Exam 2

Solutions

Multiple Choice Questions

1. The graph of $y = x^2$ is shifted up 9 and to the left 1. Write the resulting function.

   Solution: $y = (x + 1)^2 + 9$

2. Determine whether the number -7 is a zero of $f(x) = x^3 + 4x^2 - 49x - 196$. If it is, find the other real zeros.

   Solution: $f(-7) = (-7)^3 + 4(-7)^2 - 49(-7) - 196 = 0$ so $x = -7$ is a root. Dividing $f(x)$ by $x + 7$, we get $f(x) = (x + 7)(x^2 - 3x - 28) = (x + 7)(x - 7)(x + 4)$. -7 is a zero and the other zeroes are -4 and 7.

3. For the rational function $f(x) = \dfrac{-4x^2 + 18x + 16}{x^2 + 8x + 7}$, find all vertical and horizontal asymptotes.

   Solution:
$$f(x) = \frac{-4x^2 + 18x + 16}{x^2 + 8x + 7} = \frac{-4x^2 + 18x + 16}{(x + 1)(x + 7)}$$
   and $\lim_{x\to\infty} f(x) = -4$ so the vertical asymptotes are $x = -1$ and $x = -7$ and the horizontal asymptote is $y = -4$.

4. The quadratic function $f(x) = 0.0042x^2 - 0.42x + 36.05$ models the median, or average, age, y, at which U.S. men were first married x years after 1900. In which year was this average age at a minimum? (Round to the nearest year.) What was the average age at first marriage for that year? (Round to the nearest tenth.)

   Solution: Plot the function and we find the minimum occurs closest to 1950 and $f(50) = 25.55 \approx 25.6$.

5. Solve the following inequality:

$$\frac{x^2(x-11)(x+1)}{(x-4)(x+9)} \geq 0$$

Solution: $\frac{x^2(x-11)(x+1)}{(x-4)(x+9)} \geq 0$, so the places at which we have to divide the real line are 0, 11, -1, 4, and -9.

	$x < -9$	$-9 < x < -1$	$-1 < x < 0$	$0 < x < 4$	$4 < x < 11$	$11 < x$
$x+9$	$-$	$+$	$+$	$+$	$+$	$+$
$x+1$	$-$	$-$	$+$	$+$	$+$	$+$
x^2	$+$	$+$	$+$	$+$	$+$	$+$
$x-4$	$-$	$-$	$-$	$-$	$+$	$+$
$x-11$	$-$	$-$	$-$	$-$	$-$	$+$
Quotient	$+$	$-$	$+$	$+$	$-$	$+$

The function is positive for $x < -9$, $-1 < x < 0$, $0 < x < 4$ and $x > 11$. We may not include $x = -9$ as this is a vertical asymptote. We include $x = -1$, $x = 0$ and $x = 11$ but must exclude $x = 4$. Thus, the inequality is greater than or equal to zero on $(-\infty, -9) \bigcup [-1, 4) \bigcup [11, \infty)$

6. Economists use what is called a Leffer curve to predict the government revenue for tax rates from 0% to 100%. Economists agree that the end points of the curve generate 0 revenue, but disagree on the tax rate that produces the maximum revenue. Suppose an economist produces this rational function $R(x) = 10x(100-x)/(15+x)$, where R is revenue in millions at a tax rate of x percent. What tax rate produces the maximum revenue? What is the maximum revenue?

Solution: Again graph the function to find that the maximum occurs at $x = 26.5\%$. The revenue there is \$469 million.

7. Which of the following are both factors of $p(x) = x^4 - 10x^3 + 29x^2 - 8x - 48$?

Solution: The easiest way to check is to find $p(x)$ for the different values of x. In doing so we get that $p(3) = p(4) = 0$ so factors are $x-3$, $x-4$.

8. Find a polynomial of degree 3 that has zeros of 2, -5,and 6, and where the coefficient of x^2 is 9.

Solution: The polynomial must look like $p(x) = a(x-2)(x+5)(x-6) = ax^3 - 3ax^2 - 28ax + 60a$. We need $-3a = 9$ so $a = -3$ and the polynomial is$-3x^3 + 9x^2 + 84x - 180$.

9. Evaluate the expression $(4+9i)(11-10i)$ and write the result in the form $a+bi$.

Solution: $(4+9i)(11-10i) = (44-90i^2) + (99-40)i = 134 + 59i$.

10. Find the inverse function of $f(x) = \dfrac{x-7}{x-8}$.

Solution:

$$\begin{aligned} y &= \frac{x-7}{x-8} \\ y(x-8) &= x-7 \\ xy - 8y &= x - 7 \\ xy - x &= 8y - 7 \\ x(y-1) &= 8y-7 \\ x &= \frac{8y-7}{y-1} \\ f^{-1}(x) &= \frac{8x-7}{x-1} \end{aligned}$$

Free Response Questions

11. The average temperature in Denver, CO, in the spring time is given by the function $T(x) = -0.65x^2 + 14.5x - 26.8$, where T is the temperature in degrees Fahrenheit and x is the time of day in military time and is restricted to $6 < x < 18$ (sunrise to sunset). What is the temperature at 11 A.M.? What is the temperature at 4 P.M.?

Solution: Evaluate $T(11)$ and $T(16)$.
$T(11) = -0.65(11)^2 + 14.5(11) - 26.8 = 54.05°\text{F}$.
$T(16) = -0.65(16)^2 + 14.5(16) - 26.8 = 38.8°\text{F}$.

12. Evaluate the expression and write the result in the form $a+bi$.

$$\frac{(1+4i)(3-i)}{2+i}.$$

Solution:

$$\begin{aligned}\frac{(1+4i)(3-i)}{2+i} &= \frac{7+11i}{2+i} \\ &= \frac{7+11i}{2+i}\frac{2-i}{2-i} \\ &= \frac{25+15i}{5} \\ &= 5-3i\end{aligned}$$

13. Given that -1 is a zero of the polynomial $f(x)=x^3+10x^2+29x+20$, determine all other zeros and write the polynomial in terms of a product of linear factors.

Solution: Since -1 is a root, then $x+1$ divides the polynomial and

$$f(x)=(x+1)(x^2+9x+20)=(x+1)(x+5)(x+4).$$

14. For the rational function $f(x)=\dfrac{9x^3+6x^2+2x-6}{3x^2+4x+2}$, find the equation of the slant asymptote.

Solution: We need to use long division to divide the denominator into the numerator. In doing so we get

$$\begin{array}{r|l} & 3x-2 \\ \hline 3x^2+4x+2 & 9x^3+6x^2+2x-6 \\ & -9x^3-12x^2-6x \\ \hline & -6x^2-4x-6 \\ & 6x^2+8x+4 \\ \hline & 4x-2 \end{array}$$

So the slant asymptote is $y=3x-2$.

15. A rare species of insect was discovered in the rain forest of Costa Rica. Environmentalists transplant the insect into a protected area. The population of the insect t months after being transplanted is

$$P(t) = 45\left(\frac{1+0.6t}{3+0.02t}\right).$$

(I) What was the population when t = 0?

Solution: $P(0) = 45\left(\frac{1+0.6\cdot 0}{3+0.02\cdot 0}\right) = 15.$

(II) What will the population be after 10 years?

Solution: 10 years is 120 months. $P(120) = 45\left(\frac{1+0.6\cdot 120}{3+0.02\cdot 120}\right) = 608.333.$

(III) What is the end behavior of this population?

Solution: As $t \to \infty$, $P(t) \to 45 * \frac{0.6}{0.02} = 1350.$

16. Find the intercepts and asymptotes of

$$R(x) = \frac{3x(x+2)}{(x-1)(x-6)}.$$

(I) The x-intercept(s) are

Solution: where the function crosses the x-axis which will be when the top is 0. This occurs at $x = 0$ and $x = -2$.

(II) The y-intercept is

Solution: the value of the function at $x = 0$ and $R(0) = 0$.

(III) The vertical asymptote(s) are

Solution: where the denominator is 0 and these are $x = 1$ and $x = 6$.

(IV) The horizontal asymptote(s) are

Solution: The horizontal asymptote is the end behavior and since the numerator and denominator have the same degree (2) the horizontal asymptote is $y = 3$.

17. Find the formula for a quadratic function with vertex $(1,4)$ and y-intercept $(0,3)$.

Solution: Since the vertex is $(1,4)$, the standard form of the quadratic function is $y = a(x-1)^2 + 4$. We need the y-intercept to be $(0,3)$ and putting $x = 0$ and $y = 3$ gives us that $a = -1$, so the formula for this quadratic function is $y = -(x-1)^2 + 4$.

18. Given that $f(x) = 1 + x$ and $g(x) = x^2 - x$, find

(I) $(f \circ g)(x)$

Solution: $(f \circ g)(x) = f(x^2 - x) = 1 + x^2 - x.$

(II) $(g \circ f)(x)$

Solution: $(g \circ f)(x) = g(1 + x) = (1 + x)^2 - (1 + x)\ [= x^2 + x].$

(III) $(f \circ f)(x)$

Solution: $(f \circ f)(x) = f(1 + x) = 1 + (1 + x)\ [= 2 + x].$

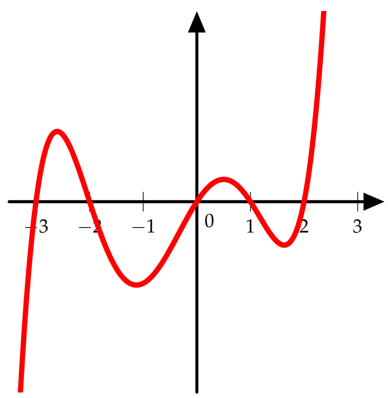
(IV) $(g \circ g)(x)$

Solution: $(g \circ g)(x) = g(x^2 - x) = (x^2 - x)^2 - (x^2 - x)\ [= x^4 - 2x^3 + x].$

(V) $g(f(2) + 5)$

Solution: $g(f(2) + 5) = g((1 + 2) + 5) = g(8) = 64 - 8 = 56.$

19. The graph is of a polynomial function $f(x)$ of degree 5 whose leading coefficient is 1. The graph is not drawn to scale. Find the polynomial.

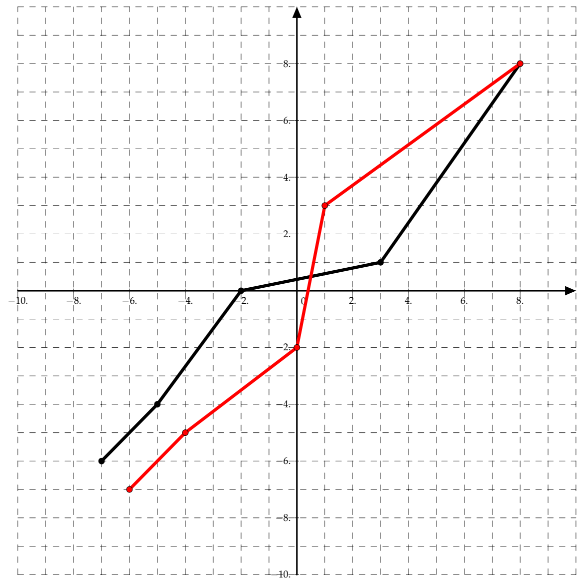


Solution: The polynomial has roots at $x = -3,\ -2,\ 0.\ 1,\ 2.$ Thus a formula for this polynomial is

$$P(x) = (x + 3)(x + 2)x(x - 1)(x - 2)\ [= x^5 + 2x^4 - 7x^3 - 8x^2 + 12x].$$

20. The graph of a function f is given. Sketch the graph of the inverse function of f. (Graph segments with closed endpoints only.)

Solution:



END OF TEST