5 Linear Mathematical Models

Concepts:

- Construct a Linear Model
- Gauge the Accuracy of a Linear Model Using Residuals
- Use Linear Regression

(Section 2.5)

1. A teacher sent students out to find *round* object and to measure the diameter and the circumference of each item. When the class returned and the data were put on the board, we have the following table:

Object	Diameter (cm)	Circumference (cm)
glass	8.3	26.5
flashlight	5.2	16.7
Aztec calendar	20.2	61.6
Tylenol bottle	3.4	11.6
Popcorn can	13	41.4
Salt shaker	6.3	20.1
Coffee canister	11.3	35.8
Cat food bucket	33.5	106.5
Dinner plate	27.3	85.6
Ritz cracker	4.9	15.5

   (a) Find the equation for the least squares regression line for this data.

   (b) What is the slope of this line? What does it measure?

   (c) What is the y-intercept of this line? What should it be?

2. A 190° cup of coffee is placed on a desk in a 72ř room. The data in the following are from a simulated experiment of gathering temperature readings from a cup of coffee in a 72° room at 20 one-minute intervals.

Time	Temp	Time	Temp
1	184.3	11	140.0
2	178.5	12	136.1
3	173.5	13	133.5
4	168.6	14	130.5
5	164.0	15	127.9
6	159.2	16	125.0
7	155.1	17	122.8
8	151.8	18	119.9
9	147.0	19	117.2
10	143.7	20	115.2

(a) Produce a scatter plot of the temperature (y) as a function of time (x).

(b) Find the linear regression equation for this data. Round the coefficients to the nearest 0.001.

(c) It is known that this phenomenon is **not** linear. What are the x-intercepts and the y-intercepts?

(d) What does the x-intercept mean, physically?

3. The average hourly earnings of U. S. production workers for 1990–2007 are shown in the table below.

Year	Average Hourly Earnings ($)
1990	10.20
1991	10.52
1992	10.77
1993	11.05
1994	11.34
1995	11.65
1996	12.04
1997	12.51
1998	13.01
1999	13.49
2000	14.02
2001	14.54
2002	14.97
2003	15.37
2004	15.69
2005	16.13
2006	16.76
2007	17.42

(a) Produce a scatter plot of the hourly earnings (y) as a function of years since 1990 (x).

(b) Find the linear regression equation for the years 1990–1998. Round the coefficients to the nearest 0.001.

(c) Find the linear regression equation for the years 1990–2007. Round the coefficients to the nearest 0.001.

(d) Use both lines to predict the hourly earnings for the year 2010. How different are the estimates? Which do you think is a safer prediction of the true value?

(e) Look online and find the average hourly earnings of U. S. production workers for 2010. Which was a better estimate?

4. The table shows the size of a room air conditioner (in BTUs) needed to cool a room of the given area (in square feet).

Room size	BTUs
150	5000
175	5500
215	6000
250	6500
280	7000
310	7500
350	8000
370	8500
420	9000
450	9500

(a) Find a linear model for the data.

(b) Use the model to find the number of BTUs required to cool a rooms of size 150 sq ft, 280 sq ft, and 420 sq ft. How well do the model estimates agree with the actual data values?

(c) Use the model to estimate how many BTUs are needed to cool a 235 sq ft room. If air conditioners are available only with the BTU choices in the table, which size should be chosen?

5. The projected number of scheduled passengers on U. S. commercial airlines (in billions) is given in the following table.

Year	2002	2003	2004	2005	2006	2007	2008	2009
Passengers	.63	.64	.69	.72	.77	.79	.8	.84

(a) Find a linear model for this data, with $x = 2$ corresponding to 2002.

(b) Estimate the number of passengers in 2012 and 2016.

(c) Find the equation of the line through the first data point and the last data point.

(d) Compute the sum of the squares of the residuals for this line.

(e) Compute the sum of the squares of the residuals for the line of best fit. Compare with above. Was the work worth it?