15 Logarithmic Functions

Concepts:

- Logarithms
  - Logarithms as Functions
  - Logarithms as Exponent Pickers
  - Inverse Relationship between Logarithmic and Exponential Functions.
  - The Common Logarithm
    - Definition and Graphs
    - Exponential Notation vs. Logarithmic Notation
    - Evaluating Common Logarithms
  - The Natural Logarithm
    - Definition and Graphs
    - Exponential Notation vs. Logarithmic Notation
    - Evaluating Common Logarithms
  - Logarithms with Different Bases
    - Definition and Graphs
    - Exponential Notation vs. Logarithmic Notation
    - Evaluating Different Base Logarithms

(Section 5.3)

1. Find the exact value of the following logarithms. Do NOT use your calculator.

   (a) $\log_3(27)$

   (b) $\log(\sqrt[3]{100})$

   (c) $\log_5\left(\frac{1}{625}\right)$

   (d) $\ln\left(\frac{1}{\sqrt[5]{e^3}}\right)$

   (e) $10^{\log(53)}$

   (f) $e^{2\ln(x)}$

2. Using your knowledge of exponents, estimate between which two integer values the following expressions will be. Use your calculator to find approximate values for each. Was your estimation accurate?

   (a) $\log(1008)$

   (b) $\ln(3)$

   (c) $\ln(7)$

   (d) $\log(53)$

3. Convert each exponential statement to an equivalent logarithmic statement.

   (a) $8^2 = 64$

   (b) $2^x = 16$

   (c) $3^4 = y$

   (d) $x^{-5} = \left(\frac{1}{32}\right)$

4. Convert each logarithmic statement to an equivalent exponential statement.

   (a) $\log_2(32) = 5$

   (b) $\ln(x) = 3$

   (c) $\log(9) = x$

   (d) $\log_x(\frac{1}{64}) = -3$

5. Find all real solutions or state that there are none. Your answers should be exact.

   (a) $\ln(x) = 2$

   (b) $10^{x+2} = 376$

   (c) $9e^{x-8} = 2$

   (d) $\log_8(x-5) - \log_8(2x+2) = 0$

6. Find the approximate solutions. (HINT: Try using a graphing approach.)

   (a) $\ln(x) = x - 5$

   (b) $\log_5(6) = x$

   (c) $\log_2(21) = x$

7. Let $f(x) = \ln(3x+7)$. Find $f^{-1}(x)$.

8. Let $f(x) = 2^{5x+3} - 1$. Find $f^{-1}(x)$.

9. Find the domain of $f(x) = \ln(2-3x)$

10. Find the domain of $g(x) = \dfrac{x}{\ln(5x+4)}$

11. Find the domain of $h(x) = \ln(x^2 - 2x - 15)$

12. List the transformations that will change the graph of $f(x) = \ln(x)$ into the graph of the given function.

    (a) $g(x) = 3\ln(x)$

    (b) $h(x) = \ln(x) - 5$

    (c) $k(x) = \ln(x-3)$

    (d) $l(x) = \ln(x+2) - 7$

13. Sketch the graph of the function.

    (a) $f(x) = \log(x + 4)$

    (b) $g(x) = 2\log(x) - 5$

14. (Question # 75 from Hungerford 5.3 exercises) Show that $g(x) = ln\left(\frac{x}{1-x}\right)$ is the inverse function of $f(x) = \frac{1}{1+e^{-1}}$

15. (Question # 77 from Hungerford 5.3 exercises) Suppose $f(x) = A\ln(x) + B$, where A and B are constants. If $f(1) = 10$ and $f(e) = 1$, what are A and B?

16. (Question # 78 from Hungerford 5.3 exercises) $f(x) = A\ln(x) + B$, where A and B are constants. If $f(e) = 5$ and $f(e^2) = 8$, what are A and B?

17. (Question # 49 from Hungerford 5.3 exercises) Do the graphs of $f(x) = \log(x^2)$ and $g(x) = 2\log(x)$ appear to be the same? How do they differ?

18. (Question # 50 from Hungerford 5.3 exercises) Do the graphs of $h(x) = \log(x^3)$ and $k(x) = 3\log(x)$ appear to be the same? How do they differ?

19. (Question # 82 from Hungerford 5.3 exercises) Students in a precalculus class were given a final exam. Each month thereafter, they took an equivalent exam. The class average on the exam taken after t months is given by:

$$F(t) = 82 - 8\ln(t + 1).$$

    (a) What was the class average after six months?

    (b) After a year?

    (c) When did the class average drop below 55?

20. (Question # 83 from Hungerford 5.3 exercises) One person with a flu virus visited the campus. The number T of days it took for the virus to infect x people was given by:

$$T = -.93\ln\left[\frac{7000 - x}{6999x}\right].$$

    (a) How many days did it take for 6000 people to become infected?

    (b) After two weeks, how many people were infected?

    (c) How large was this campus population? (HINT: Think about the domain!)