

21 Trigonometric Identities Worksheet

Concepts:

- Trigonometric Identities
 - Reciprocal Identities
 - Pythagorean Identities
 - Periodicity Identities
 - Negative Angle Identities

(Sections 6.6 & 7.1)

1. Find the period, phase shift, and sketch the graph of $y = 2 \csc(2x - \frac{\pi}{2})$.

2. Find all solutions to the equation $\tan(\theta) = \cot(\theta)$ for $0 < \theta < \pi$

3. Use the Pythagorean identity for each of the following problems.
 - (a) Find $\sin(t)$ if $\cos(t) = \frac{2}{\sqrt{10}}$ and $\frac{3\pi}{2} < t < 2\pi$.
 - (b) Find $\cos(t)$ if $\sin(t) = \frac{3}{5}$ and $\frac{\pi}{2} < t < \pi$.
 - (c) Find $\cot(t)$ if $\sin(t) = -\frac{2}{5}$ and $\pi < t < \frac{3\pi}{2}$.

4. Determine each of the following:
 - (a) Find $\cos x$ if $\sin x = -\frac{5}{13}$ and $\tan x > 0$.
 - (b) Find $\tan x$ if $\cos x = \frac{1}{4}$ and $\sin x < 0$.
 - (c) Find $\tan x$ if $\sec x = \frac{\sqrt{10}}{3}$ and $\sin x > 0$.

5. Use the periodicity and negative angle identities to solve each of the following problems.

- (a) Find $\sin(-t)$ if $\sin(t) = \frac{1}{\sqrt{5}}$.
- (b) Find $\sin(t - 2\pi)$ if $\sin(t) = \frac{1}{\sqrt{2}}$.
- (c) Find $\cos(-t + 4\pi)$ if $\cos(t) = -\frac{\sqrt{6}}{3}$.
- (d) Find $\cot(7\pi - t)$ if $\tan(t) = \sqrt{15}$.

6. Use basic identities to simplify the expression.

- (a) $\cot \theta \sec \theta \sin \theta$
- (b) $\frac{\cos^2 x}{\sin^2 x} + \csc x \sin x$
- (c) $\frac{1}{\cot^2 x} + \sec x \cos x$
- (d) $\sin^2 x + \tan^2 x + \cos^2 x$
- (e) $\frac{1 - \sin^2 x}{\sin x - \csc x}$
- (f) $\frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x}$
- (g) Prove the identity $\sin x (\tan x \cos x - \cot x \cos x) = 1 - 2 \cos^2 x$

7. State whether or not the equation might be an identity. If it appears to be, prove it.

- (a) $\cot x = \frac{\csc x}{\sec x}$
- (b) $\frac{\sin(-x)}{\cos(-x)} = -\tan x$
- (c) $1 + \sec^2 x = \tan^2 x$
- (d) $\sin^2 x (\cot x + 1)^2 = \cos^2 x (\tan x + 1)^2$
- (e) $\frac{1 + \sin x}{1 - \sin x} = \frac{\sec x + \tan x}{\sec x - \tan x}$
- (f) $(1 - \cos^2 x) \csc x = \sin x$
- (g) $\frac{\sec x}{\csc x} + \frac{\sin x}{\cos x} = 2 \tan x$
- (h) $\frac{\csc^2 x - 1}{\csc^2 x} = \cos^2 x$
- (i) $\frac{\sin x - \cos x}{\tan x} = \frac{\tan x}{\sin x + \cos x}$