

	Alex	Blair	Casey	Devin	Emerson	Gayl	Lee	Marcus	Nikki	Sonia	Toni
1st	Ovid's	Ovid's	Ovid's	Fazoli's	Fazoli's	Ovid's	Fazoli's	K-Lair	Ovid's	Fazoli's	K-Lair
2nd	Panera	Panera	Panera	Panera	Panera	Panera	Panera	Subway	Panera	Panera	Subway
3rd	Subway	Subway	Subway	Subway	Subway	Subway	Subway	Fazoli's	Subway	Subway	Fazoli's
4th	K-Lair	K-Lair	K-Lair	K-Lair	K-Lair	K-Lair	K-Lair	Ovid's	K-Lair	K-Lair	Ovid's
5th	Fazoli's	Fazoli's	Fazoli's	Ovid's	Ovid's	Fazoli's	Ovid's	Panera	Fazoli's	Ovid's	Panera

(a) Convert this set of ballots into a preference schedule.

Your answer:

	5	4	0	2
1st	Ovid's	Fazoli's	Panera	K-Lair
2nd	Panera	Panera	Subway	Subway
3rd	Subway	Subway	Fazoli's	Fazoli's
4th	K-Lair	K-Lair	Ovid's	Ovid's
5th	Fazoli's	Ovid's	K-Lair	Panera

(The 5 is Alex, Blair, Casey, Gayl, Nikki)

(b) How many voters are needed to form a majority in this election?

More than half. 11 total voters,  $\frac{11}{2} = 5.5$ , so 6 or more voters.

(c) If you eliminate Panera, what does the new preference schedule look like?

	5	4	0	2
O	F	S	K	
S	S	F	S	
K	K	O	F	
F	O	K	O	

(You can leave the  $\frac{0}{S}$  column out)

(d) In Fazoli's versus Panera head-to-head matchup, whose votes does each restaurant get?

Fazoli's vs Panera: 6 to 5  
 (Devin, Emerson, Lee, Marcus, Sonia, Toni) vs (Alex, Blair, Casey, Gayl, Nikki)

(e) Name one restaurant that is NOT a Condorcet winner. Explain why.

Panera loses a head-to-head match (to Fazoli's) so is not a Condorcet winner, since a Condorcet winner wins every head-to-head match they are in.

(f) Name one restaurant that is NOT a Majority winner. Explain why.

Subway received 0 first place votes, which is not more than half of the first place votes (6 or more) as required by Majority winner.

2. Here is a preference schedule.

(a) Show work: Who wins using plurality? 1 pt for 1st, none otherwise.

$$\begin{aligned} A: 5+0+0 &= 5 \\ B: 0+4+0 &= 4 \\ C: 0+0+0 &= 0 \\ D: 0+0+0 &= 0 \\ E: 0+0+2 &= 2 \end{aligned} \quad A \text{ wins}$$

	5	4	2
1st	A	B	E
2nd	D	D	C
3rd	C	C	B
4th	E	E	A
5th	B	A	D

(b) Show work: Who wins using plurality with elimination?

C and D are eliminated first (0 votes). No votes change.

E is eliminated next, and their 2 votes go to B (not C, C is eliminated)

A is eliminated and their 5 votes go to B (not D, C, or E they are eliminated)

B wins 11 to 0! (Well after everyone else is gone :))

(c) Show work: Who wins using pairwise comparison?

A vs B	A vs C	A vs D	A vs E	B vs C	B vs D	B vs E	C vs D	C vs E	D vs E
5 4	5 4	5 4	5 4	4 5	4 5	4 5	2 5	5 2	5 2
+2	2	2	2	2	2	2	4	4	4
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
5 6	5 6	7 4	5 6	4 7	6 5	4 7	2 9	9 2	9 2
B	C	A	E	C	B	E	D	C	D

A: 1 win  
B: 2 win  
C: 3 win ← C wins overall  
D: 2 win  
E: 2 win

(d) Show work: Who wins using the full Borda count?

(5 points for first, 4 points for second, 3 points for third, 2 points for fourth, 1 point for last; or faster 2 points for first, 1 point for second, 0 points for third, -1 point for fourth, -2 for last)

	(5)	(4)	(2)	Total	Full 5/4/3/2/1
A	10	-8	-2	= 0	+33 = 33
B	-10	+8	+0	= -2	= 31
C	0	0	+2	= +2	= 35
D	5	+4	-4	= +5	= 38 ←
E	-5	-4	+4	= -5	= 28

D wins

(e) Show work: Who wins using "the simple combination"?

(1 point for 1st, -1 point for last, no points for the rest)

	(5)	(4)	(2)	Total
A	5	-4	0	= 1
B	-5	4	0	= -1
C	0	0	0	= 0
D	0	0	-2	= -2
E	0	0	2	= 2 ←

E wins

3. (a) Borda Count declares K-Lair the winner of this election. Which fairness criterion does this violate and why?

Ovid's is a majority candidate.

The majority criterion says that Ovid's

should win. Since Borda Count declares someone else (K-Lair) the winner, Borda Count violates the majority criterion.

	65	35
1st	Ovid's	K-Lair
2nd	K-Lair	Subway
3rd	Subway	Ovid's

(b) Borda Count declares K-Lair the winner of the top election, but Ovid's the winner of the bottom election. Which fairness criterion does this violate and why?

IIA says that if a losing candidate is removed, the winner should not change.

	40	35	25
1st	Ovid's	K-Lair	Subway
2nd	K-Lair	Subway	Ovid's
3rd	Subway	Ovid's	K-Lair

O:  $40 - 35 = 5$   
K:  $35 - 25 = 10$   
S:  $25 - 40 = -15$

Subway loses (badly) according to Borda Count, but when it is removed, the winner changes.

	45	35	25
1st	Ovid's	K-Lair	Ovid's
2nd	K-Lair	Ovid's	K-Lair

K wins.  
S is awful

IIA says the winners should stay the same, so Borda Count violates IIA.

(c) Plurality with Elimination declares A the winner of the top election, but C the winner of the bottom election. Which fairness criterion does this violate and why?

In the top election the last 3 people are sad: their least favorite candidate, A, won.

In the bottom election, those 3 people vote A best (leaving  $D > C > B$  alone). But now A loses! I'll point out that those 3 couldn't make their top choice (D) win, but they did get their second choice (C).

	46	18	17	16	3
1st	A	B	C	D	D
2nd	B	C	A	C	C
3rd	C	A	D	B	B
4th	D	D	B	A	A

C elim, +17 total  
B elim, +18 to A  
D elim, +19 to A  
A wins

	46	18	17	16	3
1st	A	B	C	D	A
2nd	B	C	A	C	D
3rd	C	A	D	B	C
4th	D	D	B	A	B

D elim, +16 total  
B elim, +18 to C  
A elim, +19 to C  
C wins

Monotonicity says that if voters move the winning candidate up (leaving the losers in the same order) then the winner should still win. "Votes are good." But here, A got 3 more votes so they lost. That means plurality with elimination violates Monotonicity.

4. (a) Create an example of a preference schedule where Borda Count disagrees with one of the other methods [ name the other method, say who won both methods, and explain "how" you figured out your example ].

#2 and #3a are great for this.

(b) Pick one of the fairness criteria we discussed in class. Explain what it means and name one voting method that violates it.

#3 a, b, or c works.

(c) Why is a restaurant with more than half of the first place votes always a Condorcet winner?

In a head-to-head you always get your first place votes (your true fans) and you even get some more (people who don't like you best, but do like you better than the other candidate). So you get at least "more than half" in a head-to-head, so the other candidate gets "less than half" and you

(d) Why does a restaurant with more than half of the first place votes always win a plurality election?

Similar to #4c. Plurality only counts 1st place votes, so "more than half" is all you get, but the other candidates have to split/share "less than half", so you have the most.

wid every head-to-head.