Ma 110 Exam 3 Review: Sections 5.4-5.6, 6.1-6.2, 6.4-6.6, 7.1-7.3

Do not rely solely on this work sheet! **Make sure to study homework problems, other work sheets, lecture notes, and the book!!!**

1. Section 5.4 (suggested problems from HW23 - #'s 3, 6, 7, 10)
   (a) Write as a single logarithm. $2\log(x-1)+\log(x^2-1)-\log(y-1)$
   (b) Simplify. $e^{x\ln(5)}$
   (c) Express $\ln\left(\frac{2z^3}{2x^4\sqrt{y}}\right)$ in terms of $\ln(x), \ln(y)$ and $\ln(z)$.
2. Section 5.5 (suggested problems from HW24 - #'s 2, 5, 9, 10)
   (a) Solve for x exactly. $\ln(x+5)=3$.
   (b) Solve for x exactly. $e^{x^2-2x-3}=1$
   (c) Section 5.5, question 37: Solve $\log(3x-1)+\log(2)=\log(4)+\log(x+2)$
   (d) How long until $10,000 doubles in a bank account with a yearly interest rate of $r=7\%$ compounded continuously?
   (e) Section 5.5, question 69: The concentration of carbon dioxide in the atmosphere is 364 parts per million (ppm) and is increasing exponentially at a continuous rate of .4%. How many years will it take for the concentration to reach 500 ppm?
3. Section 5.6 (suggested problems from HW25 - #'s 3, 4, 6)
   (a) A culture starts with 8600 bacteria. After one hour the count is 10,000.
      i. Find a function that models the number of bacteria after t hours.
      ii. Find the number of bacteria after 2 hours.
      iii. How long will it take for the number of bacteria to double?
   (b) The population of California was $10,586,223$ in 1950 and $23,668,562$ in 1980. Assume the population grows exponentially.
      i. Find a function that models the population t years after 1950.
      ii. Find the time required for the population to double.
      iii. In what year was the population $1,000,000$?
   (c) The half-life of radium-226 is 1600 years. Suppose we have a 22-mg sample.
      i. Find a function that models the mass remaining after t years.
      ii. How much of the sample will remain after 4000 years?
      iii. After how long will only 18 mg of the sample remain?

4. Section 6.1 (suggested problems from HW26 - #'s 1, 2, 5, 6, 9)

   (a) Find the radian measure of a $-450°$ angle.

   (b) What quadrant does the angle with measure $\frac{26\pi}{3}$ lie in.

   (c) Suppose that an angle of measure θ radians intersects the unit circle at the point $\left(\frac{-\sqrt{3}}{2}, \frac{-1}{2}\right)$. Give two possibilities for θ.

5. Section 6.2 (suggested problems from HW27 - #'s 3, 5, 6)

   (a) Find the point P on the unit circle corresponding to the angle $\frac{-5\pi}{6}$.

   (b) Evaluate the $\sin, \cos,$ and $\tan$ functions at $t = \frac{5\pi}{2}$.

   (c) Evaluate $\sin, \cos,$ and $\tan$ if the terminal ray of an angle contains the point $(\sqrt{5}, -7)$.

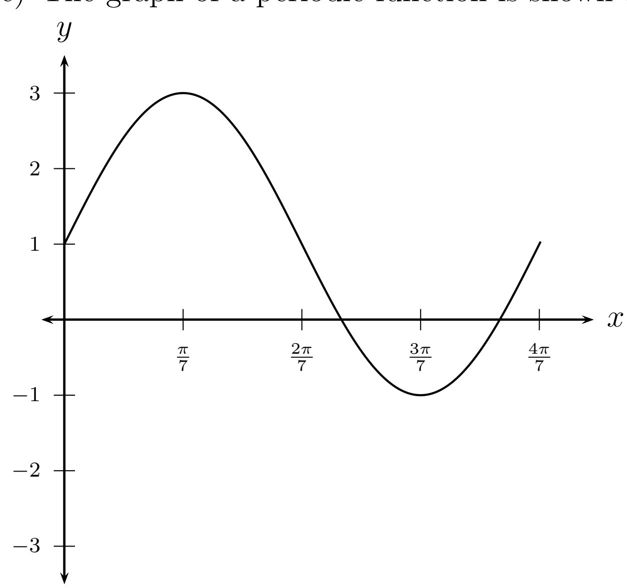
   (d) Approximate $\sin(2.6)$.

6. Section 6.4 (suggested problems from HW28 - #'s 1, 3, 6, 7)

   (a) List the transformations needed to apply to the graph of $y = \sin(x)$ to sketch the graph of $y = 3\sin\left(\frac{x}{\pi}\right)$. Sketch the graph. Be sure that the graph is well-labeled.

   (b) List the transformations needed to apply to the graph of $y = \cos(x)$ to sketch the graph of $y = 2\cos\left(x - \frac{\pi}{3}\right)$. Sketch the graph. Be sure that the graph is well-labeled.

   (c) The graph of a periodic function is shown below. Find a rule for the function.



7. Section 6.5 (suggested problems from HW29 - #'s 2, 3, 4)

   (a) For each state the amplitude, period, phase shift and sketch the graph.

      i. $f(x) = 2\cos\left(3x - \frac{\pi}{2}\right)$

      ii. $f(x) = \tan\left(x + \frac{\pi}{4}\right)$

   (b) How many solutions for x between 0 and 2π does $\sin(x) = -0.2$ have?

8. Section 6.6 (suggested problems from HW30 - #'s 2, 5, 6)

   (a) Evaluate the six trigonometric functions at $t = -\dfrac{7\pi}{3}$.

   (b) Evaluate the six trigonometric functions if the terminal ray of an angle contains the point $(.6, .8)$.

   (c) Answer as True or False.

      i. $\cos^2(t) = 1 + \sin^2(t)$.

      ii. $\cos(t - 2\pi) = \cos(t)$.

      iii. $\csc(t) = \dfrac{1}{\sin(t)}$.

      iv. $\tan(-t) = \tan(t)$.

   (d) Find $\pi < t < 2\pi$ such that $\sec(t) = \sqrt{2}$ exactly.

   (e) Find the other five trigonometric functions exactly if $\cos(t) = \dfrac{1}{3}$ and $\pi < t < 2\pi$.

9. Section 7.1 (suggested problems from HW31 - #'s 4, 8, 9, 10)

   (a) Simplify the following:

      i. $\tan(x)\Big(\sin(x) + \cot(x)\cos(x)\Big)$

      ii. $\Big(\cos(\theta) - \sin(\theta)\Big)^2$

      iii. $\dfrac{\sec(t)}{\sin(t)} - \dfrac{\sin(t)}{\cos(t)}$

   (b) Prove the following:

      i. $\csc^2(x) - \cos^2(x)\csc^2(x) = 1$

      ii. $\big(\sec(\theta) + 1\big)\big(\sec(\theta) - 1\big) = \tan^2(\theta)$

      iii. $\dfrac{\cos(\alpha)}{1 - \sin(\alpha)} = \sec(\alpha) + \tan(\alpha)$

      iv. $\dfrac{\sin(t)}{1 - \cos(t)} + \dfrac{1 - \cos(t)}{\sin(t)} = 2\csc(t)$

      v. $\sec^4(x) - \tan^4(x) = \sec^2(x) + \tan^2(x)$

10. Section 7.2 (suggested problems from HW32 - #'s 5, 6, 8, 10)

   (a) Find the exact value of each of the following expressions:

      i. $\cos\left(\frac{11\pi}{12}\right)$

      ii. $\sin\left(\frac{19\pi}{12}\right)$

      iii. $\tan\left(\frac{17\pi}{12}\right)$

   (b) Write the following expression as a trigonometric function of one number, and find its exact value.

$$\cos\left(\frac{3\pi}{7}\right)\cos\left(\frac{2\pi}{21}\right)+\sin\left(\frac{3\pi}{7}\right)\sin\left(\frac{2\pi}{21}\right)$$

   (c) Prove the cofunction identity: $\sin\left(\frac{\pi}{2}-\theta\right)=\cos(\theta)$

   (d) Section 7.2, Question 45: If x is in the first and y is in the second quadrant, $\sin(x)=\frac{24}{25}$, and $\sin(y)=\frac{4}{5}$, find the exact value of $\sin(x+y)$ and $\tan(x+y)$ and the quadrant in which $x+y$ lies.

11. Section 7.3 (suggested problems from HW33 - #'s 1, 2, 3, 4)

   (a) Find the exact value of each of the following expressions:

      i. $\cos\left(\frac{3\pi}{8}\right)$

      ii. $\sin\left(\frac{5\pi}{8}\right)$

      iii. $\tan\left(\frac{7\pi}{8}\right)$

   (b) Given $\tan(t)=-\frac{4}{3}$ and $\frac{\pi}{2}<t<\pi$, find $\sin(2t), \cos(2t)$, and $\tan(2t)$.

   (c) Given $\tan(x)=1$ and x is in Quadrant III, find $\sin\left(\frac{x}{2}\right), \cos\left(\frac{x}{2}\right)$, and $\tan\left(\frac{x}{2}\right)$.

   (d) Simplify. $\dfrac{1-\cos(4\theta)}{\sin(4\theta)}$