28 Plane Curves & Parametric Equations

Concepts:

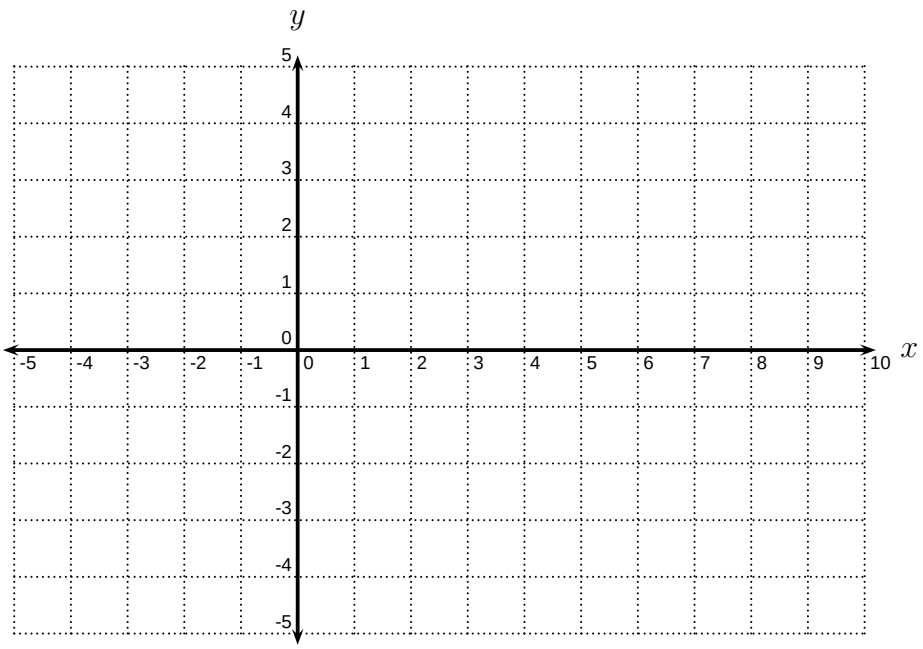
- Sketching Graphs of Parametric Equations
- Eliminating the Parameter to Find an Equation in x and y
- Finding Parametric Equations for Graphs in x and y

(Section 10.5)

1. Graph each set of parametric equations and indicate the points on the graph for the given values of t labeled with the t-value.

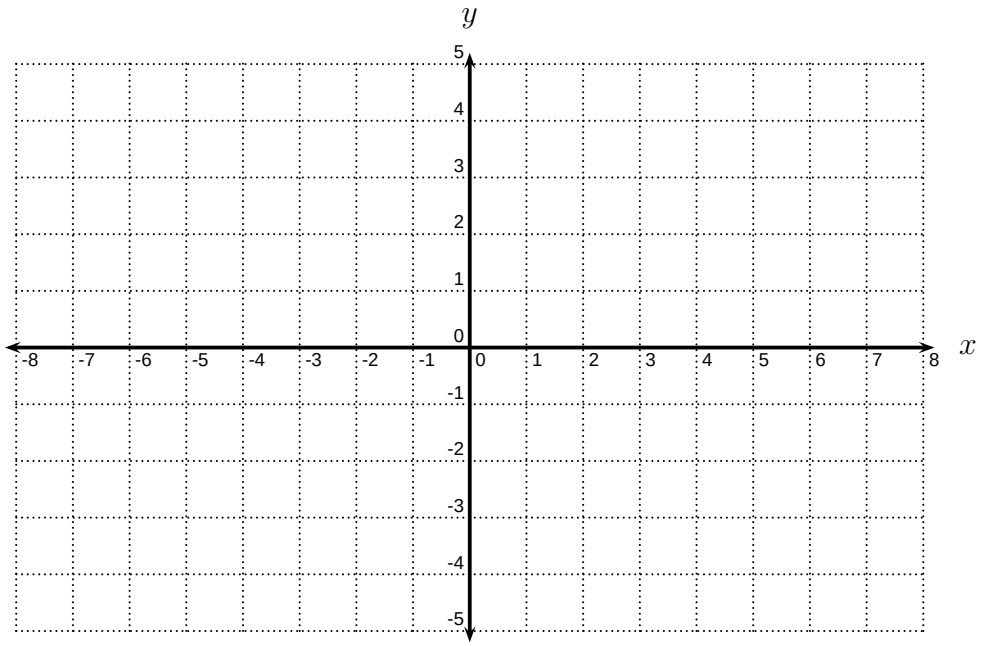
   (a) $x(t) = t^2 - 1, \quad y(t) = t - 1, \qquad -2 \leq t \leq 3$

   $t = -2, -1, 0, 1, 2, 2.25$



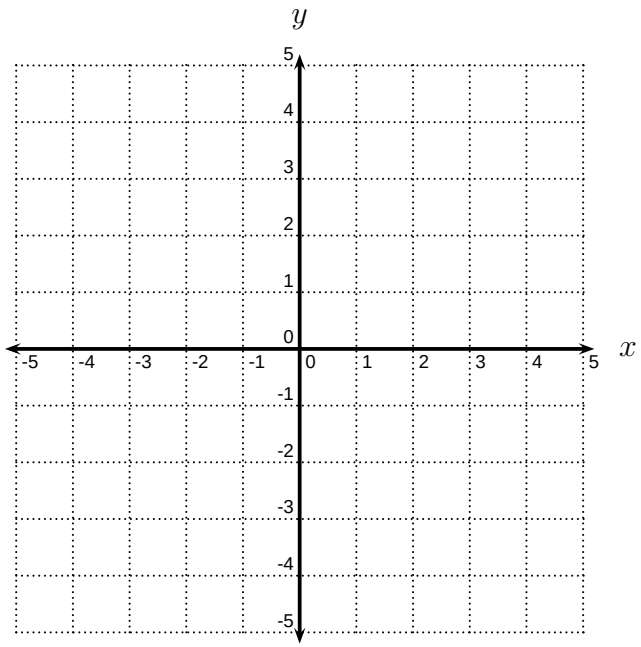
(b) $x(t) = t^3 - t, \quad y(t) = t^2 - 2, \qquad -2 \le t \le 2$

$t = -1.75, -1, 0, 0.5, 1, 1.75$



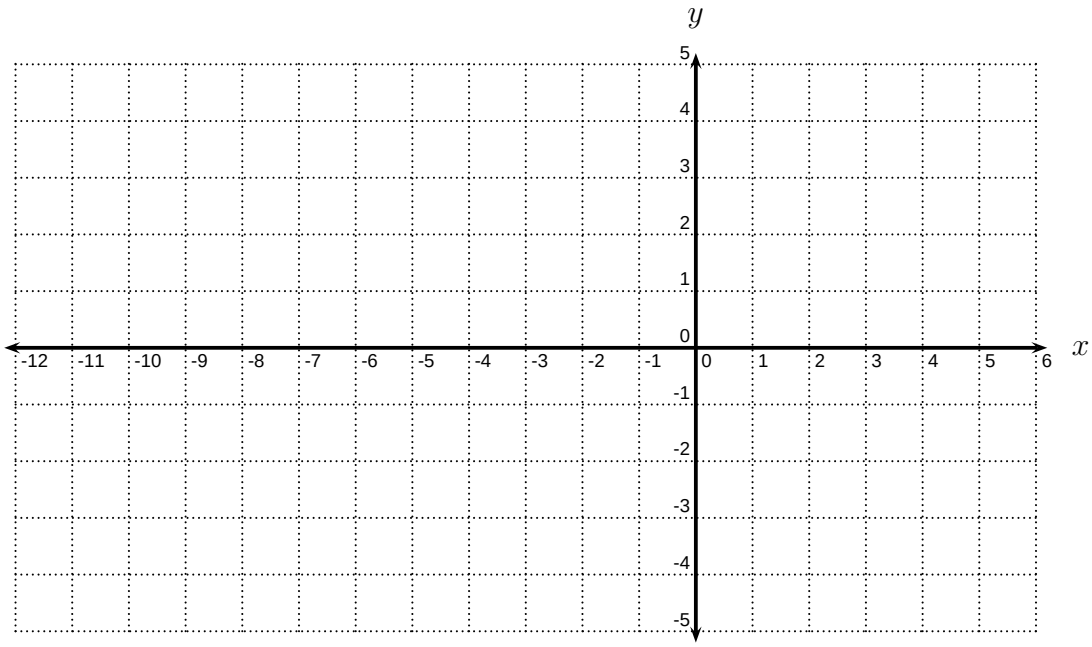
(c) $x(t) = 3\cos t, \quad y(t) = 4\sin t, \qquad 0 \le t \le 2\pi$

$t = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{2}, \pi, \frac{5\pi}{2}, 2\pi$



(d) $x(t) = (t^3 - 3t^2 - 4)/2, \quad y(t) = t, \qquad -2 \le t \le 4$

$t = -2, -1, 0, 1, 2, 3, 4$



2. The given curve is part of the graph of an equation in x and y. Find the equation by eliminating the parameter.

   (a) $x(t) = t + 5, \quad y(t) = \sqrt{t} - 1, \qquad t \ge 0$

   (b) $x(t) = t^2 - 1, \quad y(t) = t^2 + 1, \qquad t$ is any real number

   (c) $x(t) = 2e^t, \quad y(t) = 1 - e^t, \qquad t \ge 0$

   (d) $x(t) = 4\sin 2t, \quad y(t) = 2\cos 2t, \qquad 0 \le t \le 2\pi$

   (e) $x(t) = e^t, \quad y(t) = \ln(t) - 1, \qquad t > 0$

3. Find a parameterization for each of the following. Confirm your answer by graphing.

   (a) circle with center (2, -5) and radius 4

   (b) $4x^2 + 4y^2 + 8x - 24y + 4 = 0$

4. Find a parameterization of the line segment joining the two points. Confirm your answer by graphing.

   (a) $(-6, 12)$ and $(12, -10)$

   (b) $(14, -5)$ and $(5, -14)$