# 1 A Bit of Review Worksheet

**Concepts:**

- Square roots and principal square roots.
- Negation.
- Scientific notation.
- Absolute Value.

**(Section 1.1)**

1. **TRUE or FALSE**

   (a) ____________ 11 is the only square root of 121.

   (b) ____________ $\sqrt{121} = \pm 11$

   (c) ____________ $\sqrt{3^2 + 4^2} = \sqrt{3+4}$

2. Simplify.

   (a) $\sqrt{75}\sqrt{12}$

   (b) $\dfrac{\sqrt{567}}{\sqrt{45}}$

   (c) $\sqrt{2535} - \sqrt{135}$.

3. Given real numbers $b, c, d$ such that $b < 0, c > 0$, and $d < 0$. Determine which of the expressions are positive?

   (a) $b - c$

   (b) $bc - bd$

   (c) $b^2c - c^2d$

4. Find the exact value of the expression. You may not use parentheses in your answer. Which of the expressions are positive?

   (a) $-(\sqrt{245} - 13)$

   (b) $-(x - 6)$ if $x > 6$

   (c) $-(x - 6)$ if $x < 6$

   (d) $-((\pi - 3) - 1)$

5. Express the given statement in symbols.

   (a) $x$ is nonnegative.

   (b) $d$ is not greater than 7.

6. For each arithmetic statement, write a corresponding geometric statement.
   (a) $a \geq b$
   (b) $a + 5 = b$
   (c) $a + c > b$, $(c > 0)$
7. For each geometric statement, write a corresponding arithmetic statement.
   (a) $a$ lies 6 units to the right of $b$ on a horizontal number line.
   (b) $a$ lies at least 4 units below $b$ on a vertical number line.
8. Express the number in normal decimal notation.
   (a) There are $6.02 \times 10^{23}$ molecules in each mole.
   (b) The mass of an electron is $9.10938188 \times 10^{-31}$ kg.
9. 1 mile = ____________ inches. Write your answer in scientific notation. (**HINT:** There are 5280 feet in one mile.)
10. 1 year = ____________ seconds. Write your answer in scientific notation. (Assume that there are 365 days in a year.)
11. 1 second = ____________ years. Write your answer in normal decimal notation.
12. Simplify, and write the given number without using absolute values.
   (a) $3 - |2 - 5|$
   (b) $\left|\sqrt{2} - 2\right|$
   (c) $|3 - \pi| + 3$
13. Write the given number without using absolute values.
   (a) $|a - 5|$ if $a < 5$
   (b) $|c - d|$ if $c \geq d$
14. Translate the given algebraic statement into a geometric statement about distance.
   (a) $|x - 3| < 2$
   (b) $|x + 7| \leq 3$
15. Draw a graph representing each of the following algebraic statements.
   (a) $|x - 17| > 7$
   (b) $|x - 17| \leq 7$
16. Use a geometric approach to solve the given equation or inequality.
   (a) $|x - 2| = 1$
   (b) $|x + 2| \geq 3$

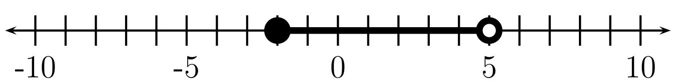
# 2 Solving Equations Worksheet

**Concepts:**

- Number Lines
- The Definitions of Absolute Value
- Absolute Value Equations and Inequalities
- Solving Equations with One Variable Type - The Algebraic Approach
- Solving Equations with a Variable in the Denominator - The Algebraic Approach
- Solving Power Equations - The Algebraic Approach
- Solving Quadratic Equations - The Algebraic Approach
  - The Zero Product Property
  - The Quadratic Formula
  - Completing the Square
- Solving Quadratic Type Equations

**(Sections 1.1-1.2)**

1. Which of the following numbers is included in the graph?

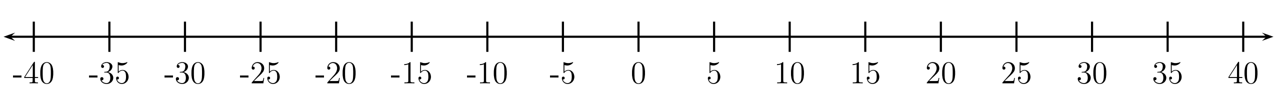



(a) $-5$ (b) $-2$ (c) $0$ (d) $5$ (e) $8$

2. Which of the following numbers are included in the interval $(-\infty, 7) \cup [20, 35)$?

(a) $-2,000,000$
(b) $0$
(c) $6.99999$
(d) $7$
(e) $7.00000001$
(f) $15$
(g) $19.99999$
(h) $20$
(i) $20.00000001$
(j) $24$
(k) $34.99999$
(l) $35$
(m) $35.00000001$
(n) $2,000,000$

3. Sketch the graph of $(-\infty, 7) \cup [20, 35)$.



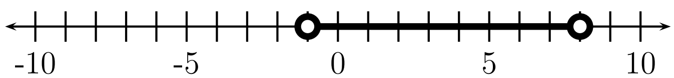
4. Complete the definition of $|x-6|$.

$$|x-6| = \begin{cases} \underline{\hspace{3cm}} & \text{if } x < \underline{\hspace{3cm}} \\ \underline{\hspace{3cm}} & \text{if } x \geq \underline{\hspace{3cm}} \end{cases}$$

5. Complete the definition of $|6-x|$.

$$|6-x| = \begin{cases} \underline{\hspace{3cm}} & \text{if } x < \underline{\hspace{3cm}} \\ \underline{\hspace{3cm}} & \text{if } x \geq \underline{\hspace{3cm}} \end{cases}$$

6. Find the exact value of $|\pi - 6|$. Your answer may not include absolute value symbols.

7. 


   (a) Write a distance sentence that corresponds to this number line.

   (b) Write an algebraic equation or inequality that corresponds to this number line.

8. Solve each equation or inequality algebraically. As you solve the equation or inequality, discuss the geometry (i.e., the number line) behind each step.

   (a) $|x-3| = 4$

   (b) $|x+5| > 2$

   (c) $|4-x| \leq 6$

9. Three pairs of equations are listed below. For each pair, determine if the two equations are equivalent.

   (a) $x + 5 = 2$ and $2x + 10 = 4$
   **CIRCLE ONE:** EQUIVALENT NOT EQUIVALENT

   (b) $x = 2$ and $x^2 = 4$
   **CIRCLE ONE:** EQUIVALENT NOT EQUIVALENT

   (c) $\frac{1}{x} = 5$ and $1 = 5x$
   **CIRCLE ONE:** EQUIVALENT NOT EQUIVALENT

10. Multiplying both sides of an equation by $x^2 + 1$ (always/sometimes/never) produces an equivalent equation.

11. Multiplying both sides of an equation by $|x|$ (always/sometimes/never) produces an equivalent equation.

12. Solve. (Describe the steps that are being applied to the variable. Think about how you will undo these to solve the equation.)

    (a) $4(x-2)^2 - 3 = 0$
    (b) $4(x-2)^2 + 3 = 0$
    (c) $4(x-2)^2 - 3 = 4x^2$
    (d) $\frac{8-2s}{5} = 13$
    (e) $-5[14 - (3x+1)^3] = 11$

13. Solve for $r$.

$$C = 2\pi r$$

14. Solve for $h$.

$$V = \frac{\pi d^2 h}{4}$$

15. Solve for $d$.

$$V = \frac{\pi d^2 h}{4}$$

    This is the formula for the volume of a cylinder. Does this simplify your solution?

16. Solve.

    (a) $\dfrac{x}{x+2} = \dfrac{1}{x-5}$
    (b) $\dfrac{3y^2 - 2y + 14}{y^2 + y - 2} = \dfrac{5}{y-1}$
    (c) $\dfrac{x}{x+2} = \dfrac{5}{x} + 1$

17. How many solutions does each equation have?

$$\textbf{(I)}\ x^3 + 5 = 0 \qquad\qquad \textbf{(II)}\ x^4 = -4$$

    **Possibilities:**

    (a) Equation **(I)** has 3 solutions, and equation **(II)** has no solutions.
    (b) Equation **(I)** has 3 solutions, and equation **(II)** has 1 solution.
    (c) Equation **(I)** has 1 solution, and equation **(II)** has 2 solutions.
    (d) Equation **(I)** has no solutions, and equation **(II)** has 2 solutions.
    (e) Equation **(I)** has 1 solution, and equation **(II)** has no solutions.

18. Solve each equation or inequality or algebraically.

   (a) $3|4x+1| = 5$
   (b) $3|4-x| + 6 = 2$
   (c) $2|x-1| + 4 \leq 8$
   (d) $|5x+7| + 4 > 10$
   (e) $|5x+7| + 4 > 1$

19. Use the Zero Product Property to solve the quadratic equation.

   (a) $x^2 - 14 = 3x + 14$
   (b) $3x^2 + 16x + 5 = 0$

20. Solve the quadratic equation by completing the square.

   (a) $x^2 - 2x = 12$
   (b) $9x^2 = 12x + 1$

21. Solve the quadratic equation by a method of your choice.

   (a) $20x + 35 = 3x^2 + 4x$
   (b) $7x^2 + x + 1 = 0$

22. Find a number $k$ such that the equation has exactly one real solution.

$$x^2 + kx + 25 = 0$$

23. For which values of $k$ does the equation have exactly two real solutions.

$$kx^2 + 8x + 1 = 0$$

24. Solve.

   (a) $2x^6 = 9x^3 + 5$
   (b) $3x^8 + x^4 - 10 = 0$
   (c) $(y-2)^2 + 5(y-2) = 3$

25. We have studied the following techniques for solving equations in class: unwrapping a variable, multiplying by a common denominator, taking roots of both sides of an equation, using the zero product property, completing the square, simplifying, using the quadratic formula, using geometry, and substituting in a quadratic type equation.

    For each of the following equations, determine which technique you could use to solve the equation. There may be more than one or zero techniques.

    (a) $3 - x + 2x^2 = 5 + x$
    (b) $3x^5 - 7 = 2$
    (c) $x^5 + 3\sqrt{x} = 7$
    (d) $\frac{5}{x+2} - \frac{5+x}{2x} = \frac{7x}{x+2}$
    (e) $-4x + 3[5(x+7) - 3x + 2] = 7(x+5)$
    (f) $\frac{1}{x+2} = 5x$
    (g) $x^4 + 2x^2 - 1 = 0$
    (h) $x^4 + 2x - 1 = 0$

    On homework, quizzes, and exams, you will not be told which technique you should use. You should practice identifying techniques that can help you solve a problem.

# 3 The Cartesian Coordinate System Worksheet

**Concepts:**

- The Cartesian Coordinate System
- Graphs of Equations in Two Variables
- $x$-intercepts and $y$-intercepts
- Distance in Two Dimensions and the Pythagorean Theorem
- Equations of Circles
  - The Distance Formula and the Standard Form for an Equation of a Circle.
  - Writing Equations of Circles
  - Identifying Equations of Circles
- Midpoints
  - Finding Midpoints
  - Verifying that a Point Is the Midpoint of a Line Segment
- Steepness
- Rates of Change
- Lines
  - The Slope
  - The Slope as a Rate of Change
  - Linear Equations
  - Point-Slope Form
  - Vertical Lines
  - Horizontal Lines
  - Parallel Lines and Perpendicular Lines
- Using 2-Dimensional Graphs to Approximate Solutions of Equations in One Variable.
  - The Intersection Method
  - The Intercept Method

**(Sections 1.3-1.4)**

1. Is $(3, 2)$ on the graph of $x^2 - y^3 = 1$?
2. Is $(0, 1)$ on the graph of $x^2 - y^3 = 1$?
3. Is $(0, -1)$ on the graph of $x^2 - y^3 = 1$?
4. Find the intercepts of the graph of $x^2 - y^3 = 1$.
5. Find the point on the $x$-axis that is equidistant to $(2, 5)$ and $(-1, 3)$.
6. Find the point on the $y$-axis that is equidistant to $(2, 5)$ and $(-1, 3)$.
7. Find the perimeter of the triangle with vertices $A(-2, -5)$, $B(-2, 7)$, and $C(10, 10)$.
8. Find the area of the triangle with vertices $A(-2, -5)$, $B(-2, 7)$, and $C(10, 10)$.
9. Sketch the graph of the circle defined by $(x + 3)^2 + y^2 = 9$. What are the center and radius of this circle?
10. Is the graph of $x^2 + 6x + y^2 - 10y + 26 = 0$ a circle? If so, find its center and radius.
11. Is the graph of $4x^2 - 8x + 4y^2 + 4y - 23 = 0$ a circle? If so, find its center and radius.
12. Is the graph of $x^2 - 2x + y^2 + 8y + 26 = 0$ a circle? If so, find its center and radius.
13. Describe the graph of $x^2 + 4x + y^2 + 10y + 29 = 0$.
14. A diameter of a circle has endpoints $(1, -2)$ and $(3, 6)$. Find an equation for the circle.
15. The center of a circle is $(5, \frac{1}{4})$, and circle passes through the point $(-2, 3)$. Find an equation for the circle.
16. The center of a circle is $(4, -5)$ and the circle intersects the $x$-axis at 2 and 6. Find an equation for the circle.
17. For each point, determine if the point is inside, outside, or on the circle

$$(x + 5)^2 + (y - 3)^2 = 36.$$

    (a) $(4, 2)$
    (b) $(-5, 0)$
    (c) $(1, 2)$

18. Which of the following are equations for the line through the points $P(1,5)$ and $Q(2,-3)$?

(a) $y+3=-8(x-2)$

(b) $y=-8x-4$

(c) $y=-8(x-1)+5$

(d) $y+3=\dfrac{-1}{8}(x-2)$

(e) $y+3=\dfrac{1}{8}(x-2)$

(f) $y-5=\dfrac{-1}{8}(x-1)$

(g) $y-5=\dfrac{1}{8}(x-1)$

(h) $y-5=-8(x-1)$

(i) $y+5=-8(x+1)$

(j) $y-5=-8x-1$

(k) $y-5=\dfrac{-1}{8}x-1$

19. **TRUE or FALSE:** The line through the points $(0,-1)$ and $(-1,4)$ is perpendicular to the line through the points $(2,-8)$ and $(7,-7)$.

20. **TRUE or FALSE:** The line through the points $(-5,-7)$ and $(-8,-5)$ is parallel to the line through the points $(-7,0)$ and $(-10,2)$.

21. Find an equation for the line that is parallel to $y=\dfrac{5}{6}x+4$ and passes through the point (0,12).

22. Find an equation for the line that is parallel to $y=\dfrac{5}{6}x+7$ and contains the point (3,21).

23. Find an equation for the line that is perpendicular to $y=\frac{5}{6}x+4$ and contains the point (0,14).

24. How many intersection points could a circle and a line have?

25. How many intersection points could the graphs of two lines have?

26. Find the points of intersection between the graphs of $y=x$ and $y=x^2+5$. Sketch the graphs.

27. Find the points of intersection between the graphs of $y=2$ and $y=x^2-3x$. Sketch the graphs.

28. Find the points of intersection between the graphs of $3x+5y=1$ and $2x-10y=7$. Sketch the graphs.

29. This problem was taken from the Precalculus textbook by David H. Collingwood and K. David Prince. It is available at `http://www.math.washington.edu/~m120/`.

    Two planes flying opposite directions (North and South) pass each other 80 miles apart at the same altitude. The northbound plane is flying 200mph (miles per hour) and the southbound plane is flying 150 mph. How far apart are the planes in 20 minutes? When are the planes 300 miles apart?

30. This problem was taken from the Precalculus textbook by David H. Collingwood and K. David Prince. It is available at `http://www.math.washington.edu/~m120/`.

A spider is located at the position $(1, 2)$ in a coordinate system where units on each axis are feet. An ant is located at the position $(15, 0)$ in the same coordinate system. Assume the location of the spider after $t$ minutes $s(t) = (1 + 2t, 2 + t)$ and the location of the ant after $t$ minutes is $a(t) = (15 - 2t, 2t)$.

(a) Sketch a picture of the situation indicating the locations of the spider and ant at times $t = 0, 1, 2, 3, 4, 5$ minutes. Label the locations of the bugs in your picture using the notation $s(0), s(1), \ldots, s(5)$, $a(0), a(1), \ldots, a(5)$.

(b) When will the $x$-coordinate of the spider equal 5? When will the $y$-coordinate of the ant equal 5?

(c) Where is the spider located when its $y$-coordinate is 3?

(d) Where is each bug located when the $y$-coordinate of the spider is twice as large as the $y$-coordinate of the ant?

(e) How far apart are the bugs when their $x$-coordinates coincide? Draw a picture indicating the locations of each bug when their $x$-coordinates coincide.

(f) A sugar cube is located at the position $(9, 6)$. Explain why each bug will pass through the position of the sugar cube. Which bug reaches the sugar cube first?

# 4 Using Technology Wisely Worksheet

**Concepts:**

- Advantages and Disadvantages of Graphing Calculators
- How Do Calculators Sketch Graphs?
- When Do Calculators Produce Incorrect Graphs?
- The Greatest Integer Function
- Graphing Calculator Skills
  - Locating the Graph (TRACE AND TABLE)
  - Changing the Viewing Window (WINDOW)
  - Connected Mode vs. Dot Mode
  - The ZOOM Features
  - Finding Approximate Coordinates for Intersection Points
  - Finding Approximate Coordinates for $x$-intercepts
- The Intersection Method Revisited
- The Intercept Method Revisited

**(Sections 2.1-2.2)**

1. Sketch a complete graph of each equation. Make sure that you label the axes for each graph. Which graphs can you draw without the assistance of a calculator? If you used your calculator to help you sketch a graph, what feature(s) of the calculator helped you to sketch a reasonable graph?

   (a) $y = 3x + 7$

   (b) $4x^2 + 9y^2 = 36$

   (c) $y = \dfrac{2}{x^2 - 4}$

   (d) $y = 5x^7 - 20x^4 + 3x^2 - 8$

   (e) $y = 3^{x^2+5}$

   (f) $y = [\![\sqrt{x}]\!]$

   (g) $y = \dfrac{1}{\sqrt{x-7}}$

2. Which of the following equations should be solved algebraically and which should be solved graphically? Solve each equation. If the solution you find is approximate, be sure to indicate that it is approximate with $\approx$. If you use a graph to find a solution, be sure to sketch the graph and label it.

   (a) $3x^6 = 5x^3 + 2$

   (b) $\dfrac{x}{x^5+1} = 2$

   (c) $\sqrt{x+7} = x$

   (d) $x^3 + 15 = 5x^2 + 3x$

   (e) $x^3 + 15 = 5x^2 + 3$

3. Compare and contrast each pair of graphs.

   (a) $y = x^3$ and $y = (x+4)^3$

   (b) $y = |x|$ and $y = |x-1|$

   (c) $y = |x|$ and $y = |x|$

   (d) $y = [\![x]\!]$ and $y = [\![2x]\!]$

   (e) $y = [\![x]\!]$ and $y = 2[\![x]\!]$

   (f) $y = x^2$ and $y = |x^2|$

4. We can use algebraic properties to prove that

$$(x+3)^2 = x^2 + 6x + 9$$

for all $x$ values. We can see further evidence of this property by looking at the graphs of $y = (x+3)^2$ and $y = x^2 + 6x + 9$. Graph $y = (x+3)^2$ and $y = x^2 + 6x + 9$ in the same viewing rectangle. What do you notice?

You should note, that we needed Algebra to prove that $(x+3)^2 = x^2 + 6x + 9$ for all $x$ values. The graphs do not prove the identity. They only provide some encouraging evidence that the identity is true.

On the other hand, consider the graphs of $y = x^2 + 4$ and $y = (x+2)^2$. Sketch these graphs in the same viewing rectangle. These are very different graphs. They do intersect at the point $(0, 2)$, but they are very different at all other points. In general $x^2 + 4 \neq (x+2)^2$ unless $x = 0$.

Use your knowledge of algebra and graphs to determine which of the following equations definitely are identities, which might be identities and which definitely are not identities. Use graphical evidence to support your conclusion.

| | Definitely An Identity | Might Be An Identity | Not An Identity |
|---|---|---|---|
| $(x+1)^3 = x^3 + 3x^2 + 3x + 1$ | | | |
| $\|x-3\| = \|x\| + 3$ | | | |
| $\|x-3\| = \|x\| - 3$ | | | |
| $\sqrt{x^2} = x$ | | | |
| $\sqrt{x^2+1} = x + 1$ | | | |
| $\sqrt{x^2+1} = x + 1$ | | | |
| $\sqrt{x^2+1} = \|x+1\|$ | | | |
| $\sqrt{x^2+2x+1} = \|x+1\|$ | | | |
| $\frac{1}{x^2} + \frac{1}{x} = \frac{1}{x^2+x}$ | | | |
| $\frac{1}{x^2} + \frac{1}{x} = \frac{1+x}{x^2}$ | | | |

5. Show that the equation $x^2 - 2x - 6 = \sqrt{2x+7}$ has two real solutions by graphing the left and right sides in the standard window and counting the number of intersection points. Approximate the solutions. (Make sure you use $\approx$ for approximate solutions.)

6. Look for solutions of
$$\sqrt{x^4 - 5x^2 + 1} = 0$$
graphically. Solve the equation algebraically.

7. What would you enter in your calculator if you wanted the graph of $2x - 4(y + 3)^4 + 1 = 0$?

# 5 Linear Mathematical Models

**Concepts:**

- Construct a Linear Model
- Gauge the Accuracy of a Linear Model Using Residuals
- Use Linear Regression

**(Section 2.5)**

1. A teacher sent students out to find *round* object and to measure the diameter and the circumference of each item. When the class returned and the data were put on the board, we have the following table:

| Object | Diameter (cm) | Circumference (cm) |
|---|---|---|
| glass | 8.3 | 26.5 |
| flashlight | 5.2 | 16.7 |
| Aztec calendar | 20.2 | 61.6 |
| Tylenol bottle | 3.4 | 11.6 |
| Popcorn can | 13 | 41.4 |
| Salt shaker | 6.3 | 20.1 |
| Coffee canister | 11.3 | 35.8 |
| Cat food bucket | 33.5 | 106.5 |
| Dinner plate | 27.3 | 85.6 |
| Ritz cracker | 4.9 | 15.5 |

   (a) Find the equation for the least squares regression line for this data.
   (b) What is the slope of this line? What does it measure?
   (c) What is the $y$-intercept of this line? What should it be?

2. A 190° cup of coffee is placed on a desk in a 72ř room. The data in the following are from a simulated experiment of gathering temperature readings from a cup of coffee in a 72° room at 20 one-minute intervals.

| Time | Temp | Time | Temp |
|---|---|---|---|
| 1 | 184.3 | 11 | 140.0 |
| 2 | 178.5 | 12 | 136.1 |
| 3 | 173.5 | 13 | 133.5 |
| 4 | 168.6 | 14 | 130.5 |
| 5 | 164.0 | 15 | 127.9 |
| 6 | 159.2 | 16 | 125.0 |
| 7 | 155.1 | 17 | 122.8 |
| 8 | 151.8 | 18 | 119.9 |
| 9 | 147.0 | 19 | 117.2 |
| 10 | 143.7 | 20 | 115.2 |

(a) Produce a scatter plot of the temperature $(y)$ as a function of time $(x)$.

(b) Find the linear regression equation for this data. Round the coefficients to the nearest 0.001.

(c) It is known that this phenomenon is **not** linear. What are the $x$-intercepts and the $y$-intercepts?

(d) What does the $x$-intercept mean, physically?

3. The average hourly earnings of U. S. production workers for 1990–2007 are shown in the table below.

| Year | Average Hourly Earnings ($) |
|---|---|
| 1990 | 10.20 |
| 1991 | 10.52 |
| 1992 | 10.77 |
| 1993 | 11.05 |
| 1994 | 11.34 |
| 1995 | 11.65 |
| 1996 | 12.04 |
| 1997 | 12.51 |
| 1998 | 13.01 |
| 1999 | 13.49 |
| 2000 | 14.02 |
| 2001 | 14.54 |
| 2002 | 14.97 |
| 2003 | 15.37 |
| 2004 | 15.69 |
| 2005 | 16.13 |
| 2006 | 16.76 |
| 2007 | 17.42 |

(a) Produce a scatter plot of the hourly earnings $(y)$ as a function of years since 1990 $(x)$.

(b) Find the linear regression equation for the years 1990–1998. Round the coefficients to the nearest 0.001.

(c) Find the linear regression equation for the years 1990–2007. Round the coefficients to the nearest 0.001.

(d) Use both lines to predict the hourly earnings for the year 2010. How different are the estimates? Which do you think is a safer prediction of the true value?

(e) Look online and find the average hourly earnings of U. S. production workers for 2010. Which was a better estimate?

4. The table shows the size of a room air conditioner (in BTUs) needed to cool a room of the given area (in square feet).

| Room size | BTUs |
|---|---|
| 150 | 5000 |
| 175 | 5500 |
| 215 | 6000 |
| 250 | 6500 |
| 280 | 7000 |
| 310 | 7500 |
| 350 | 8000 |
| 370 | 8500 |
| 420 | 9000 |
| 450 | 9500 |

(a) Find a linear model for the data.

(b) Use the model to find the number of BTUs required to cool a rooms of size 150 sq ft, 280 sq ft, and 420 sq ft. How well do the model estimates agree with the actual data values?

(c) Use the model to estimate how many BTUs are needed to cool a 235 sq ft room. If air conditioners are available only with the BTU choices in the table, which size should be chosen?

5. The projected number of scheduled passengers on U. S. commercial airlines (in billions) is given in the following table.

| Year | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 |
|---|---|---|---|---|---|---|---|---|
| Passengers | .63 | .64 | .69 | .72 | .77 | .79 | .8 | .84 |

(a) Find a linear model for this data, with $x = 2$ corresponding to 2002.

(b) Estimate the number of passengers in 2012 and 2016.

(c) Find the equation of the line through the first data point and the last data point.

(d) Compute the sum of the squares of the residuals for this line.

(e) Compute the sum of the squares of the residuals for the line of best fit. Compare with above. Was the work worth it?

# 6 Functions and Functional Notation

**Concepts:**

- The Definition of A Function
- Function Notation
- Piecewise-defined Functions
  - Evaluating Piecewise-defined Functions
  - Sketching the Graph of a Piecewise-defined Functions
- The Domain of a Function

**(Sections 3.1-3.2)**

1. The amount of postage required to mail a first-class letter is determined by its weight. In this situation, is weight a function of postage? Or vice versa? Or both?

2. An epidemiological study of the spread of malaria in a rural area finds that the total number $P$ of people who contracted malaria $t$ days into an outbreak is modeled by the function
$$P(t) = -\frac{1}{4}t^2 + 7t + 180, \qquad 1 \leq t \leq 14.$$
   (a) How many people have contracted malaria 14 days into the outbreak?
   (b) How many people have contracted malaria 6 days into the outbreak?

3. In the following identify the independent variable (input) and the dependent variable (output).
   (a) The amount of property tax you owe is a function of the assessed value of your home in dollars.
   (b) The length of your fingernails is a function of the amount of time that has passed since your last manicure.
   (c) The cost of mailing a letter is a function of the weight of the package in ounces.
   (d) The amount of water required for your lawn (in gallons) is a function of the temperature (in degrees).
   (e) A person's blood alcohol level is a function of the number of alcoholic drinks consumed in a 2-hour period.

4. The number of recreational visits to the National Parks of the United States is displayed in the table. The number of visits to the national parks, $p$, is a function of the year, $t$.

| Year | Recreational Visits to US National Parks (millions of people) |
|---|---|
| 1990 | 258.7 |
| 1995 | 269.6 |
| 1999 | 287.1 |
| 2000 | 285.9 |
| 2001 | 279.9 |
| 2002 | 277.3 |
| 2003 | 266.1 |
| 2004 | 276.4 |

Source: www.census.gov

   (a) Solve $p(t) = 277.3$ for $t$ and explain the meaning of the solution.

   (b) Evaluate $p(2000)$ and write a sentence explaining what the numerical value you find means in its real-world context.

   (c) Estimate $p(2010)$ and discuss the accuracy of your prediction.

   (d) Estimate the solution to $p(t) = 300$ and discuss the accuracy of your approximation.

5. Evaluate the given function at the given values:

   (a) $f(x) = x^3 + 2x$; $f(-2), f(-1), f(0), f(\frac{1}{2})$

   (b) $g(t) = \dfrac{t+2}{t-2}$; $g(-2), g(2), g(0), g(a), g(a^2-2), g(a+1)$

   (c) $h(u) = 2|u-1|$; $h(-2), h(0), g(\frac{1}{2}), h(2), h(x+1), h(x^2+2)$

   (d) $f(x) = \dfrac{|x|}{x}$; $f(-2), f(-1), f(0), f(5), f(w^2), f\left(\dfrac{1}{w}\right))$

6. Evaluate the given piecewise defined function at the given values:

   (a) $f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ x+1 & \text{if } x \geq 0 \end{cases}$; $f(-2), f(-1), f(0), f(1), f(2)$

   (b) $g(u) == \begin{cases} u^2 + 2u & \text{if } u \leq -1 \\ u & \text{if } -1 < u \leq 1 \\ -1 & \text{if } u > 1 \end{cases}$; $g(-4), g(-\frac{3}{2}), f(-1), f(0), f(25)$

7. According to `http://revenue.ky.gov/`, the tax brackets for the 2015 Kentucky state taxes are described below.

   If your taxable income on Form 740, line 11 is:

| more than | but not more than | then your tax is | plus: |
|---|---|---|---|
| \$0 | \$3,000 | 2.00% of your taxable income | \$0 |
| \$3,001 | \$4,000 | 3.00% of the amount over \$3,000 | \$60 |
| \$4,001 | \$5,000 | 4.00% of the amount over \$4,000 | \$90 |
| \$5,001 | \$8,000 | 5.00% of the amount over \$5,000 | \$130 |
| \$8,001 | \$75,000 | 5.80% of the amount over \$8,000 | \$280 |
| \$75,001 | | 6.00% of the amount over \$75,000 | \$4,160 |

   They give the following example.

   Taxable income \$6,800. Tax = (\$6,800-\$5,000) $\times$ .05(5%) + \$130 = \$220.

   Use this tax table to write a piecewise-defined function $KYTax(I)$ where $I$ is the adjusted gross income on Form 740 line 11 of the Kentucky tax form 740, and $KYTax(I)$ is the amount of tax owed by a resident of Kentucky.

8. Let $f(x) = x^2 + 1$.
   (a) What is $f(a+b)$?
   (b) What is $f(x-1)$?

9. Let $g(x) = x^2 + x$.
   (a) What is $\dfrac{g(2x)}{2g(x)}$?
   (b) What is $g(x^2)$?
   (c) What is $(g(x))^2$?
   (d) What is $\dfrac{g(x+h) - g(x)}{h}$?

10. Let

$$h(x) = \begin{cases} 10 & \text{if } x < -4 \\ x^2 + 10 & \text{if } -4 \leq x \leq 6 \\ x + 15 & \text{if } x > 6 \end{cases}$$

   (a) Find $h(5)$.
   (b) Find $h(-4)$.
   (c) Find $h(-6)$.
   (d) Find $h(6)$.
   (e) Find $h(10)$.

11. Find the domain of each of the following functions. Write the domain in interval notation.
    (a) $a(x) = x^5 + 2x^2 - 6$
    (b) $b(x) = \dfrac{x+1}{x-5} + \dfrac{x+4}{2x+1}$
    (c) $c(x) = \sqrt[3]{x+7}$
    (d) $d(x) = \sqrt{x+7}$
    (e) $e(x) = \dfrac{1}{\sqrt[3]{10-x}}$
    (f) $f(x) = \dfrac{1}{\sqrt[4]{10-x}}$
    (g) $g(x) = \sqrt{x+7} - \dfrac{1}{x^2-5}$
    (h) $h(x) = \begin{cases} \dfrac{1}{x} & \text{if } x \leq -2 \\ \dfrac{1}{x+3} & \text{if } x > -2 \end{cases}$

12. To graph the function $f$ we plot the points $(x, \_\_\_)$ in a coordinate plane.To graph $f(x) = x^2 - 2$, we plot the points $(x, \_\_\_)$. So the point $(3, \_\_\_)$ is on the graph of $f$. The height of the graph of $f$ above the $x$-axis when $x = 3$ is ___.

13. Sketch graphs of the following functions:
    (a) $f(x) = |x| + x$
    (b) $g(x) = |x| - x$
    (c) $h(x) = x|x|$
    (d) $f(x) = x/|x|$
    (e) $g(x) = x - [|x|]$
    (f) $h(x) = x[|x|]$
    (g) $f(x) = \begin{cases} -1 & \text{if } x < -1 \\ x & \text{if } -1 \leq x \leq 1 \\ 1 & \text{if } x > 1 \end{cases}$

# 7 Graphing Functions

**Concepts:**

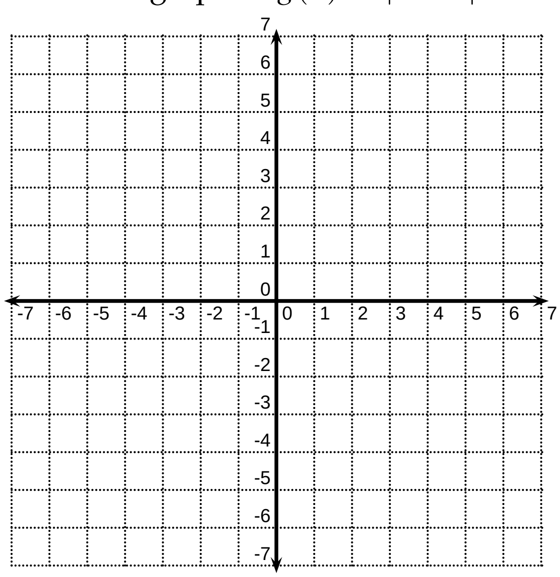
- The Domain of a Function
- Graphs of Functions
  - Identifying Graphs of Functions (Vertical Line Test)
  - Interpreting Graphs of Functions
  - Sketching Graphs of Functions
  - Relative Maximums and Minimums

**You are responsible for graphs of basic functions.** You will need to know how to graph some basic functions without the help of your calculator.

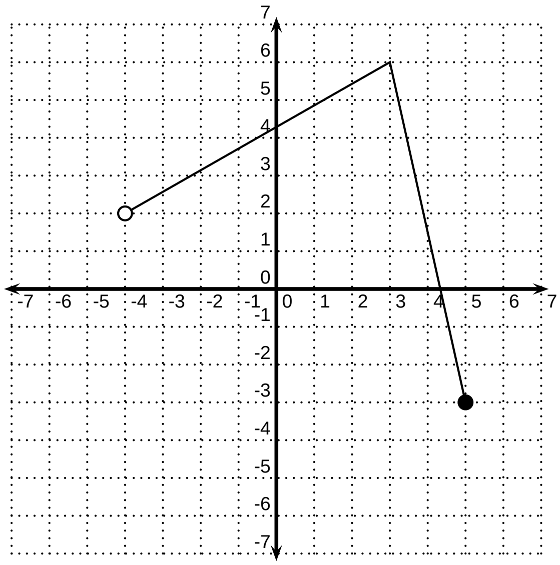
- Linear Functions ($f(x) = mx + b$)
- Power Functions ($f(x) = x^n$) where $n$ is a positive integer.
- Square Root Function ($f(x) = \sqrt{x}$)
- Greatest Integer Function ($f(x) = [\![x]\!]$)
- Absolute Value Function ($f(x) = |x|$)
- Piecewise-defined Functions.

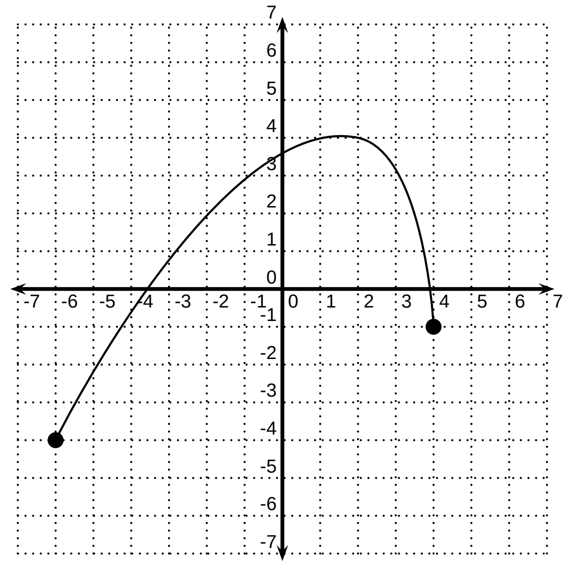
**(Section 3.3)**

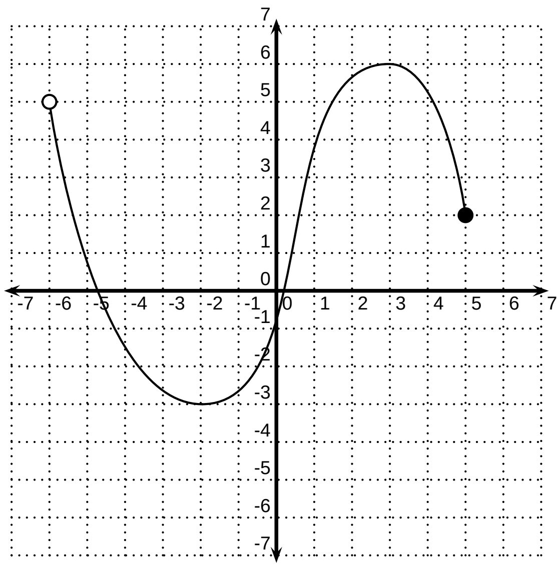
1. Sketch the graph of $g(x) = |x+2|$.

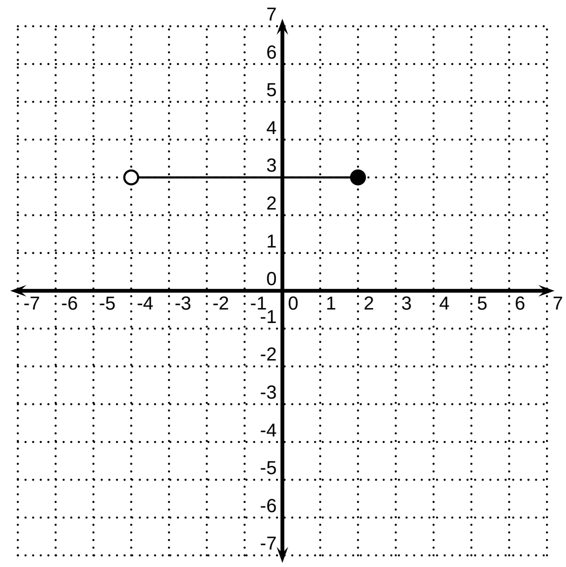


2. Find the domain and range of each of the following functions.



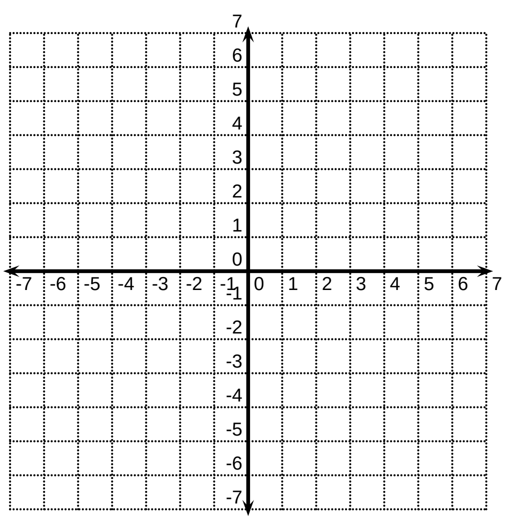




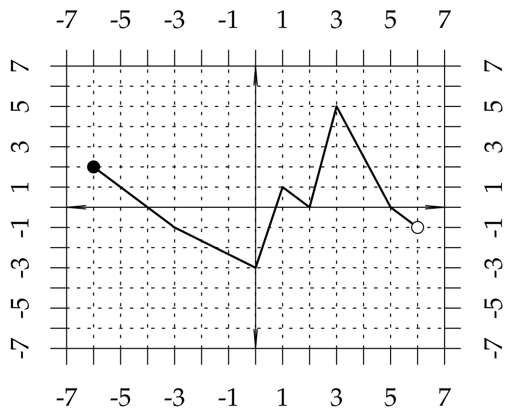


3. Sketch the graph of

$$f(x) = \begin{cases} 3x - 2 & \text{if } x \leq -3 \\ -x^2 + 4 & \text{if } x > -3 \end{cases}$$



4. The graph of $y = f(x)$ is shown below.



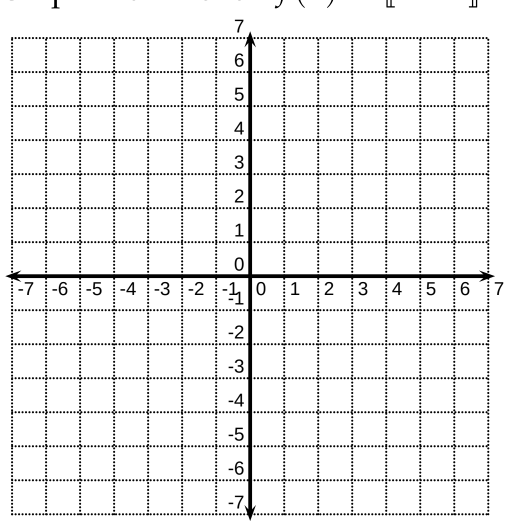
(a) For what $x$ values is $f(x) \geq 0$? Write your answer in interval notation.

(b) For what $x$ values is $f(x) < 0$? Write your answer in interval notation.

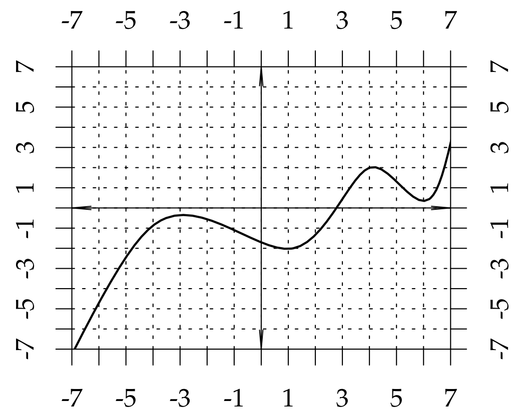
(c) For what $x$ values is $f(x) \leq -1$? Write your answer in interval notation.

(d) What is $\dfrac{f(3) - f(2)}{2f(-6)}$?

5. Graph the function $f(x) = [\![x - 2]\!]$.



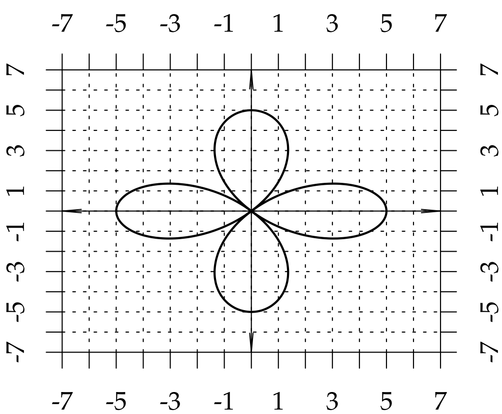
6. Find the $x$ values of all local maxima and minima.

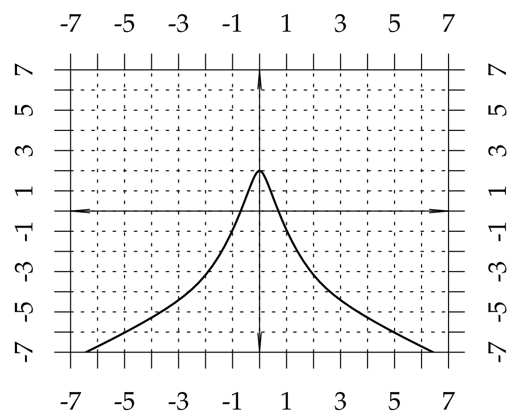


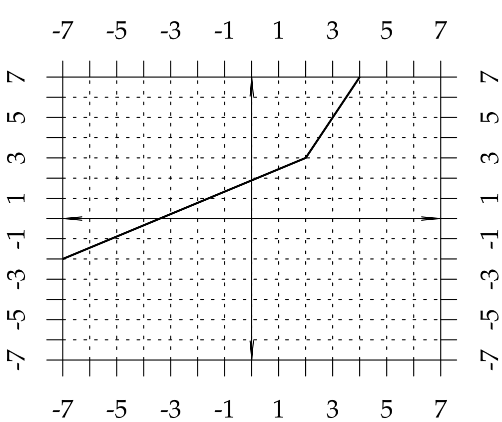
7. For each of the graphs below, answer the following questions:

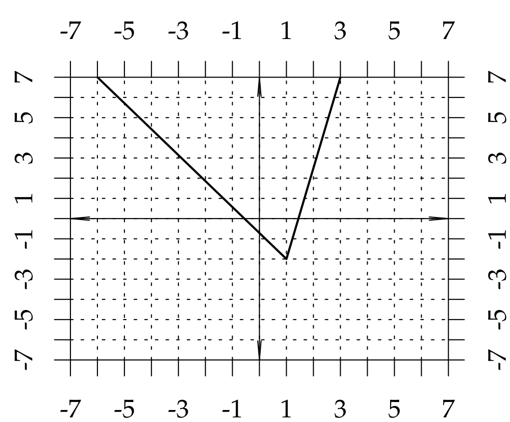
   A. Is $y$ a function of $x$?

   B. Is $x$ a function of $y$?









**Ma 110 Exam 1 Review:** Sections 1.1 - 1.4, 2.1 - 2.2, 2.5, 3.1 - 3.3

**Do not rely solely on this work sheet!** **Make sure to study homework problems, other work sheets, lecture notes, and the book!!!**

1. Section 1.1
   (a) Translate the geometric statement, "the distance from $x$ to $-5$ on the number line is greater than 3", into an algebraic statement with absolute values.
   (b) Translate the algebraic statement, $|x+3| \le 4$, into a geometric statement about distance on the number line.
   (c) Indicate the solution to $|x+3| \le 4$ on the number line.
   (d) Write the number $|\sqrt{6} - 5|$ with out using absolute values.
   (e) Find the distance between $-\frac{1}{5}$ and $-6$ exactly.
2. Section 1.2
   (a) Solve algebraically: $|x+3| \le 4$
   (b) Solve for $y$ **exactly**: $7[(2y-1)^3 - 5] = -21$
   (c) Solve by completing the square: $3x^2 + 5 = 12x$
   (d) Solve using any method: $4x - 10 = 14 - x^2$
   (e) Solve: $\frac{3}{x} = 1 - \frac{x}{x-3}$
   (f) Solve: $3z^2 = 2 - 4z^4$
3. Section 1.3
   (a) Find the distance in the Cartesian plane between the points $(-1, 5)$ and $(3, -9)$
   (b) Find the center and radius of the circle $x^2 + 4x + y^2 - 3x - 10 = 0$.
   (c) Find the midpoint of the line segment joining $(1, 3)$ and $(-3, 5)$.
   (d) Find the $x$- and $y$-intercepts of the graph of $x^2 - 2xy + 3y^2 = 1$.
   (e) Find the equation of the circle if the endpoints of the diameter are $(-3, 5)$ and $(7, -5)$.
4. Section 1.4
   (a) Find the equation of the line through $(-1, 5)$ and perpendicular to the line $2x+3y-2 = 0$.
   (b) Find the equation of the line through $(-1, 5)$ and parallel to the line $2x + 3y - 2 = 0$.
   (c) Find the rate of change $\frac{\Delta y}{\Delta x}$ for the line $y - 5 = -2(x+6)$
   (d) Find the equation of the line through the points $(-3, -5)$ and $(2, -6)$ in point slope form.
   (e) Find a number $k$ such that the slope of the line passing through the two points $(k, -3)$; $(-5, 8)$ is equal to $-7$.

5. Section 2.1 & 2.2

   (a) Sketch a complete graph of the equation, $3x^2 + 5y^2 = 15$, making certain to label your axes.

   (b) Determine the number of solutions to the equation $\sqrt{3x+8} + 3x = x^2 - 7$, and find the approximate solutions graphically.

   (c) Justify whether the equation $x^3 - 2\sqrt{x} = 5$ should be solved graphically or algebraically.

   (d) Find an approximate solution to the equation $\sqrt[4]{x^4 + 3x^2 - 3} = 0$ graphically and find an exact answer algebraically.

6. Section 2.5

   (a) Advertising expenditures in the United States (in billions of dollars) in selected years are shown in the table below. Two models are $y = 12x + 215$ and $y = 11x + 218$, where $x = 0$ corresponds to 2000.

| Year | 2001 | 2002 | 2003 | 2004 |
|---|---|---|---|---|
| Amount | 231 | 237 | 245 | 264 |

      i. Find the residuals and their sum;
      ii. Find the sum of the squares of the residuals;
      iii. Determine which model is the better fit.

   (b) Enrollment in public colleges (in thousands) in selected years is shown in the table below.

| Year | 2000 | 2001 | 2002 | 2004 | 2006 | 2008 |
|---|---|---|---|---|---|---|
| Amount | 15,313 | 15,928 | 16,612 | 17,095 | 17,664 | 18,350 |

      i. Find a linear model for this data, with $x = 0$ corresponding to 2000.
      ii. Use the model to estimate public college enrollment in 2005 and 2010.
      iii. According to this model, when will public college enrollment reach 21 million?

7. Section 3.1

   (a) Does the equality $x^2 - 3y = 6$ express $x$ as a function of $y$ or $y$ as a function $x$.

   (b) Does the table represent a function? If so find the domain and range.

| input | -1 | 5 | -4 | $\pi$ | 0 |
|---|---|---|---|---|---|
| output | 0 | 1.1 | 1.1 | $\theta$ | 3 |

   (c) Find an equation that expresses the area $A$ of a circle as a function of the radius $r$.

   (d) A group of students drives from Cleveland to Seattle, a distance of 2350 miles, at an average speed of 52 mph.

      i. Express their distance from Cleveland as a function of time.
      ii. Express their distance from Seattle as a function of time.

8. Section 3.2

   (a) For the function $f(x) = 5x^2 + 1$, find $f(1)$, $f(x+1)$, and $f(\Omega)$.

   (b) Find the difference quotient for the function $f(x) = \sqrt{x}$ and simplify.

   (c) Find the difference quotient for the function $f(x) = 2 - x^2$ and simplify.

(d) For the function:

$$f(n) = \begin{cases} 2x+3 & \text{if } x < 4 \\ x^2 - 1 & \text{if } 4 \le x \le 10 \end{cases}$$

Find $f(-5)$, $f(8)$, $f(k)$, and the domain of $f$.

(e) Find the domain of the function $f(x) = \dfrac{1}{x-2}$

(f) Find the domain of the function $f(x) = \dfrac{1}{\sqrt{x+2}}$

(g) Find the domain of the function $f(x) = \dfrac{\sqrt{x-2}}{x}$.

9. Section 3.3

(a) Make a sketch of the graphs $f(x) = x^n$ for $n$ even and for $n$ odd. Do not use a scale on your graphs.

(b) Sketch the graphs of $f(x) = mx + b$, $f(x) = |x|$, the Greatest Integer Function, and $f(x) = \sqrt{x}$

(c) Which of the following graphs pass the vertical line test?

i. $y = 3x^2 + 2$

ii. $x = 3y - 6$

iii. $x = 3y^2 + 2$

(d) Find the domain and range for each of the functions whose graphs are shown below.