KEY

Vowels: (mod 5)
Consonants: (mod 21)

Vowel(and y)s: (mod 6)
Consonant(not y)s: (mod 20)

1. For each number, list 4 equivalent numbers of the same type

1 (mod 5): 1, 6, 11, 16, 21, 26, ..., −4, −9, −14, −19, ...

1 (mod 6): 1, 7, 13, 19, 25, 31, ..., −5, −11, −17, −23, ...

1 (mod 21): 1, 22, 43, 64, 85, 106, ..., −20, −41, −62, −83, ...

1 (mod 20): 1, 21, 41, 61, 81, 101, ..., −19, −39, −59, −79, ...

40 (mod 5): 40, 45, 50, 55, 60, ..., 35, 30, 25, 20, 15, 10, 5, ...

40 (mod 6): 40, 46, 52, 58, 64, ..., 34, 28, 22, 16, 10, 4, ...

40 (mod 21): 40, 61, 82, 103, 124, ..., 19, −2, ...

40 (mod 20): 40, 60, 80, 100, 120, ..., 20, 0, −20, ...

2. For any particular number $x \pmod{n}$, there is a unique number y between 1 and n with $x \equiv y \pmod{n}$. This basically means "find the letter," except we stick with numbers. Find this number y for each x:

0 (mod 5): 5

0 (mod 6): 6

0 (mod 21): 21

0 (mod 20): 20

−1 (mod 5): 4

−1 (mod 6): 5

−1 (mod 21): 20

−1 (mod 20): 19

40 (mod 5): 5

40 (mod 6): 4

40 (mod 21): 19

40 (mod 20): 20

This number is called the **standard representative** of the number.

3. For each arithmetic problem, write down the standard representative of the answer.

$20 + 30 \pmod{5}$ $0 \equiv 5$

$0 + 0 = 0$

$20 + 30 \pmod{6}$ 2

$2 + 0 = 2$

$20 \times 30 \pmod{5}$ 5

$0 \times 0 = 0$

$20 \times 30 \pmod{6}$ 6

$2 \times 0 = 0$

$20 - 30 \pmod{5}$ 5

$0 - 0 = 0$

$20 - 30 \pmod{6}$ 2

$2 - 0 = 2$

4. For each number, list ALL standard representatives of numbers that work:

Double to 2 (mod 5): 1 2 3 4 5 / ×2: 2 4 1 3 5 → 1

Double to 2 (mod 6): 1 2 3 4 5 6 / ×2: 2 4 6 2 4 6 → 1, 4

Double to 1 (mod 5): → 3

Double to 1 (mod 6): None!

How many answers can there be? 0, 1, or 2

How do you tell how many answers there are?

mod 5: always 1 / mod 6: 2 for even, 0 for odd

How do consonants (mod 21) and consonant(y)s (mod 20) work?

mod 21: same as mod 5 / mod 20: same as mod 6

4. For each number, list ALL standard representatives of numbers that work:

Triple to 3 (mod 5): 1 2 3 4 5 / ×3: 3 1 4 2 5 → 1

Triple to 3 (mod 6): 1 2 3 4 5 6 / ×3: 3 6 3 6 3 6 → 1, 3, 5

Triple to 1 (mod 5): → 2

Triple to 1 (mod 6): None

Triple to 2 (mod 5): → 4

Triple to 2 (mod 6): None

How many answers can there be? 0, 1, or 3

How do you tell how many answers there are?

mod 5: always 1 / mod 6: divis by 3 has 3, otherwise 0

How do consonants (mod 21) and consonant(y)s (mod 20) work?

mod 21: same as mod 6 / mod 20: same as mod 5

5. Instead of triple, maybe it is quadruple, quintuple, or even multiply by 23.

How many answers can there be?

0, 1, 2 or 4 / 0, 1 or 5 / 0, 1 or 23

How do you tell how many answers there are?

Think about. A fun problem.