26   Circles & Ellipses

Concepts:

- Circles & Ellipses:

    - sketch a graph from an equation
    - find an equation given information about a graph

- Solving Applied Problems Involving Ellipses

(Section 10.1)

1. Match each of the following equations with the appropriate graph.
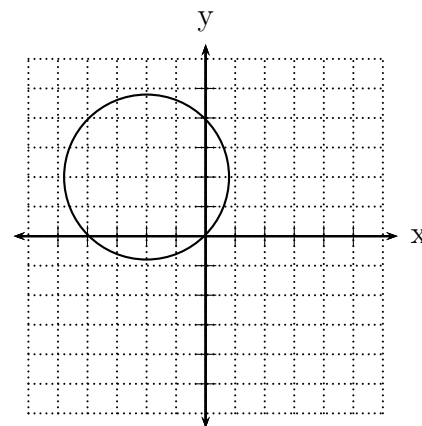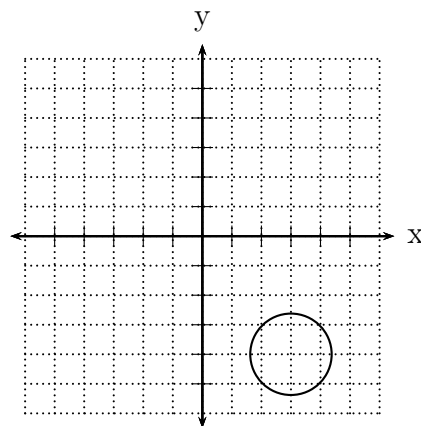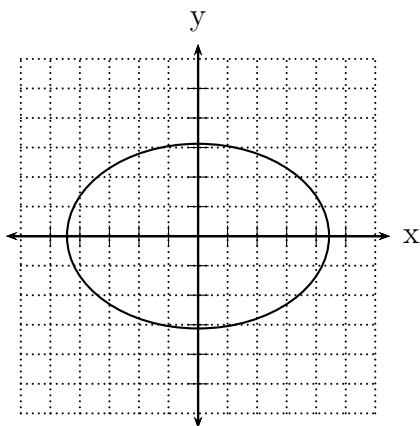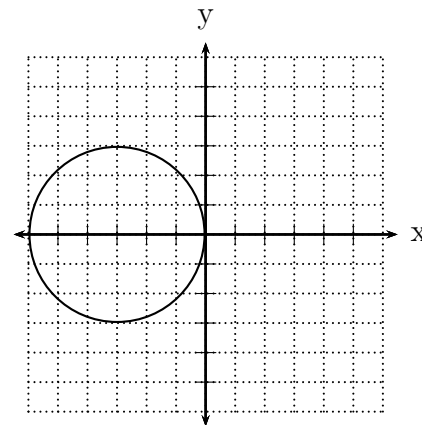
(a) $(x+2)^2 + (y-2)^2 = 8$

(b) $25x^2 + 9y^2 = 225$

(c) $(x+3)^2 + y^2 = 9$

(d) $(x-3)^2 + (y+4)^2 = 2$

(e) $x^2 + (y-3)^2 = 4$

(f) $x^2 + 2y^2 = 20$
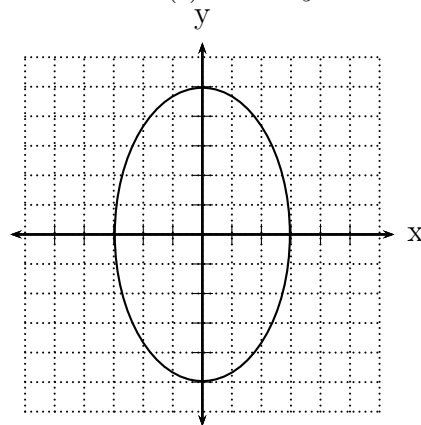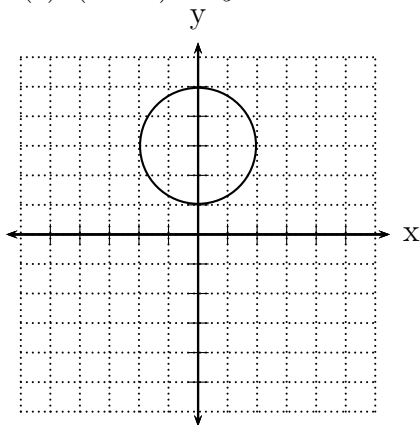
2. For each of the following, identify which conic section is represented by the equation. If it is a circle, give its center and radius. If it is an ellipse, give its center, vertices, and foci. Sketch the graph of each.

   (a) $5x^2 + 5y^2 + 20x + 20 = 30y$.

   (b) $\dfrac{x^2}{8} + \dfrac{y^2}{8} = 1$.

   (c) $5x^2 + 4y^2 = 60$.

   (d) $\dfrac{(x-1)^2}{8} + \dfrac{(y+3)^2}{8} = 1$.

   (e) $\dfrac{x^2}{8} - 1 = -\dfrac{y^2}{36}$.

   (f) $\dfrac{(x-1)^2}{36} + \dfrac{(y-6)^2}{8} = 1$.

   (g) $x^2 + 5y^2 - 2x + 30y + 26 = 0$.

3. Find the equation of the ellipse that satisfies the given conditions. HINT: there may be more than one equation.

   (a) Center $(0,0)$; foci on y-axis; major axis of length 8; minor axis of length 4.

   (b) Center $(0,0)$; vertices $(4,0)$ and $(-4,0)$; minor axis of length 6

   (c) Center $(-4,5)$; endpoints of major and minor axes: $(0,5), (-4,16), (-8,5), (-4,-6)$.

   (d) Center $(-3,-9)$; foci on the line $y = -9$; major axis of length 11; minor axis of length 5.

   (e) Center at $(3,-6)$; major axis of length 17; minor axis of length 5.

4. (Question 65, Section 10.1) The bottom of a bridge is shaped like half an ellipse and is 20 feet above the river at the center. Find the height of the bridge bottom over a point on the river 25 feet from the center of the river.

5. (Question 62, Section 10.1) Halley's Comet has an elliptical orbit with the sun as one focus and a major axis that is 1,636,484,848 miles long. The closest the comet comes to the sun is 54,004,000 miles. What is the maximum distance from the comet to the sun?

6. (Question 61, Section 10.1) The orbit of the moon around the earth is an ellipse with the earth as one focus. If the length of the major axis of the orbit is 477,736 miles and the length of the minor axis is 477,078 miles, find the minimum and maximum distances from the earth to the moon.

7. An elliptically shaped garden is surrounded by a stone walkway. The garden is 15 meters long and 8 meters wide. The walkway is 2 meters wide. Find the equation describing the ellipse that includes both the garden and the walkway.