

9 Transformations and Composition

Concepts:

- Graphs of functions
- Sketching graphs of functions
- Applying transformations to the graph of a function
- How does a graph transformation move a point on a graph?
- Operations on functions
- The domain of a composition of functions.

(Section 3.4 & 3.5)

1. (Do you understand graph transformations?) Suppose that the graph of f contains the point $(-4, 7)$. Find a point that must be on the graph of g . Explain how you had to move the point on the original graph f to obtain a point on the new graph g .

(a) The graph of $g(x) = f(x) + 5$ must contain the point _____.

(b) The graph of $g(x) = f(x + 5)$ must contain the point _____.

(c) The graph of $g(x) = f(x) - 5$ must contain the point _____.

(d) The graph of $g(x) = f(x - 5)$ must contain the point _____.

(e) The graph of $g(x) = 5f(x)$ must contain the point _____.

(f) The graph of $g(x) = f(5x)$ must contain the point _____.

(g) The graph of $g(x) = \frac{1}{5}f(x)$ must contain the point _____.

(h) The graph of $g(x) = f\left(\frac{1}{5}x\right)$ must contain the point _____.

(i) The graph of $g(x) = f(7x) + 5$ must contain the point _____.

(j) The graph of $g(x) = 3f(7x + 1) + 5$ must contain the point _____.

(k) **A Challenge:** The graph of $g(x) = 3(f(7(x + 1))) + 5$ must contain the point _____.

2. Let $f(x) = x^2 + 3$ and $g(x) = 2 - x$.
- Find $f(g(x))$.
 - Find $g(f(x))$.
 - Find $f(f(x))$.
 - Find $g(g(x))$.
 - Find $g(g(g(x)))$.
3. Let $f(x) = \frac{x}{\sqrt{x+1}}$ and $g(x) = 2x + 5$.
- Find $f(g(x))$.
 - Find the domain $f(g(x))$.
 - Find $g(f(x))$.
 - Find the domain $g(f(x))$.
4. Write $h(x)$ as a composition of three simpler functions. (**HINT:** Think of placing x in a box. What happens first? second? *etc.*? There may be more than one correct answer.)
- $h(x) = \sqrt{x^3 + 5}$
 - $h(x) = \frac{3}{x^5 - 7}$
 - $h(x) = 3(x + 5)^2$
 - $h(x) = (3x + 5)^2$
5. Write $g(x) = 2(f(3(x + 1))) - 6$ as a composition of five simpler functions (Hint: One of these functions should be f .)
6. You have a 20% off coupon from the manufacturer good for the purchase of a new cell phone. Your cell provider is also offering a 10% discount on any new phone. You make two trips to cell phone stores to look at various phones. On your first trip, you speak with Miranda. Miranda tells you that you can take advantage of both the coupon and the discount. She will apply the discount and then apply the coupon to the reduced price. On your second trip, you talk to Ariel. She also says that you can take advantage of both deals, but she tells you that she will apply the coupon and then apply the discount.
- Let x represent the original sticker price of the cell phone.
- Suppose that only the 20% discount applies. Find a function f that models the purchase price of the cell phone as a function of the sticker price x .
 - Suppose that only the 10% coupon applies. Find a function g that models the purchase price of the cell phone as a function of the sticker price x .
 - If you can take advantage of both deals, then the price you will pay is either $f(g(x))$ or $g(f(x))$, depending on the order in which the coupon and the discount are applied to the price. Find $f(g(x))$ and $g(f(x))$.
 - The price that Miranda is offering you is modeled by _____.
 - The price that Ariel is offering you is modeled by _____.