6 Functions and Functional Notation

Concepts:

- The Definition of A Function
- Function Notation
- Piecewise-defined Functions
  - Evaluating Piecewise-defined Functions
  - Sketching the Graph of a Piecewise-defined Functions
- The Domain of a Function

(Sections 3.1-3.2)

1. The amount of postage required to mail a first-class letter is determined by its weight. In this situation, is weight a function of postage? Or vice versa? Or both?

2. An epidemiological study of the spread of malaria in a rural area finds that the total number P of people who contracted malaria t days into an outbreak is modeled by the function
$$P(t) = -\frac{1}{4}t^2 + 7t + 180, \qquad 1 \leq t \leq 14.$$
   (a) How many people have contracted malaria 14 days into the outbreak?
   (b) How many people have contracted malaria 6 days into the outbreak?

3. In the following identify the independent variable (input) and the dependent variable (output).
   (a) The amount of property tax you owe is a function of the assessed value of your home in dollars.
   (b) The length of your fingernails is a function of the amount of time that has passed since your last manicure.
   (c) The cost of mailing a letter is a function of the weight of the package in ounces.
   (d) The amount of water required for your lawn (in gallons) is a function of the temperature (in degrees).
   (e) A person's blood alcohol level is a function of the number of alcoholic drinks consumed in a 2-hour period.

4. The number of recreational visits to the National Parks of the United States is displayed in the table. The number of visits to the national parks, p, is a function of the year, t.

Year	Recreational Visits to US National Parks (millions of people)
1990	258.7
1995	269.6
1999	287.1
2000	285.9
2001	279.9
2002	277.3
2003	266.1
2004	276.4

Source: www.census.gov

(a) Solve $p(t) = 277.3$ for t and explain the meaning of the solution.

(b) Evaluate $p(2000)$ and write a sentence explaining what the numerical value you find means in its real-world context.

(c) Estimate $p(2010)$ and discuss the accuracy of your prediction.

(d) Estimate the solution to $p(t) = 300$ and discuss the accuracy of your approximation.

5. Evaluate the given function at the given values:

(a) $f(x) = x^3 + 2x$; $f(-2), f(-1), f(0), f(\frac{1}{2})$

(b) $g(t) = \dfrac{t+2}{t-2}$; $g(-2), g(2), g(0), g(a), g(a^2-2), g(a+1)$

(c) $h(u) = 2|u-1|$; $h(-2), h(0), g(\frac{1}{2}), h(2), h(x+1), h(x^2+2)$

(d) $f(x) = \dfrac{|x|}{x}$; $f(-2), f(-1), f(0), f(5), f(w^2), f\left(\dfrac{1}{w}\right)$)

6. Evaluate the given piecewise defined function at the given values:

(a) $f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ x+1 & \text{if } x \geq 0 \end{cases}$; $f(-2), f(-1), f(0), f(1), f(2)$

(b) $g(u) == \begin{cases} u^2 + 2u & \text{if } u \leq -1 \\ u & \text{if } -1 < u \leq 1 \\ -1 & \text{if } u > 1 \end{cases}$; $g(-4), g(-\frac{3}{2}), f(-1), f(0), f(25)$

7. According to `http://revenue.ky.gov/`, the tax brackets for the 2015 Kentucky state taxes are described below.

   If your taxable income on Form 740, line 11 is:

more than	but not more than	then your tax is	plus:
\$0	\$3,000	2.00% of your taxable income	\$0
\$3,001	\$4,000	3.00% of the amount over \$3,000	\$60
\$4,001	\$5,000	4.00% of the amount over \$4,000	\$90
\$5,001	\$8,000	5.00% of the amount over \$5,000	\$130
\$8,001	\$75,000	5.80% of the amount over \$8,000	\$280
\$75,001		6.00% of the amount over \$75,000	\$4,160

   They give the following example.

   Taxable income \$6,800. Tax = (\$6,800-\$5,000) $\times$ .05(5%) + \$130 = \$220.

   Use this tax table to write a piecewise-defined function $KYTax(I)$ where I is the adjusted gross income on Form 740 line 11 of the Kentucky tax form 740, and $KYTax(I)$ is the amount of tax owed by a resident of Kentucky.

8. Let $f(x) = x^2 + 1$.
   (a) What is $f(a+b)$?
   (b) What is $f(x-1)$?

9. Let $g(x) = x^2 + x$.
   (a) What is $\dfrac{g(2x)}{2g(x)}$?
   (b) What is $g(x^2)$?
   (c) What is $(g(x))^2$?
   (d) What is $\dfrac{g(x+h)-g(x)}{h}$?

10. Let

$$h(x) = \begin{cases} 10 & \text{if } x < -4 \\ x^2 + 10 & \text{if } -4 \le x \le 6 \\ x + 15 & \text{if } x > 6 \end{cases}$$

    (a) Find $h(5)$.
    (b) Find $h(-4)$.
    (c) Find $h(-6)$.
    (d) Find $h(6)$.
    (e) Find $h(10)$.

11. Find the domain of each of the following functions. Write the domain in interval notation.

(a) $a(x) = x^5 + 2x^2 - 6$

(b) $b(x) = \dfrac{x+1}{x-5} + \dfrac{x+4}{2x+1}$

(c) $c(x) = \sqrt[3]{x+7}$

(d) $d(x) = \sqrt{x+7}$

(e) $e(x) = \dfrac{1}{\sqrt[3]{10-x}}$

(f) $f(x) = \dfrac{1}{\sqrt[4]{10-x}}$

(g) $g(x) = \sqrt{x+7} - \dfrac{1}{x^2-5}$

(h) $h(x) = \begin{cases} \dfrac{1}{x} & \text{if } x \leq -2 \\ \dfrac{1}{x+3} & \text{if } x > -2 \end{cases}$

12. To graph the function f we plot the points $(x, \_\_\_\_)$ in a coordinate plane.To graph $f(x) = x^2 - 2$, we plot the points $(x, \_\_\_\_)$. So the point $(3, \_\_\_\_)$ is on the graph of f. The height of the graph of f above the x-axis when $x = 3$ is $\_\_\_\_$.

13. Sketch graphs of the following functions:

(a) $f(x) = |x| + x$

(b) $g(x) = |x| - x$

(c) $h(x) = x|x|$

(d) $f(x) = x/|x|$

(e) $g(x) = x - [|x|]$

(f) $h(x) = x[|x|]$

(g) $f(x) = \begin{cases} -1 & \text{if } x < -1 \\ x & \text{if } -1 \leq x \leq 1 \\ 1 & \text{if } x > 1 \end{cases}$