12 Rational Functions & Polynomial and Rational Inequalities Worksheet

Concepts:

- The Definition of a Rational Function

- Identifying Rational Functions

- Finding the Domain of a Rational Function

- The Big-Little Principle

- The Graphs of Rational Functions

- Vertical, Horizontal, and Oblique Asymptotes

- Holes in the Graphs of Rational Functions

- Equivalent Inequalities

- Solving Polynomial and Rational Inequalities Algebraically

- Approximating Solutions to Inequalities Graphically

(Section 4.5 & 4.6)

1. Describe the end behavior of the following rational functions.

   (a) $f(x) = \dfrac{3x - 1}{2 - 5x}$

   (b) $g(x) = \dfrac{2x}{x + 7}$

   (c) $h(x) = \dfrac{x + 7}{x^2 - 6x + 8}$

   (d) $k(x) = \dfrac{x + 7}{x^2 + 6x - 7}$

   (e) $l(x) = \dfrac{x^2 - 6x + 8}{x + 7}$

   (f) $m(x) = \dfrac{x^2 + 6x - 7}{x + 7}$

   (g) $n(x) = \dfrac{7x^2 - 3x + 2x^3 + 6}{4x - x^2 - 2 - 5x^3}$

   (h) $o(x) = \dfrac{(2x + 5)^4(6 - x)^3}{(3x - 1)(x - 2)^6}$

   (i) $p(x) = \dfrac{(2x + 5)^4(6 - x)^3}{(3x - 1)(x - 2)^7}$

2. Find all vertical, horizontal, oblique asymptotes, holes, x-intercepts, and y-intercepts for the following rational functions. Show the algebra that justifies your answer. Graph these functions.

   (a) $f(x) = \dfrac{3x - 1}{2 - 5x}$

   (b) $g(x) = \dfrac{2x}{x + 7}$

   (c) $h(x) = \dfrac{x + 7}{x^2 - 6x + 8}$

   (d) $k(x) = \dfrac{x + 7}{x^2 + 6x - 7}$

   (e) $l(x) = \dfrac{x^2 - 6x + 8}{x + 7}$

   (f) $m(x) = \dfrac{x^2 + 6x - 7}{x + 7}$

3. Solve the inequalities. Find exact solutions when possible and approximate ones otherwise.

   (a) $x^3 - x \geq 0$

   (b) $x^2 + 8x + 20 < 0$

   (c) $2x^4 + 3x^3 < 2x^2 + 4x - 2$

   (d) $\dfrac{2x^2 + x - 1}{x^2 - 4x + 4} \geq 0$

   (e) $\dfrac{1}{x - 1} < -\dfrac{1}{x + 2}$

   (f) $\dfrac{2x^2 + 6x - 8}{2x^2 + 5x - 3} < 1$

4. If it is possible to solve the inequality algebraically, do so and give exact solutions. If it is not possible to solve it algebraically, find an approximate solution graphically. Be sure to sketch the graph and label it.

   (a) $7x - 3 < 10x + 2$

   (b) $x^2 + 7x \geq -10$

   (c) $(x + 2)(x - 3)^2 > 0$

   (d) $x^3 > 1$

   (e) $x^4 - 2x \geq 5$

   (f) $x^4 - 2x \geq 0$

   (g) $\dfrac{1}{x - 2} \geq -1$

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5. It costs a craftsman $ 5 in materials to make a medallion. He has found that if he sells the medallions for $50x$ dollars each, where x is the number of medallions produced each week, then he can sell all that he makes. His fixed costs are $ 350 per week. If he wants to sell all he makes and show a profit each week, what are the possible numbers of medallions he should make?

6. Emma and Aidan currently pay $ 60 per month for phone service from AT& T. This fee gets them 900 minutes per month. They look at their phone bills and realize that, at most, they talk for 100 minutes per month. They find out that they can go with Virgin Mobile and pay 18 cents per minute. If they choose to switch services, they will have to buy two new phones at $ 40 each, and pay a $ 175 "cancellation fee" to AT& T.

   (a) Assuming that they talk for 100 minutes per month, how many months would they have to talk before they would be saving money?

   (b) Assume they make the switch, and talk between zero and 100 minutes per month. What is the range of possible savings?