10 Inverse and Quadratic Functions

Concepts:

- Identifying Inverse Functions
- The Definition of an Inverse Function
- Inverse Function Notation
- Finding Formulas for Inverse Functions
- Evaluating Inverse Functions
- Graphs of Inverse Functions
- The Round-Trip Theorem
- Determine Graph and Algebraic Form of a Quadratic
- Understand the Meaning of the Vertex

(Section 3.7 & 4.1)

1. Which of the following functions are one-to-one?
   (a) The function that maps a word to the number of letters in the word.

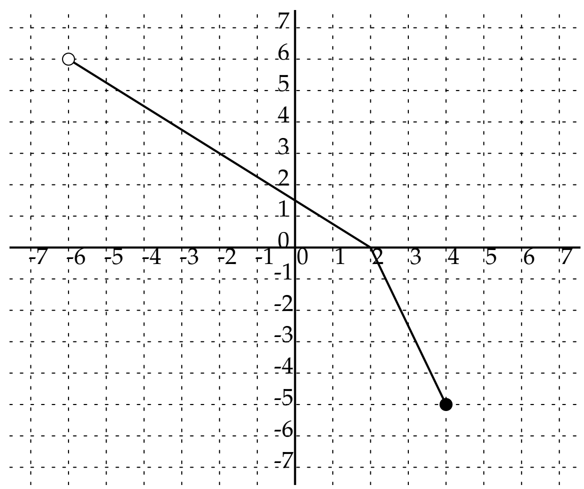
   (b) The function that maps the year of a Summer Olympics to the winner of the marathon in that Olympics.

   (c) The function that maps a U.S. state to its two letter postal code.

   (d) The function that maps a person to his or her name.

   (e) The function that maps a person to his or her address.

2. The graph of a one-to-one function is shown below. (How do you know that this is a one-to-one function?) Sketch the graph of its inverse on the same set of axes.



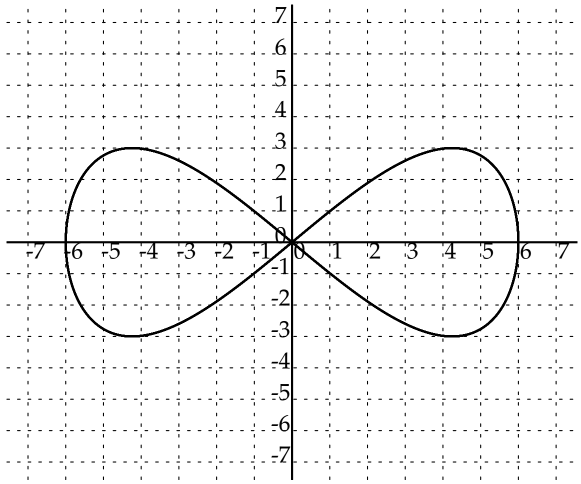
3. Find the inverse of the one-to-one functions below. Find the domains and ranges of the function and its inverse.

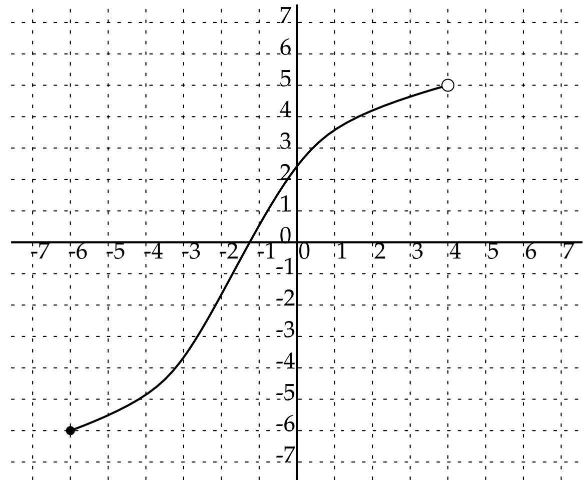
   (a) $f(x) = \dfrac{2 - x^3}{7}$

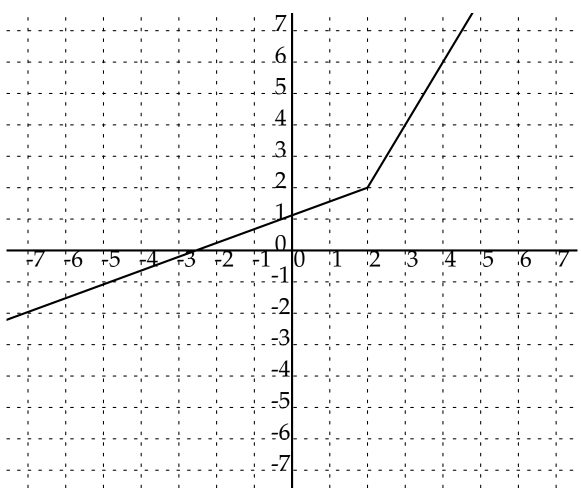
   (b) $g(x) = \dfrac{x + 7}{x + 5}$

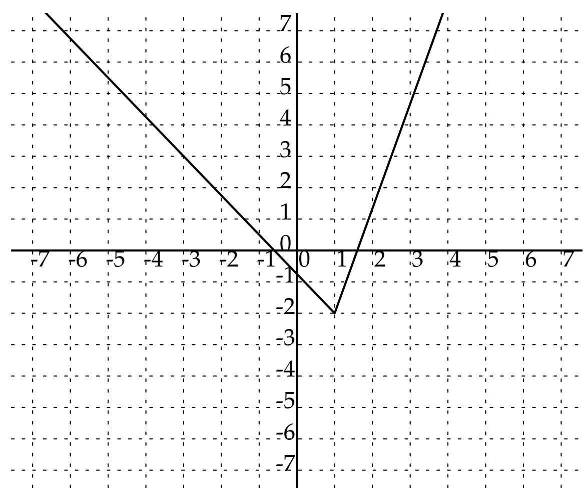
   (c) $h(x) = \sqrt{x^5 - 2}$

4. The following graphs display a one-to-one function? If the graph displays a one-to-one function, sketch its inverse.









5. Use composition of functions to determine if the pair of functions are inverses of each other.

   (a) $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x}$

   (b) $h(x) = \frac{3x+2}{7}$ and $j(x) = \frac{7x-2}{3}$

   (c) $k(x) = \sqrt{3}x - 3$ and $m(x) = x^3 + 27$

6. Find the equation of the unique quadratic function with the following properties and graph the function.

   (a) passes through $(-2, 0)$ and $(5, 0)$ and has a leading coefficient of 5.

   (b) passes through the points $(3, 0)$, $(-2, 0)$ and $(0, 12)$.

   (c) passes through $(0, 0)$, $(1, -1)$, and $(2, 0)$.

   (d) passes through $(0, 0)$, $(1, -2)$, and $(2, 0)$.

   (e) passes through $(0, 0)$, $(1, -3)$, and $(2, 0)$.

   (f) has vertex $(2, 4)$ and passes through the point $(0, -2)$.

7. A golf ball is hit so that its height h in feet after t seconds is $h(t) = -16t^2 + 60t$.
   (a) What is the initial height of the golf ball?

   (b) How high is the golf ball after 1.5 seconds?

   (c) Find the maximum height of the golf ball *algebraically*.

8. The accompanying table gives the number of homicides per 100,000 population in the United States (*Federal Bureau of Investigation*, `www.fbi.gov`).
   (a) Use quadratic regression on a graphing calculator to express the number of homicides as a function of x, where x is the number of years after 1994.

   (b) Plot the data and the quadratic function on your calculator. Judging from what you see, does the function appear to be a good model for the data?

   (c) Use the quadratic function to find the year in which the number of homicides was at a minimum.

   (d) Use the quadratic function to find the year in which there will be 10 homicides per 100,000 population.

Year	Homicides per 100,000	Year	Homicides per 100,000
1994	9.0	2001	5.6
1995	8.2	2002	5.6
1996	7.4	2003	5.7
1997	6.8	2004	5.5
1998	6.3	2005	5.6
1999	5.7	2006	5.7
2000	5.5	2007	5.6

9. **[Challenge]** When a basketball player shoots a foul shot, the ball follows a parabolic arc. This arc depends on both the angle and velocity with which the basketball is released. If a person shoots the basketball overhand from a position 8 feet above the floor, then the path can sometimes be modeled by the parabola $y = \dfrac{-16x^2}{0.434v^2} + 1.15x + 8$, where v is the velocity of the ball in feet per second, as illustrated in the first figure. (Source: C. Rist, *The Physics of Foul Shots.*)

   (a) If the basketball hoop is 10 feet high and located 15 feet away, what initial velocity v should the basketball have?

   (b) Check your answer from part (a) graphically. Plot the point $(0, 8)$ where the ball is released and the point $(15, 10)$ where the basketball hoop is. Does your graph pass through both points?

   (c) What is the maximum height of the basketball?

   (d) If a person releases a basketball underhand from a position 3 feet above the floor, it often has a steeper arc than if it is released overhand and the path sometimes may be modeled by $y = \dfrac{-16x^2}{0.117v^2} + 2.75x + 3$. See the second figure below. Complete parts (a), (b), and (c) from the first part. Then compare the paths for an overhand shot and an underhand shot.