts a,b,c)

I'll also look at your answers on the back (the derivative is linear, its null space is blank, its image is blank).