MA111: Contemporary mathematics

Entrance Slip (due 5 min past the hour):

A group of people uses plurality with elimination to decide where to get catering.

Everyone casts their ballots, but Jared is unhappy with the result: K-Lair.

Jared wants to vote again, "we never go to Subway, so I want to make Subway my last place vote." His friend Jordan agrees.

(1) If Jared and Jordan change their Subway vote from 1st to worst, can Subway go from being a loser to being a winner?

(2) Why?

Schedule:

- HW 2 is due 7am Wednesday, Sep 17th, 2014
- HW 3 is due 7am Tuesday, Sep 23rd, 2014
- Exam 1 is in-class on Thursday, Sep 25th, 2014

Today we explore anonymity, neutrality, and monotonicity

While we are passing out the worksheet...

- Please turn in your entrance slips. We will do this every non-exam day. Please bring your own 3x5 index cards.

- To be fair: votes should count.

- **Everyone's** votes should count

- Everyone's votes should count **equally** [**Anonymity**]

- **Monotonicity** is the idea that votes should count the right direction

- If people hate on Subway and Subway already lost, then why should Subway win?

Old words

- ballot, preference schedule,

- voting method, majority winner,

- plurality method, soccer rule, Borda count $=$ Thomas's rule, Daisia's rule

- standard elimination (plurality with elimination)

- pairwise comparison, Condorcet candidate, bracket method, agenda/seed, shape

- Anonymous, Neutral

New words: Neutral

- Jared proposes the rule: Subway gets two points for every first place vote, K-Lair and Ovid's get one point for every first place vote. Most points wins.

- This is not **neutral** since one restaurant is treated differently, but it is anonymous and monotone

- A rule that is not neutral is pretty obviously unfair.

- The most unfair version is "imposed rule" – "Subway always wins, no matter how anyone votes"

- This is anonymous
  (everyone's votes count equally, that is, not at all)

- This is monotone
  (changing your vote cannot change the outcome the wrong way, because it doesn't change the outcome at all)

- But this is not neutral

New words: Monotone

- Jared proposes the rule: A restaurant gets one point per last place vote. Most points wins.

- This is anonymous – everyone's vote counts equally

- This is neutral – each restaurant is treated equally

- This is silly – if people vote how they feel, then they will always be unhappy

- **Monotone** says that changing your vote from a loser to a winner should not have any effect (winner should still win)

- Another version is "changing your ranking of Subway from first to last shouldn't make Subway win"

May's theorem

- Kenneth May, 1952: The only anonymous, neutral, monotone voting method on two restaurants is plurality = majority rule

- Anonymous, so we don't need ballots, just a preference schedule

- Only two restaurants (say Ovid's and K-Lair), so only need to know how many total voters (say 5) and how many vote for Ovid's

- Neutral, so there must be a number where Ovid's wins and where K-Lair wins

- Monotone, so higher numbers should be better for Ovid's

- So 5 = Ovid's wins, and 0 = K-Lair wins

- Neutral, so switching 4 and 1 (or 3 and 2) should switch who wins

- So 5,4,3 are Ovid's wins, and 2,1,0 are K-Lair wins

- Majority rule

Monotone for more candidates

- Monotone means "if K-Lair wins with one set of Ballots, then K-Lair should still win, even if one ballot is changed so that K-Lair ranks higher"

- (And the same thing for any restaurant)

- Simple version "votes are good"

- Point systems (plurality, Borda count, Soccer, Daisia's rule) are monotone (assuming the points go the right way; "most last place" is not monotone)

- Head-to-head systems (pairwise comparison, bracket, Smith, Beahtpath) are monotone

- Elimination systems (plurality with elimination, survivor) are NOT monotone

Exit quiz

- A group wants to decide between four restaurants

- Jordan's method is to count only second and third place votes;
  1 point for each 2nd or 3rd place vote; most points wins

- Is Jordan's method anonymous?

- Neutral?

- Monotone?

- Give an example group of people (by Ballots) where Jordan's
  method doesn't work well; explain why its answer is wrong