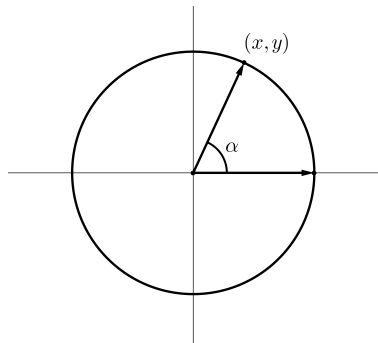


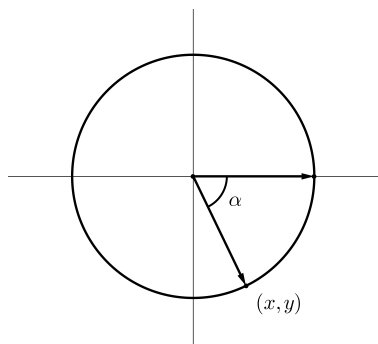
The Unit Circle and Basic Trigonometric Identities

A circle of radius one is called a *unit circle*. A unit circle with center at the origin of the Cartesian plane is often called *the unit circle*. The trigonometric functions sine, cosine, tangent, secant, cosecant, and cotangent, can be defined using the unit circle.

Let α be the radian measure of an angle. Place a ray r from the origin along the x axis. If $\alpha \geq 0$, rotate the ray by α radians in the counterclockwise direction.



If $\alpha < 0$, rotate the ray by $|\alpha|$ radians in the clockwise direction.



Determine the point (x, y) where r intersects the unit circle. We define

$$\cos \alpha = x$$

and

$$\sin \alpha = y.$$

Define also

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha}.$$

$$\sec \alpha = \frac{1}{\cos \alpha},$$

$$\csc \alpha = \frac{1}{\sin \alpha},$$

$$\cot \alpha = \frac{\cos \alpha}{\sin \alpha}.$$

1. Use the definitions for the sine, cosine, and tangent functions to evaluate $\sin \alpha$, $\cos \alpha$ and $\tan \alpha$ when α equals
 - (a) 0
 - (b) $\frac{\pi}{2}$
 - (c) π
 - (d) $\frac{3\pi}{2}$
 - (e) 2π
 - (f) $\frac{\pi}{3}$
 - (g) $\frac{\pi}{4}$
 - (h) $\frac{\pi}{6}$
 - (i) $\frac{n\pi}{3}$ for all possible integer values of n
 - (j) $\frac{n\pi}{4}$ for all possible integer values of n
 - (k) $\frac{n\pi}{6}$ for all possible integer values of n
2. Drawing on the definitions for the sine and cosine functions, sketch the graphs of the functions $f(\alpha) = \sin \alpha$ and $f(\alpha) = \cos \alpha$, and explain how you can deduce these naturally from the unit circle definition.

3. Continuing to think about the unit circle definition, complete the following formulas and give brief explanations, including a diagram, for each.

(a) $\sin(-\alpha) = -\sin(\alpha)$.

(b) $\cos(-\alpha) =$

(c) $\sin(\pi + \alpha) =$

(d) $\cos(\pi + \alpha) =$

(e) $\sin(\pi - \alpha) =$

(f) $\cos(\pi - \alpha) =$

(g) $\sin(\pi/2 + \alpha) =$

(h) $\cos(\pi/2 + \alpha) =$

(i) $\sin(\pi/2 - \alpha) =$

(j) $\cos(\pi/2 - \alpha) =$

(k) $\sin^2(\alpha) + \cos^2(\alpha) =$

4. Use the sine and cosine functions to determine the coordinates of the vertices of the following. In each case except the last two, choose one vertex to be the point $(1, 0)$.

(a) A regular triangle with vertices having a distance of 1 from the origin.

(b) A regular square with vertices having a distance of 1 from the origin.

(c) A regular pentagon with vertices having a distance of 1 from the origin.

(d) A regular hexagon with vertices having a distance of 1 from the origin.

(e) A regular heptagon with vertices having a distance of 3 from the origin.

(f) A regular n -gon with vertices having a distance of r from the origin.