17 Logarithmic Properties, Solving Exponential & Logarithmic Equations & Models

Concepts:

- Properties of Logarithms
- Simplifying Logarithmic Expressions
- Proving the Quotient Rule for Logarithms
- Using the Change of Base Formula to Find Approximate Values of Logarithms
- Solving Exponential and Logarithmic Equations Algebraically
  - Strategies:
    - Same base exponential expressions that are equal must have equal exponents.
    - Same base logarithmic expressions that are equal must have equal arguments.
    - Isolate exponential expression, rewrite equation in logarithmic form.
    - Isolate logarithmic expression, rewrite equation in exponential form.
- Solving Exponential and Logarithmic Models/Applications

(Section 5.4 - 5.6)

1. Prove the *Quotient Rule for Logarithms.*

$$\log_a\left(\frac{u}{v}\right) = \log_a(u) - \log_a(v)$$

Proof:

2. Write each expression in terms of $\log(x)$, $\log(y)$, and $\log(z)$ if possible. If it is not possible, explain why.

   (a) $\log\left(\dfrac{x^3y^7}{\sqrt{z}}\right)$

   (b) $\log\left(\dfrac{x^2+y^2}{z}\right)$

   (c) $\log\left(x^5\sqrt[3]{yz}\right)$

3. Use your calculator to find approximate values for the following.

   (a) $\ln(7)$

   (b) $\log(53)$

   (c) $\log_5(6)$

   (d) $\log_2(21)$

4. Given the magnitude of an earthquake on the Richter scale is given by $R(i) = \log(\frac{i}{i_0})$, where i is the amplitude of the ground motion of the earthquake and i_0 is the amplitude of the ground motion of the "zero" earthquake,

   (a) Find the magnitude on the Richter scale of an earthquake that is 10000 times stronger than the zero quake.

   (b) Find the magnitude on the Richter scale of an earthquake that is 25 times stronger than the zero quake.

5. The half-life of a certain radioactive substance is 2,365 years.

   (a) Find the decay rate constant r.

   (b) How much substance will be left in 100 years if there is currently 500 grams of the substance?

6. Find how long it takes for a deposit to double in value if the annual interest rate is 3.5% and the interest is compounded continuously.

7. The antibiotic clarithromycin is eliminated from the body according to the formula $A(t) = 500e^{-0.1386t}$, where A is the amount remaining in the body (in milligrams) t hours after the drug reaches peak concentration.

   (a) How much time will pass before the amount of drug in the body is reduced to 100 milligrams?

   (b) Find the inverse of $A(t)$ and explain what the inverse function models.

8. You are given models for the population (in millions) of different countries t years after 2005. For each part, determine the year in which the models predict the populations will be equal.

(Source: World Health Organization's 2006 World Health Statistics)

   (a) Rwanda: $R(t) = 9.04(1.05)^t$ and Hungary: $H(t) = 10.1(0.98)^t$.
   (b) Cambodia: $C(t) = 14.07(1.02)^t$ and Kazakhstan: $K(t) = 14.83(0.93)^t$.

9. Find the solution(s) of the following exponential equations. Your answers should be exact.

   (a) $10^{2x^2-3} = 10^{9-x^2}$
   (b) $2^{3x+1} = 3^{x-2}$
   (c) $\dfrac{10}{1+e^{-x}} = 2$
   (d) $3^{4x} - 3^{2x} - 6 = 0$
   (e) $9e^{x-8} = 2$

10. Find the solution(s) of the following logarithmic equations. Your answers should be exact.

   (a) $\log_4(x+2) + \log_4 3 = \log_4 5 + \log_4(2x-3)$
   (b) $\log_3(x+15) - \log_3(x-1) = 2$
   (c) $\log_2(\log_3 x) = 4$
   (d) $\log(x+3) = \log x + \log 3$
   (e) $\log_8(x-5) + \log_8(x+2) = 1$

11. Suppose you're driving your car on a cold winter day (20° F outside) and the engine overheats (at about 220° F). When you park, the engine begins to cool down. The temperature U of the engine t minutes after you park satisfies the equation

$$\ln\left(\frac{U-20}{200}\right) = -0.11t.$$

   (a) Solve the equation for U.
   (b) Use part (a) to find the temperature of the engine after 20 min ($t = 20$).

12. Joni invests \$5000 at an interest rate of 5% per year compounded continuously. How much time will it take for the value of the investment to quadruple.

13. Joni invests \$5000 at an interest rate of 5% per year compounded monthly. How much time will it take for the value of the investment to quadruple.