Math 110: Algebra for Trig and Calculus
Tuesday, November 14, 2017
Exam 3 Fall 2017

Name: KEY

Section: ______________________

Last 4 Digits of Student ID #: __________

This exam has twelve multiple choice questions (5 points each), five true/false questions (2 points each), and three free response questions (10 points each). Additional blank sheets are available for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has scientific or graphing capabilities.

On the multiple choice and true/false choice problems:

1. You must give your final answer in the multiple choice and true/false answer boxes on the front page of your exam. See the "EXAMPLE" row for a correct shading example.
2. Carefully check your answers. No credit will be given for answers other than those indicated in the answer boxes.

On the free response problems:

1. Write your solutions neatly in the space below the question (unsupported answers may not receive credit). You are not expected to write your solution next to the statement of the question.
2. Give exact answers, rather than decimal approximations (unless otherwise stated).

Multiple Choice Answers

EXAMPLE	A	B	C	D	E
Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E

True/False Choice Answers

Question		
13	T	F
14	T	F
15	T	F
16	T	F
17	T	F

Exam Scores

Question	Score	Total
MC		60
TF		10
18		10
19		10
20		10
Total		100

<u>Record the correct answer to the following problems on the front page of this exam.</u>

1. Convert $450°$ to radians.

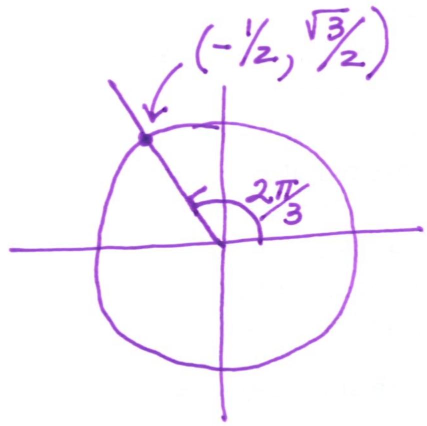
   (a) $\frac{7\pi}{2}$
   (b) $\frac{3\pi}{2}$
   (c) 3π
   (d) $\frac{5\pi}{2}$
   (e) None of the other choices.

$$450^{\circ} \times \frac{\pi \text{ radians}}{180^{\circ}} = \frac{\overset{(5)(90)}{450\pi}}{\underset{(2)(90)}{180}} \text{ radians}$$

$$= \frac{5\pi}{2} \text{ radians}$$

2. Determine the value of $\cos\left(\frac{2\pi}{3}\right)$.

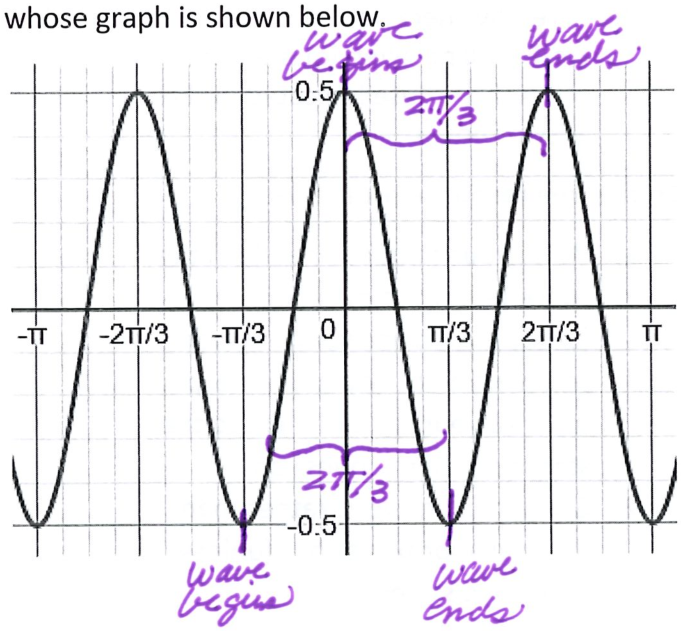
   (a) $\frac{1}{2}$
   (b) $-\frac{1}{2}$
   (c) $\frac{\sqrt{3}}{2}$
   (d) $-\frac{\sqrt{3}}{2}$
   (e) None of the other choices.



$\cos(\theta) = x$
$\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$

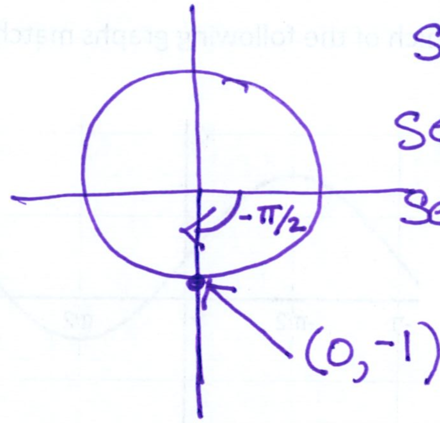
3. Determine the period of the function whose graph is shown below.

   (a) $\frac{2\pi}{3}$
   (b) $\frac{\pi}{3}$
   (c) 2π
   (d) 0.5
   (e) None of the other choices.



Record the correct answer to the following problems on the front page of this exam.

4. Determine the value of $\sec\left(-\frac{\pi}{2}\right)$.
   - (a) 1
   - (b) -1
   - (c) 0
   - (d) $-\frac{1}{2}$
   - (e) None of the other choices.



$$\sec(\theta) = \frac{1}{\cos(\theta)}$$
$$\sec(\theta) = \frac{1}{x}$$
$$\sec(-\pi/2) = \frac{1}{0}$$
$$= \text{undefined}$$

5. If $\csc(\theta) = -4$ and θ is in Quadrant III, then compute the value of $\cos(\theta)$.
   - (a) $-\frac{\sqrt{15}}{4}$
   - (b) $\frac{1}{4}$
   - (c) $-\frac{4}{\sqrt{15}}$
   - (d) $\sqrt{15}$
   - (e) None of the other choices.

$$\cos^2(\theta) + \sin^2(\theta) = 1$$
$$\cos^2(\theta) = 1 - \sin^2(\theta)$$
$$\cos(\theta) = \cancel{\pm}-\sqrt{1-\sin^2(\theta)}$$

$\boxed{\cos(\theta) < 0 \text{ in } Q\ \text{III}}$

$$= -\sqrt{1-\sin^2(\theta)}$$
$$= -\sqrt{1-\frac{1}{\csc^2(\theta)}}$$
$$= -\sqrt{1-\frac{1}{(-4)^2}}$$
$$= -\sqrt{1-\frac{1}{16}}$$
$$= -\sqrt{\frac{16}{16}-\frac{1}{16}}$$
$$= -\sqrt{\frac{15}{16}}$$
$$= -\frac{\sqrt{15}}{4}$$

6. If the terminal side of an angle of θ radians passes through the point $(5,3)$, then determine the value of $\sin(\theta)$.
   - (a) $\frac{3}{\sqrt{34}}$
   - (b) $\frac{5}{\sqrt{34}}$
   - (c) $\frac{3}{5}$
   - (d) $\frac{5}{3}$
   - (e) None of the other choices.

a, b

$$\sin(\theta) = \frac{b}{r} \ \& \ r = \sqrt{a^2+b^2}$$
$$r = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$$
$$\sin(\theta) = \frac{3}{\sqrt{34}}$$

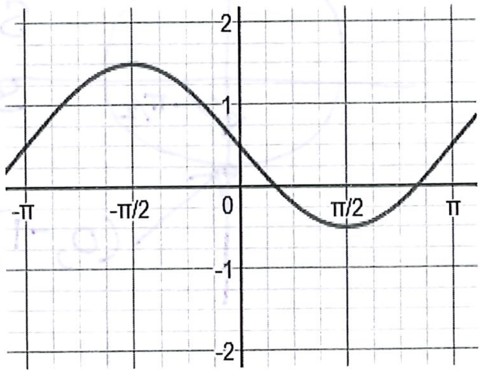
Record the correct answer to the following problems on the front page of this exam.

vertical shift up $\frac{1}{2}$ unit

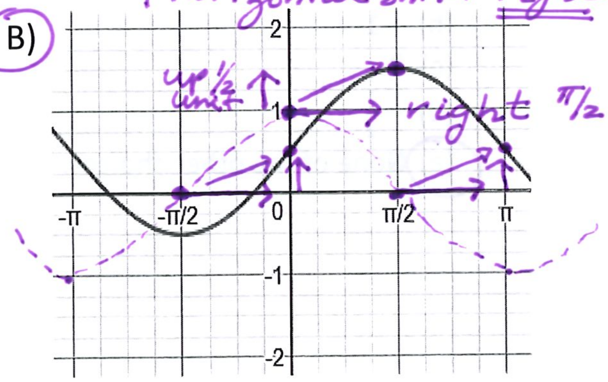
7. Which of the following graphs matches $f(x) = \cos\left(x - \frac{\pi}{2}\right) + \frac{1}{2}$?

horizontal shift right $\pi/2$ units

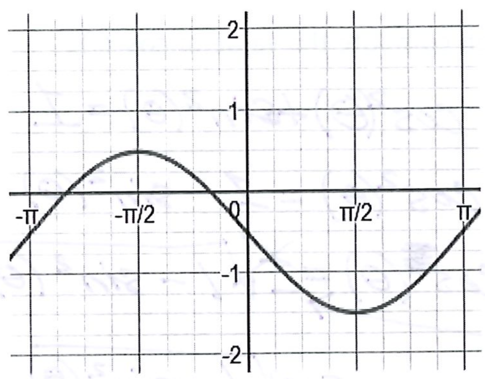
A)



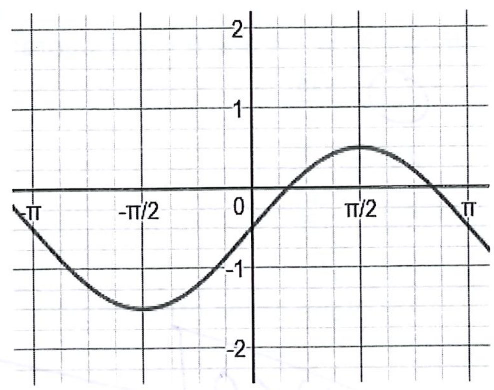
B)



C)



D)



8. The expression $\dfrac{4\sin^3(x)\cot^2(x)}{\cos^2(x)}$ simplifies to which of the following?

(a) 4
(b) $4\sin(x)$
(c) $4\cos(x)$
(d) $4\csc(x)$
(e) $4\sec(x)$

$$\frac{4\sin^3(x)\cot^2(x)}{\cos^2(x)} = \frac{4\sin(x)\cdot\cancel{\sin^2(x)}\cdot\frac{\cos^2(x)}{\cancel{\sin^2(x)}}}{\cos^2(x}$$

$$= \frac{4\sin(x)\cancel{\cos^2(x)}}{\cancel{\cos^2(x)}}$$

$$= 4\sin(x)$$

Record the correct answer to the following problems on the front page of this exam.

9. Compute the exact value of $\cos\left(\frac{\pi}{12}\right)$.

   (a) $\frac{\sqrt{2}-\sqrt{6}}{4}$
   (b) $\frac{\sqrt{6}-\sqrt{2}}{4}$
   (c) $\frac{\sqrt{2}+\sqrt{6}}{4}$
   (d) $\frac{1-\sqrt{2}}{2}$
   ~~(e) None of the other choices.~~

Using ~~addition~~ subtraction identity...

$$\cos(\pi/12) = \cos\left(4\pi/12 - 3\pi/12\right) = \cos\left(\pi/3 - \pi/4\right) = \cos(\pi/3)\cos(\pi/4) + \sin(\pi/3)\sin(\pi/4)$$
$$= \left(1/2\right)\left(\sqrt{2}/2\right) + \left(\sqrt{3}/2\right)\left(\sqrt{2}/2\right) = \sqrt{2}/4 + \sqrt{6}/4 = \frac{\sqrt{2}+\sqrt{6}}{4}$$

these are equivalent!!

Using half angle identity...

$$\cos(\pi/12) = \cos\left(\frac{\pi/6}{2}\right) = \cancel{\pm}\sqrt{\frac{1+\cos(\pi/6)}{2}} = +\sqrt{\frac{1+\sqrt{3}/2}{2}\cdot\frac{2}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

10. The expression $\cos(x+\pi)+\sin(x+\pi)$ simplifies to which of the following?

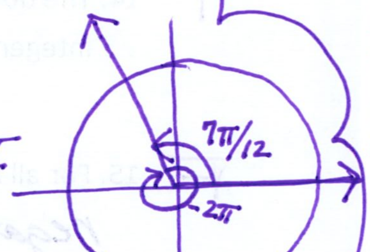
   (a) $\cos(x)+\sin(x)$
   (b) $\cos(x)-\sin(x)$
   (c) $-\cos(x)+\sin(x)$
   (d) $-\cos(x)-\sin(x)$
   (e) $\cos(x)+\sin(x)-1$

$$\cos(x+\pi)+\sin(x+\pi) =$$
$$\cos(x)\cos(\pi) - \sin(x)\sin(\pi) + \sin(x)\cos(\pi) + \cos(x)\sin(\pi)$$
$$\cos(x)\cdot(-1) - \sin(x)\cdot 0 + \sin(x)\cdot(-1) + \cos(x)\cdot 0$$
$$-\cos(x) - 0 - \sin(x) + 0$$
$$-\cos(x) - \sin(x)$$

11. Which of the following angles is coterminal with $-\frac{17\pi}{12}$?

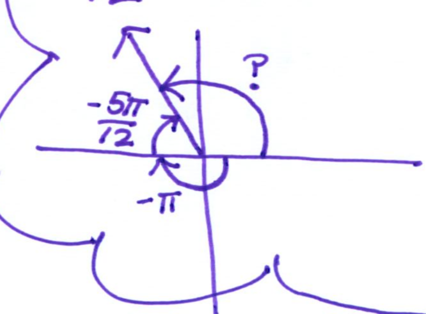
   (a) $-\frac{29\pi}{12}$
   (b) $-\frac{5\pi}{12}$
   (c) $\frac{7\pi}{12}$
   (d) $\frac{17\pi}{12}$
   (e) None of the other choices.

$$\frac{-17\pi}{12} = \frac{7\pi}{12} - \frac{24\pi}{12} = \frac{7\pi}{12} - 2\pi$$



OR

$$\frac{-17\pi}{12} = \frac{-12\pi}{12} - \frac{5\pi}{12} = -\pi - \frac{5\pi}{12}$$



$$? + \frac{5\pi}{12} + \pi = 2\pi$$
$$? = 2\pi - \pi - \frac{5\pi}{12}$$
$$? = \pi - \frac{5\pi}{12}$$
$$? = 7\pi/12$$

Record the correct answer to the following problems on the front page of this exam.

12. Which of the following identities is TRUE?

(a) $\cos^2(\theta) = \frac{1}{2} - \frac{1}{2}\cos(2\theta)$

(b) $\cos^2(\theta) = \frac{1}{2} + \frac{1}{2}\cos(2\theta)$

(c) $\cos^2(\theta) = 2\sin(\theta)\cos(\theta)$

(d) $\cos^2(\theta) = \sin^2(\theta) - 1$

(e) $\sec(\theta) + \csc(\theta) = \cot(\theta)$

$\cos(2\theta) = \cos(\theta+\theta) = \cos(\theta)\cos(\theta) - \sin(\theta)\sin(\theta)$ ← add. identity

$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$ ← properties of exponents

$\cos(2\theta) = \cos^2(\theta) - (1 - \cos^2(\theta))$ ← pythagorean ID

$= \cos^2(\theta) - 1 + \cos^2(\theta)$ ← arithmetic

$\cos(2\theta) + 1 = 2\cos^2(\theta)$ ← arithmetic

$\frac{1}{2}\cos(2\theta) + \frac{1}{2} = \cos^2(\theta)$ ← arithmetic

For questions 13-17, determine whether each statement is true or false.

T 13. If P is the point on the unit circle that lies on the terminal side of angle θ in standard position, then the coordinates of P are $(\cos(\theta), \sin(\theta))$. $\boxed{\text{DEFINITION}}$ or

unit circle equation $\Rightarrow x^2 + y^2 = 1$

pythagorean ID $\Rightarrow \cos^2(\theta) + \sin^2(\theta) = 1$

T 14. The domain of $f(x) = \tan(x)$ is all real numbers EXCEPT $x = \frac{\pi}{2} + k\pi$, where k is any integer. notice: $\tan(x) = \frac{\sin(x)}{\cos(x)}$ ← domain is all x so that $\cos(x) \neq 0$

$\cos(x) = 0$ when $x = \pi/2 + k\pi$

F 15. For all real numbers x: $\sin(-x) = \sin(x)$

negative angle identity $\sin(-x) = -\sin(x)$

F 16. For all real numbers x: $\cos(3x) = 3\cos(x)$

notice: when $x = 0 \Rightarrow \cos(3 \cdot 0) \stackrel{?}{=} 3\cos(0) \Rightarrow \cos(0) \neq 3\cos(0)$

$1 \neq 3$

T 17. For all real numbers x: $\cos^2(x) + \sin^2(x) = 1$

pythagorean identity

Free Response Questions: Show your work!!

18. Prove the following.

$$\frac{1}{1-\cos(x)} + \frac{1}{1+\cos(x)} = 2\csc^2(x)$$

THERE ARE MULTIPLE WAYS TO PROVE. THIS IS JUST ONE SUGGESTED WAY!

$$LHS = \frac{1}{1-\cos(x)} + \frac{1}{1+\cos(x)} = \frac{1}{1-\cos(x)} \cdot \frac{1+\cos(x)}{1+\cos(x)} + \frac{1}{1+\cos(x)} \cdot \frac{1-\cos(x)}{1-\cos(x)}$$ ⟸ getting common denominator

$$= \frac{1+\cos(x)}{(1-\cos(x))(1+\cos(x))} + \frac{1-\cos(x)}{(1+\cos(x))(1-\cos(x))}$$ ⟸ multiplying fractions

$$= \frac{1+\cancel{\cos(x)} + 1 - \cancel{\cos(x)}}{1-\cos^2(x)}$$ ⟸ arithmetic

$$= \frac{2}{1-\cos^2(x)}$$ ⟸ arithmetic

$$= \frac{2}{\sin^2(x)}$$ ⟸ pythagorean identity

$$= 2\csc^2(x) = RHS$$ ⟸ definition of coscecant

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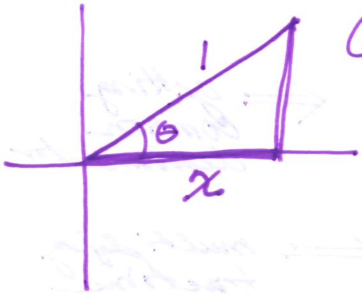
Free Response Questions: Show your work!!

19. Suppose that $\cos(\theta) = x$ and θ is in Quadrant I. Compute formulas (in terms of x) for $\sin(\theta)$ and $\tan(\theta)$. Write your answers in the spaces provided below.

THERE ARE MULTIPLE WAYS TO SOLVE THIS. THIS IS JUST ONE SUGGESTED WAY.



QI

$$\cos(\theta) = \frac{adj.}{hyp} = \frac{x}{1} = \frac{a}{r}$$

$$r = \sqrt{a^2+b^2} \Rightarrow (1)^2 = \left(\sqrt{x^2+b^2}\right)^2 \Rightarrow 1 = x^2+b^2$$

$$1 - x^2 = b^2$$

$$\pm\sqrt{1-x^2} = b$$

quadrant I $\rightarrow$ $+\sqrt{1-x^2} = b$

$a = x$

$b = \sqrt{1-x^2}$

$r = 1$

$$\sin(\theta) = \frac{b}{r} = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$$

$$\tan(\theta) = \frac{b}{a} = \frac{\sqrt{1-x^2}}{x}$$

Alternate solution:

$$\cos^2(\theta) + \sin^2(\theta) = 1$$

$$x^2 + \sin^2(\theta) = 1$$

$$\sin^2(\theta) = 1 - x^2$$

$$\sin(\theta) = \pm\sqrt{1-x^2}$$

$$\sin(\theta) = +\sqrt{1-x^2}$$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$= \frac{\sqrt{1-x^2}}{x}$$

$\sin(\theta) =$ $\sqrt{1-x^2}$

$\tan(\theta) =$ $\dfrac{\sqrt{1-x^2}}{x}$

Free Response Questions: Show your work!!

20. A 20-ft ladder leans against a building so that the angle between the ground and the ladder is 60°. Compute the exact values for how high the ladder reaches on the building and how far the bottom of the ladder is from the building. Write your answers in the spaces provided below.

$\sin(\theta) = \frac{opp}{hyp}$ $\cos(\theta) = \frac{adj}{hyp}$

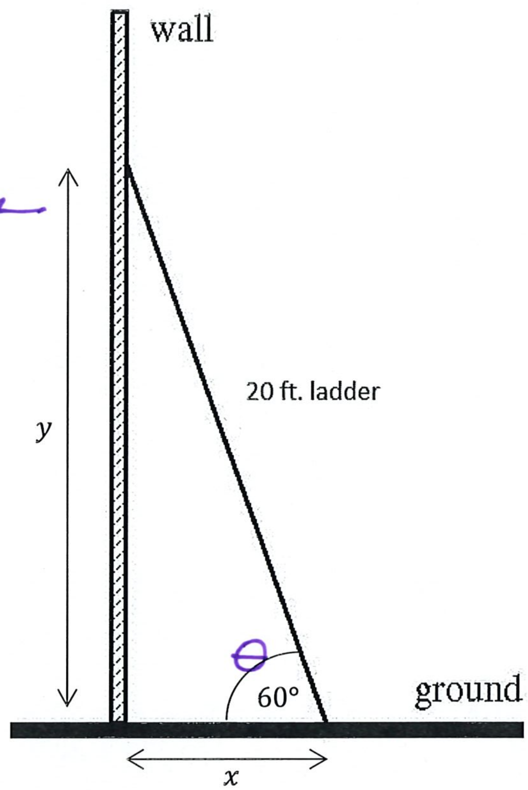
$\theta = 60° \times \frac{\pi \text{ radians}}{180°} = \frac{\pi}{3} \text{ radians}$

$\sin\left(\frac{\pi}{3}\right) = \frac{y}{20}$ $\cos\left(\frac{\pi}{3}\right) = \frac{x}{20}$

$\frac{\sqrt{3}}{2} = \frac{y}{20}$ $\frac{1}{2} = \frac{x}{20}$

$20\left(\frac{\sqrt{3}}{2}\right) = y$ $20\left(\frac{1}{2}\right) = x$

$10\sqrt{3} = y$ $10 = x$



$x =$ 10 feet

$y =$ $10\sqrt{3}$ feet