11 Polynomials Worksheet

Concepts:

- Graphs of Polynomials
- Leading Term vs. Shape of the Graph
- Continuous Graphs
- Smooth Graphs
- End Behavior of the Graph
- Multiplicity of a Root and Behavior of the Graph at x-intercepts.
- How Many Local Extrema Can a Polynomial Graph Have?

(Sections 4.2 & 4.4)

1. Evaluate $\dfrac{x^3 - 2x^2 + x - 2}{x - 4}$ and express the result in the form $P(x) = D(x)Q(x) + R(x)$.
2. Use the remainder from the above problem to decide if $x-4$ is a factor of x^3-2x^2+x-2 and to find $P(4)$.
3. What is the remainder when $f(x) = 2x^{90} - 5x^{70} - 3x^{15} + 7$ is divided by $x + 1$?
4. Completely factor $f(x) = x^3 - x^2 - 2x + 2$ by using a calculator to find one root and long division or factoring to find the others. Factors should be exact.
5. Find the zeros of the function $f(x) = 6x^2 - 19x - 36$. Use these zeros to help you factor this function.
6. (Exercise 67 from Section 4.2 of your textbook) Use the Factor Theorem to show that for every real number c, $(x - c)$ is not a factor of $x^4 + x^2 + 1$.
7. What is the maximum number of roots of the polynomial $P(x) = 5x^3 + 4x^5 - 3x + 1.2$?
8. Find the maximum value of the function $f(x) = -3x^2 + 10x + 4$.
9. Use a graphing calculator to find the local extrema of the function $f(x) = 3x^4 - 8x^3 - 6x^2 + 24x + 1$.

10. Which one of the following statements is false

    (a) The graphs of all polynomials are continuous.

    (b) The graphs of all polynomials are smooth.

    (c) The graph of a polynomial may have a vertical asymptote.

    (d) The graph of a polynomial never contains a sharp corner.

    (e) The domain of any polynomial is $(-\infty, \infty)$.

11. Describe the end behavior of each polynomial. Use correct mathematical symbols.

    (a) $P(x) = 2x^5 - 3x^2 + 76$

    (b) $Q(x) = -55x^{100} + 15x^{75} - 3$

    (c) $R(x) = (2x + 3)^4(50 - x)^{100}$ (**HINT** What is the leading term?)

    (d) $S(x) = (1 - 2x)^{11}(x + 5)^4$

12. The graph shown below is NOT the graph of $y = g(x) = -2(x + 3)(x - 2)(x - 5)$. Which of the following are clues that this is NOT the graph of g?

    (a) The graph crosses the x-axis at $(-3, 0)$, but it should not cross the x-axis at this point.

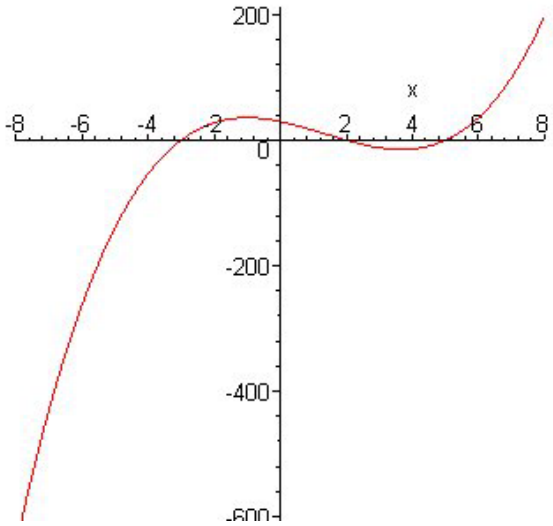
    (b) The graph crosses the x-axis at $(5, 0)$, but it should not cross the x-axis at this point.

    (c) The graph has the wrong x-intercepts.

    (d) The graph crosses the x-axis at $(2, 0)$, but it should not cross the x-axis at this point.

    (e) The graph displays the wrong end behavior.

    (f) The graph has too many local extreme points to be the graph of a polynomial of degree 3.



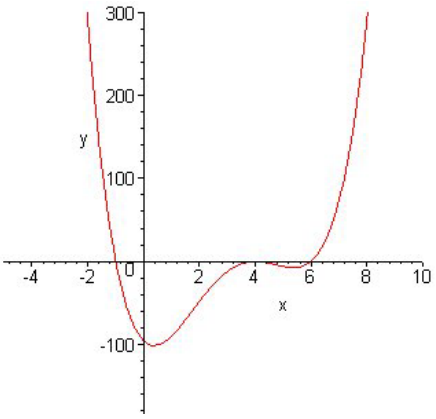
13. The graph of a polynomial $P(x)$ is shown below.

    (a) Is the degree of the polynomial even or odd?

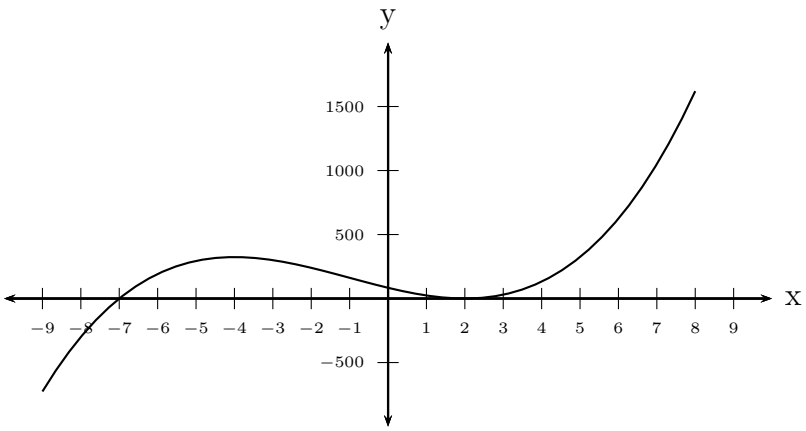
    (b) Is the leading coefficient positive or negative?

    (c) What can you say about the factors of this polynomial?

    (d) Can you find a formula for the polynomial if you know that the degree of the polynomial is less than or equal to 4 and that $P(1) = -90$



14. The graph of a polynomial $P(x)$ is shown below.



    (a) Is the degree of the polynomial even or odd?

    (b) Is the leading coefficient positive or negative?

    (c) What can you say about the factors of this polynomial?

    (d) Can you find a formula for the polynomial if you know that the degree of the polynomial is less than or equal to 4 and that $P(1) = 24$

15. The graph shown below is NOT the graph of $y = h(x) = 5(x+1)^4$. Which of the following are clues that this is NOT the graph of h?

(a) The graph crosses the x-axis at $(-1, 0)$, but it should not cross the x-axis at this point.

(b) The graph displays the wrong end behavior.

(c) The graph has the wrong x-intercepts.

(d) The graph does not have the right number of local extreme points to be the graph of a polynomial of degree 4.