13 Complex Numbers

Concepts:

- The imaginary number i.
- Complex numbers.
- Complex arithmetic.
- Solutions to quadratic equations.
- Applications.

(Section 4.7)

1. **Answer as TRUE or FALSE.**

   (a) _____ The only solution to the equation $x^2 = -1$ is i.

   (b) _____ $\sqrt{-4} = -2i$.

   (c) _____ $i^2 = -1$.

   (d) _____ $\sqrt{4 + 9i} = 2 + 3i$.

   (e) _____ Any real number c can be expressed in standard complex form.

2. Write each expression in terms of a real number and i.

   (a) $\sqrt{-49}$

   (b) $\sqrt{-\pi}$

   (c) $\sqrt{-5}$

3. Perform the indicated operation and write in standard complex form.

   (a) $\sqrt{-9}$

   (b) $(3 - 4.5i) + (-2.2 - 6.1i)$

   (c) i^{24}

   (d) $\dfrac{-5}{i}$

   (e) $i(5 - 3i)$

   (f) $\dfrac{5}{4} - (\dfrac{7}{3} - i)$

   (g) $(\sqrt{-7} + 3)(4 - \sqrt{-5})$

   (h) $2 + 3i - (5 - 4i)$

   (i) $\left(3 + \dfrac{3}{2}i\right) 4i$

   (j) $(-4 - i)(-4 + i)$

   (k) i^{42}

   (l) $\dfrac{1 + i}{3 + 2i}$

   (m) $\dfrac{4}{i}$

   (n) $(a - bi)(a + bi)$

   (o) $(a - bi) - (a + bi)$

4. Solve each equation by using the quadratic formula and express answer in standard complex form.

   (a) $x^2 = -3$

   (b) $3x^2 - 2x = -5$

   (c) $x^2 - 4x - 6 = 0$

   (d) $x^2 + 5x - 6 = 0$

   (e) $x^2 + 5x + 6 = 0$

   (f) $x^2 + 4 = 0$

   (g) $3x^2 - 2x = -5$

5. Given the quadratic equation $2x^2 + x + 3 = 0$.

   (a) Find the two solutions to the equation.

   (b) Add the two solutions together. What can you say about the result. Is this true for solutions to any quadratic?

6. Find the complete factorization into linear terms for each of the following

   (a) $f(x) = x^4 - 16$

   (b) $f(x) = x^7 - 7x^6 + 19x^5 - 43x^4 + 74x^3 - 68x^2 + 56x - 32$ (Hint: 1, 2, and 4 are roots.)

   (c) $g(x) = x^3 - 1$

   (d) $h(x0 = x^3 + 1$

7. Find a polynomial that has roots 1, 3, -7, $2i$ and $-2i$. Do not leave it in factored form.