9 Transformations and Composition

Concepts:

- Graphs of functions
- Sketching graphs of functions
- Applying transformations to the graph of a function
- How does a graph transformation move a point on a graph?
- Operations on functions
- The domain of a composition of functions.

(Section 3.4 & 3.5)

1. (Do you understand graph transformations?) Suppose that the graph of f contains the point $(-4, 7)$. Find a point that must be on the graph of g. Explain how you had to move the point on the original graph f to obtain a point on the new graph g.

   (a) The graph of $g(x) = f(x) + 5$ must contain the point ____________.

   (b) The graph of $g(x) = f(x + 5)$ must contain the point ____________.

   (c) The graph of $g(x) = f(x) - 5$ must contain the point ____________.

   (d) The graph of $g(x) = f(x - 5)$ must contain the point ____________.

   (e) The graph of $g(x) = 5f(x)$ must contain the point ____________.

(f) The graph of $g(x) = f(5x)$ must contain the point ______________.

(g) The graph of $g(x) = \frac{1}{5}f(x)$ must contain the point ______________.

(h) The graph of $g(x) = f\left(\frac{1}{5}x\right)$ must contain the point ______________.

(i) The graph of $g(x) = f(7x) + 5$ must contain the point ______________.

(j) The graph of $g(x) = 3f(7x + 1) + 5$ must contain the point ______________.

(k) **A Challenge:** The graph of $g(x) = 3(f(7(x+1)) + 5)$ must contain the point ______________.

2. Let $f(x) = x^2 + 3$ and $g(x) = 2 - x$.
   (a) Find $f(g(x))$.
   (b) Find $g(f(x))$.
   (c) Find $f(f(x))$.
   (d) Find $g(g(x))$.
   (e) Find $g(g(g(x)))$.

3. Let $f(x) = \dfrac{x}{\sqrt{x+1}}$ and $g(x) = 2x + 5$.
   (a) Find $f(g(x))$.
   (b) Find the domain $f(g(x))$.
   (c) Find $g(f(x))$.
   (d) Find the domain $g(f(x))$.

4. Write $h(x)$ as a composition of three simpler functions. (**HINT:** Think of placing x in a box. What happens first? second? *etc.*? There may be more than one correct answer.)
   (a) $h(x) = \sqrt{x^3 + 5}$
   (b) $h(x) = \dfrac{3}{x^5 - 7}$
   (c) $h(x) = 3(x+5)^2$
   (d) $h(x) = (3x+5)^2$

5. Write $g(x) = 2(f(3(x+1)) - 6)$ as a composition of five simpler functions (Hint:One of these functions should be f.)

6. You have a 20% off coupon from the manufacturer good for the purchase of a new cell phone. Your cell provider is also offering a 10% discount on any new phone. You make two trips to cell phone stores to look at various phones. On your first trip, you speak with Miranda. Miranda tells you that you can take advantage of both the coupon and the discount. She will apply the discount and then apply the coupon to the reduced price. On your second trip, you talk to Ariel. She also says that you can take advantage of both deals, but she tells you that she will apply the coupon and then apply the discount.

   Let x represent the original sticker price of the cell phone.
   (a) Suppose that only the 20% discount applies. Find a function f that models the purchase price of the cell phone as a function of the sticker price x.
   (b) Suppose that only the 10% coupon applies. Find a function g that models the purchase price of the cell phone as a function of the sticker price x.
   (c) If you can take advantage of both deals, then the price you will pay is either $f(g(x))$ or $g(f(x)$, depending on the order in which the coupon and the discount are applied to the price. Find $f(g(x))$ and $g(f(x))$.
   (d) The price that Miranda is offering you is modeled by _____________.
   (e) The price that Ariel is offering you is modeled by _____________.

10 Inverse and Quadratic Functions

Concepts:

- Identifying Inverse Functions
- The Definition of an Inverse Function
- Inverse Function Notation
- Finding Formulas for Inverse Functions
- Evaluating Inverse Functions
- Graphs of Inverse Functions
- The Round-Trip Theorem
- Determine Graph and Algebraic Form of a Quadratic
- Understand the Meaning of the Vertex

(Section 3.7 & 4.1)

1. Which of the following functions are one-to-one?

   (a) The function that maps a word to the number of letters in the word.

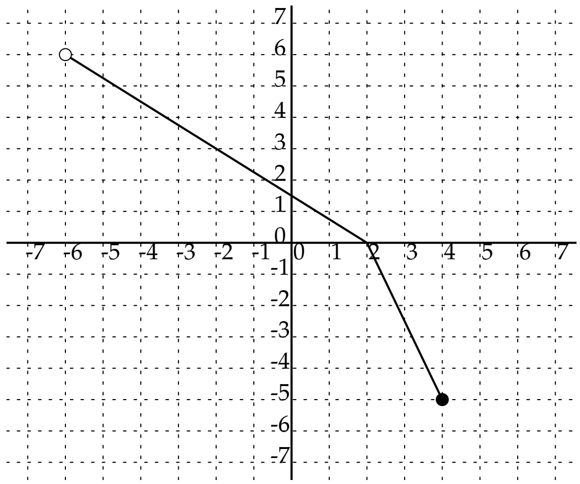
   (b) The function that maps the year of a Summer Olympics to the winner of the marathon in that Olympics.

   (c) The function that maps a U.S. state to its two letter postal code.

   (d) The function that maps a person to his or her name.

   (e) The function that maps a person to his or her address.

2. The graph of a one-to-one function is shown below. (How do you know that this is a one-to-one function?) Sketch the graph of its inverse on the same set of axes.



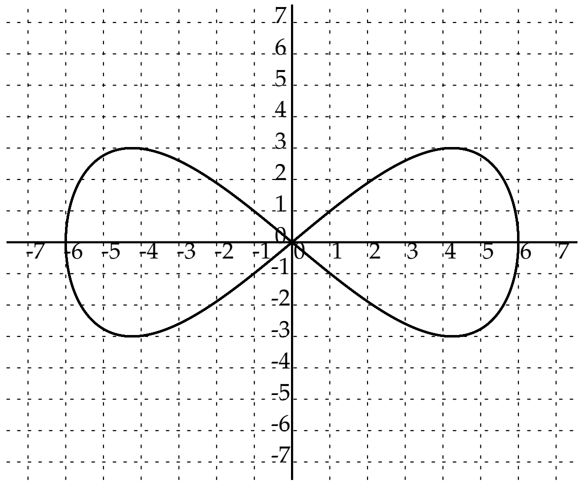
3. Find the inverse of the one-to-one functions below. Find the domains and ranges of the function and its inverse.

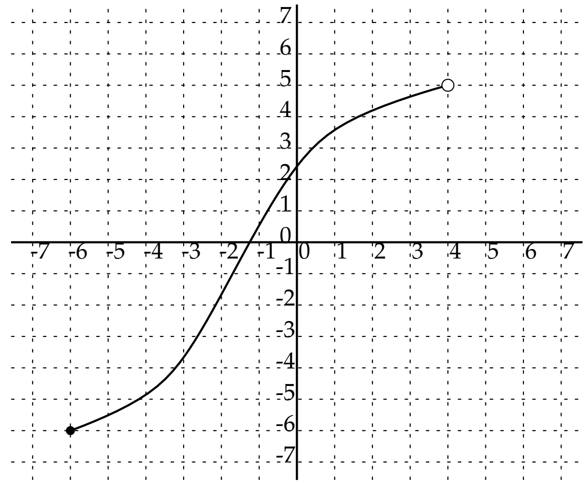
   (a) $f(x) = \dfrac{2 - x^3}{7}$

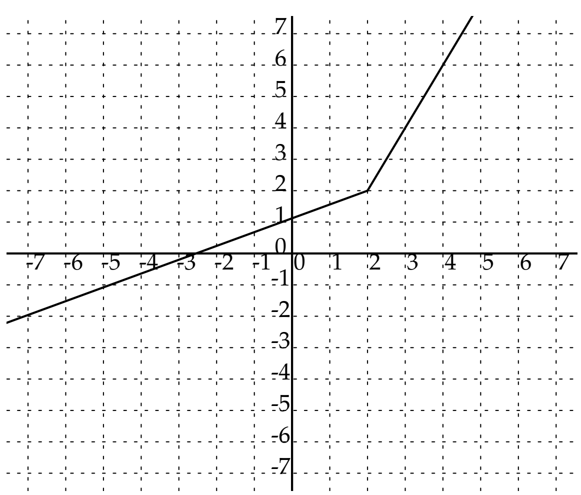
   (b) $g(x) = \dfrac{x+7}{x+5}$

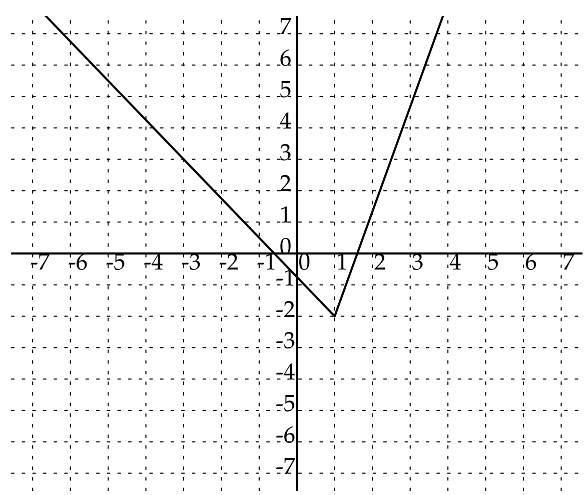
   (c) $h(x) = \sqrt{x^5 - 2}$

4. The following graphs display a one-to-one function? If the graph displays a one-to-one function, sketch its inverse.









5. Use composition of functions to determine if the pair of functions are inverses of each other.

   (a) $f(x) = \dfrac{1}{x}$ and $g(x) = \dfrac{1}{x}$

   (b) $h(x) = \dfrac{3x+2}{7}$ and $j(x) = \dfrac{7x-2}{3}$

   (c) $k(x) = \sqrt{3}x - 3$ and $m(x) = x^3 + 27$

6. Find the equation of the unique quadratic function with the following properties and graph the function.

   (a) passes through $(-2, 0)$ and $(5, 0)$ and has a leading coefficient of 5.

   (b) passes through the points $(3, 0)$, $(-2, 0)$ and $(0, 12)$.

   (c) passes through $(0, 0)$, $(1, -1)$, and $(2, 0)$.

   (d) passes through $(0, 0)$, $(1, -2)$, and $(2, 0)$.

   (e) passes through $(0, 0)$, $(1, -3)$, and $(2, 0)$.

   (f) has vertex $(2, 4)$ and passes through the point $(0, -2)$.

7. A golf ball is hit so that its height h in feet after t seconds is $h(t) = -16t^2 + 60t$.
   (a) What is the initial height of the golf ball?

   (b) How high is the golf ball after 1.5 seconds?

   (c) Find the maximum height of the golf ball *algebraically*.

8. The accompanying table gives the number of homicides per 100,000 population in the United States (*Federal Bureau of Investigation*, `www.fbi.gov`).
   (a) Use quadratic regression on a graphing calculator to express the number of homicides as a function of x, where x is the number of years after 1994.

   (b) Plot the data and the quadratic function on your calculator. Judging from what you see, does the function appear to be a good model for the data?

   (c) Use the quadratic function to find the year in which the number of homicides was at a minimum.

   (d) Use the quadratic function to find the year in which there will be 10 homicides per 100,000 population.

Year	Homicides per 100,000	Year	Homicides per 100,000
1994	9.0	2001	5.6
1995	8.2	2002	5.6
1996	7.4	2003	5.7
1997	6.8	2004	5.5
1998	6.3	2005	5.6
1999	5.7	2006	5.7
2000	5.5	2007	5.6

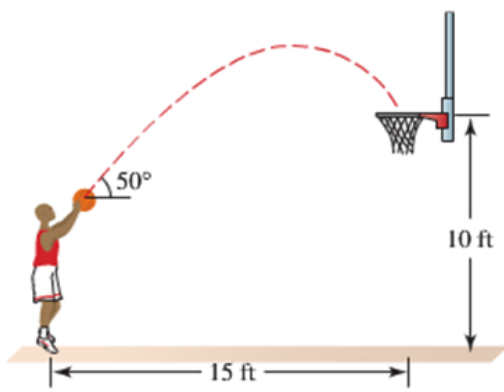
9. [**Challenge**] When a basketball player shoots a foul shot, the ball follows a parabolic arc. This arc depends on both the angle and velocity with which the basketball is released. If a person shoots the basketball overhand from a position 8 feet above the floor, then the path can sometimes be modeled by the parabola $y = \dfrac{-16x^2}{0.434v^2} + 1.15x + 8$, where v is the velocity of the ball in feet per second, as illustrated in the first figure. (Source: C. Rist, *The Physics of Foul Shots.*)

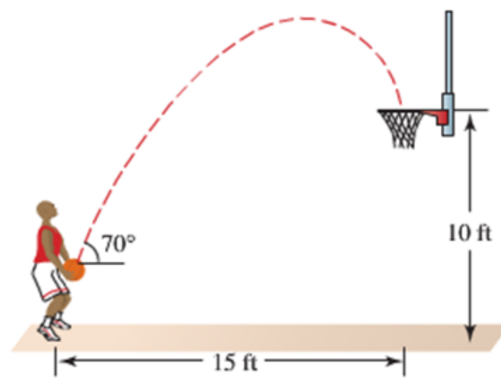
   (a) If the basketball hoop is 10 feet high and located 15 feet away, what initial velocity v should the basketball have?

   (b) Check your answer from part (a) graphically. Plot the point $(0, 8)$ where the ball is released and the point $(15, 10)$ where the basketball hoop is. Does your graph pass through both points?

   (c) What is the maximum height of the basketball?

   (d) If a person releases a basketball underhand from a position 3 feet above the floor, it often has a steeper arc than if it is released overhand and the path sometimes may be modeled by $y = \dfrac{-16x^2}{0.117v^2} + 2.75x + 3$. See the second figure below. Complete parts (a), (b), and (c) from the first part. Then compare the paths for an overhand shot and an underhand shot.





11 Polynomials Worksheet

Concepts:

- Graphs of Polynomials
- Leading Term vs. Shape of the Graph
- Continuous Graphs
- Smooth Graphs
- End Behavior of the Graph
- Multiplicity of a Root and Behavior of the Graph at x-intercepts.
- How Many Local Extrema Can a Polynomial Graph Have?

(Sections 4.2 & 4.4)

1. Evaluate $\dfrac{x^3 - 2x^2 + x - 2}{x - 4}$ and express the result in the form $P(x) = D(x)Q(x)+R(x)$.
2. Use the remainder from the above problem to decide if $x-4$ is a factor of x^3-2x^2+x-2 and to find $P(4)$.
3. What is the remainder when $f(x) = 2x^{90} - 5x^{70} - 3x^{15} + 7$ is divided by $x + 1$?
4. Completely factor $f(x) = x^3 - x^2 - 2x + 2$ by using a calculator to find one root and long division or factoring to find the others. Factors should be exact.
5. Find the zeros of the function $f(x) = 6x^2 - 19x - 36$. Use these zeros to help you factor this function.
6. (Exercise 67 from Section 4.2 of your textbook) Use the Factor Theorem to show that for every real number c, $(x - c)$ is not a factor of $x^4 + x^2 + 1$.
7. What is the maximum number of roots of the polynomial $P(x) = 5x^3 + 4x^5 - 3x + 1.2$?
8. Find the maximum value of the function $f(x) = -3x^2 + 10x + 4$.
9. Use a graphing calculator to find the local extrema of the function $f(x) = 3x^4 - 8x^3 - 6x^2 + 24x + 1$.

10. Which one of the following statements is false

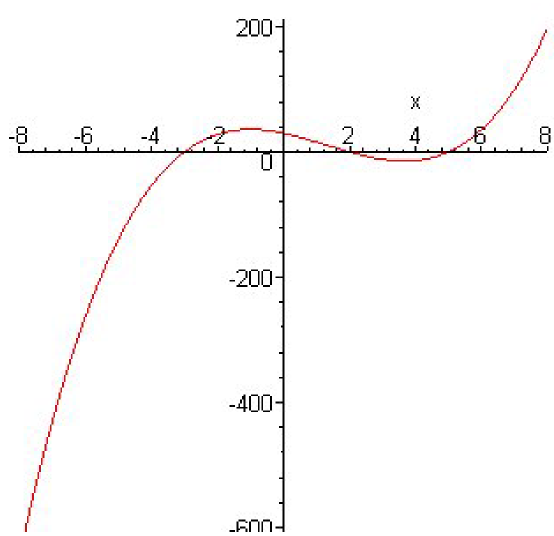
(a) The graphs of all polynomials are continuous.
(b) The graphs of all polynomials are smooth.
(c) The graph of a polynomial may have a vertical asymptote.
(d) The graph of a polynomial never contains a sharp corner.
(e) The domain of any polynomial is $(-\infty, \infty)$.

11. Describe the end behavior of each polynomial. Use correct mathematical symbols.

(a) $P(x) = 2x^5 - 3x^2 + 76$
(b) $Q(x) = -55x^{100} + 15x^{75} - 3$
(c) $R(x) = (2x+3)^4(50-x)^{100}$ (**HINT** What is the leading term?)
(d) $S(x) = (1-2x)^{11}(x+5)^4$

12. The graph shown below is NOT the graph of $y = g(x) = -2(x+3)(x-2)(x-5)$. Which of the following are clues that this is NOT the graph of g?

(a) The graph crosses the x-axis at $(-3, 0)$, but it should not cross the x-axis at this point.
(b) The graph crosses the x-axis at $(5, 0)$, but it should not cross the x-axis at this point.
(c) The graph has the wrong x-intercepts.
(d) The graph crosses the x-axis at $(2, 0)$, but it should not cross the x-axis at this point.
(e) The graph displays the wrong end behavior.
(f) The graph has too many local extreme points to be the graph of a polynomial of degree 3.



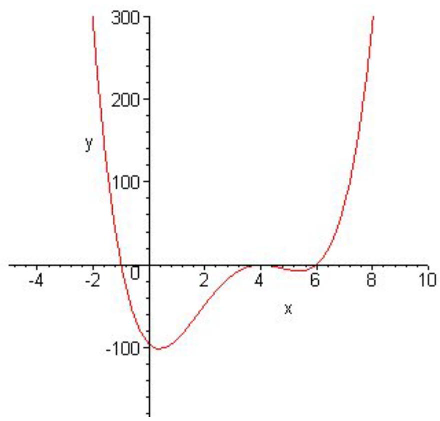
13. The graph of a polynomial $P(x)$ is shown below.

(a) Is the degree of the polynomial even or odd?

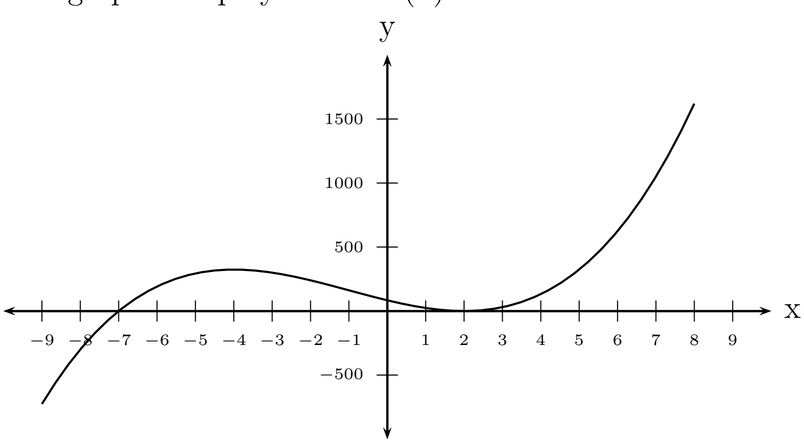
(b) Is the leading coefficient positive or negative?

(c) What can you say about the factors of this polynomial?

(d) Can you find a formula for the polynomial if you know that the degree of the polynomial is less than or equal to 4 and that $P(1) = -90$



14. The graph of a polynomial $P(x)$ is shown below.



(a) Is the degree of the polynomial even or odd?

(b) Is the leading coefficient positive or negative?

(c) What can you say about the factors of this polynomial?

(d) Can you find a formula for the polynomial if you know that the degree of the polynomial is less than or equal to 4 and that $P(1) = 24$

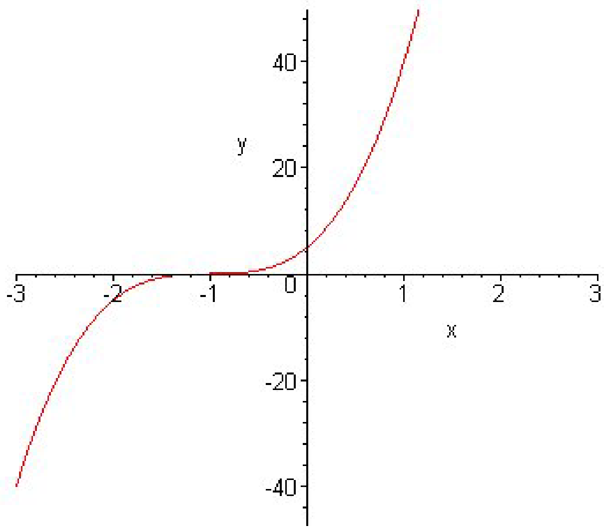
15. The graph shown below is NOT the graph of $y = h(x) = 5(x+1)^4$. Which of the following are clues that this is NOT the graph of h?

   (a) The graph crosses the x-axis at $(-1, 0)$, but it should not cross the x-axis at this point.

   (b) The graph displays the wrong end behavior.

   (c) The graph has the wrong x-intercepts.

   (d) The graph does not have the right number of local extreme points to be the graph of a polynomial of degree 4.



12 Rational Functions & Polynomial and Rational Inequalities Worksheet

Concepts:

- The Definition of a Rational Function
- Identifying Rational Functions
- Finding the Domain of a Rational Function
- The Big-Little Principle
- The Graphs of Rational Functions
- Vertical, Horizontal, and Oblique Asymptotes
- Holes in the Graphs of Rational Functions
- Equivalent Inequalities
- Solving Polynomial and Rational Inequalities Algebraically
- Approximating Solutions to Inequalities Graphically

(Section 4.5 & 4.6)

1. Describe the end behavior of the following rational functions.

   (a) $f(x) = \dfrac{3x-1}{2-5x}$

   (b) $g(x) = \dfrac{2x}{x+7}$

   (c) $h(x) = \dfrac{x+7}{x^2-6x+8}$

   (d) $k(x) = \dfrac{x+7}{x^2+6x-7}$

   (e) $l(x) = \dfrac{x^2-6x+8}{x+7}$

   (f) $m(x) = \dfrac{x^2+6x-7}{x+7}$

   (g) $n(x) = \dfrac{7x^2-3x+2x^3+6}{4x-x^2-2-5x^3}$

   (h) $o(x) = \dfrac{(2x+5)^4(6-x)^3}{(3x-1)(x-2)^6}$

   (i) $p(x) = \dfrac{(2x+5)^4(6-x)^3}{(3x-1)(x-2)^7}$

2. Find all vertical, horizontal, oblique asymptotes, holes, x-intercepts, and y-intercepts for the following rational functions. Show the algebra that justifies your answer. Graph these functions.

   (a) $f(x) = \dfrac{3x-1}{2-5x}$

   (b) $g(x) = \dfrac{2x}{x+7}$

   (c) $h(x) = \dfrac{x+7}{x^2-6x+8}$

   (d) $k(x) = \dfrac{x+7}{x^2+6x-7}$

   (e) $l(x) = \dfrac{x^2-6x+8}{x+7}$

   (f) $m(x) = \dfrac{x^2+6x-7}{x+7}$

3. Solve the inequalities. Find exact solutions when possible and approximate ones otherwise.

   (a) $x^3 - x \geq 0$

   (b) $x^2 + 8x + 20 < 0$

   (c) $2x^4 + 3x^3 < 2x^2 + 4x - 2$

   (d) $\dfrac{2x^2+x-1}{x^2-4x+4} \geq 0$

   (e) $\dfrac{1}{x-1} < -\dfrac{1}{x+2}$

   (f) $\dfrac{2x^2+6x-8}{2x^2+5x-3} < 1$

4. If it is possible to solve the inequality algebraically, do so and give exact solutions. If it is not possible to solve it algebraically, find an approximate solution graphically. Be sure to sketch the graph and label it.

   (a) $7x - 3 < 10x + 2$

   (b) $x^2 + 7x \geq -10$

   (c) $(x+2)(x-3)^2 > 0$

   (d) $x^3 > 1$

   (e) $x^4 - 2x \geq 5$

   (f) $x^4 - 2x \geq 0$

   (g) $\dfrac{1}{x-2} \geq -1$

5. It costs a craftsman \$ 5 in materials to make a medallion. He has found that if he sells the medallions for $50x$ dollars each, where x is the number of medallions produced each week, then he can sell all that he makes. His fixed costs are \$ 350 per week. If he wants to sell all he makes and show a profit each week, what are the possible numbers of medallions he should make?

6. Emma and Aidan currently pay \$ 60 per month for phone service from AT& T. This fee gets them 900 minutes per month. They look at their phone bills and realize that, at most, they talk for 100 minutes per month. They find out that they can go with Virgin Mobile and pay 18 cents per minute. If they choose to switch services, they will have to buy two new phones at \$ 40 each, and pay a \$ 175 "cancellation fee" to AT& T.

   (a) Assuming that they talk for 100 minutes per month, how many months would they have to talk before they would be saving money?

   (b) Assume they make the switch, and talk between zero and 100 minutes per month. What is the range of possible savings?

13 Complex Numbers

Concepts:

- The imaginary number i.
- Complex numbers.
- Complex arithmetic.
- Solutions to quadratic equations.
- Applications.

(Section 4.7)

1. **Answer as TRUE or FALSE.**

   (a) ____________ The only solution to the equation $x^2 = -1$ is i.

   (b) ____________ $\sqrt{-4} = -2i$.

   (c) ____________ $i^2 = -1$.

   (d) ____________ $\sqrt{4+9i} = 2+3i$.

   (e) ____________ Any real number c can be expressed in standard complex form.

2. Write each expression in terms of a real number and i.

   (a) $\sqrt{-49}$

   (b) $\sqrt{-\pi}$

   (c) $\sqrt{-5}$

3. Perform the indicated operation and write in standard complex form.

   (a) $\sqrt{-9}$

   (b) $(3-4.5i)+(-2.2-6.1i)$

   (c) i^{24}

   (d) $\dfrac{-5}{i}$

   (e) $i(5-3i)$

   (f) $\dfrac{5}{4} - (\dfrac{7}{3} - i)$

   (g) $(\sqrt{-7}+3)(4-\sqrt{-5})$

   (h) $2+3i-(5-4i)$

   (i) $\left(3+\dfrac{3}{2}i\right)4i$

   (j) $(-4-i)(-4+i)$

   (k) i^{42}

   (l) $\dfrac{1+i}{3+2i}$

   (m) $\dfrac{4}{i}$

   (n) $(a-bi)(a+bi)$

   (o) $(a-bi)-(a+bi)$

4. Solve each equation by using the quadratic formula and express answer in standard complex form.

   (a) $x^2 = -3$
   (b) $3x^2 - 2x = -5$
   (c) $x^2 - 4x - 6 = 0$
   (d) $x^2 + 5x - 6 = 0$
   (e) $x^2 + 5x + 6 = 0$
   (f) $x^2 + 4 = 0$
   (g) $3x^2 - 2x = -5$

5. Given the quadratic equation $2x^2 + x + 3 = 0$.

   (a) Find the two solutions to the equation.

   (b) Add the two solutions together. What can you say about the result. Is this true for solutions to any quadratic?

6. Find the complete factorization into linear terms for each of the following

   (a) $f(x) = x^4 - 16$
   (b) $f(x) = x^7 - 7x^6 + 19x^5 - 43x^4 + 74x^3 - 68x^2 + 56x - 32$ (Hint: 1, 2, and 4 are roots.)
   (c) $g(x) = x^3 - 1$
   (d) $h(x0 = x^3 + 1$

7. Find a polynomial that has roots 1, 3, -7, $2i$ and $-2i$. Do not leave it in factored form.

14 Radicals, Rational Exponents & Exponential Functions

Concepts:

- The nth root.
- Rational and Irrational Exponents.
- Radicals.
- Application.
- Exponential Functions
  - Power Functions vs. Exponential Functions
  - The Definition of an Exponential Function
  - Graphing Exponential Functions
  - Exponential Growth and Exponential Decay
- The Irrational Number e and Continuously Compounded Interest

(Section 5.1 & 5.2)

1. **The following are all FALSE.** Change each to make the statement true.
   (a) $\sqrt{c^2} = c$ for all real numbers c.
   (b) $(x^2)^3 = x^5$
   (c) The equation $x^n = c$ has exactly one solution when n is even and $c \geq 0$.
2. Write each expression with out using radicals.
   (a) $\dfrac{1}{\sqrt[3]{x^2}}$
   (b) $\sqrt[5]{t}\sqrt{16t^5}$
   (c) $\sqrt{\sqrt[3]{a^3b^4}}$
3. Simplify and express answers exactly (no decimal approximations).
   (a) $\sqrt[3]{40}$
   (b) $256^{-3/4}$
   (c) $\sqrt{16a^8b^{-2}}$.
   (d) $\dfrac{(x^2)^{1/3}(y^2)^{2/3}}{3x^{2/3}y^2}$

4. Compute and simplify your answer.

   (a) $x^{2/3}(x^{5/2} + y^{1/2})$

   (b) $(x^{1/2} - y^{1/2})(x^{1/2} + y^{1/2})$

5. Rationalize the denominator and simplify.

   (a) $\dfrac{5}{\sqrt{x}}$

   (b) $\dfrac{1-x}{1-\sqrt{x}}$

6. If \$10,000 is invested at an interest rate of 4% per year, compounded quarterly, find the value of the investment after the given number of years.

   (a) 5 years

   (b) 10 years

   (c) 15 years

   (d) 20 years

7. If \$10,000 is invested at an interest rate of 3% per year, find the amounts in the account after 3 years if interest is compounded

   (a) quarterly

   (b) monthly

   (c) daily

8. Joni invests \$5000 at an interest rate of 5% per year compounded continuously. How much time will it take for the value of the investment to quadruple.

9. Between 1790 and 1860, the population y of the United States (in millions) in the year x was given by $y = 3.9572(1.0299^x)$, where x is the number of years since 1790. Find the population in the year 1858. (The question similar to this on Webassign expects your answer to be expressed as an actual number, not in units of 1,000,000.)

10. According to the *Kelly Blue Book*, the factory invoice price of a 2010 Buick LaCrosse 4-door CX Sedan is \$25,945.21 and the MSRP is \$26,995.00. (The factory invoice price is the price the dealer pays to the factory. The MSRP is the Manufacturer's Suggested Retail Price.) A certain dealership is charging the MSRP for the LaCrosse, but they need to clear their inventory to make room for the 2011 models. They decide to reduce the price of the car by 1% every day after October 21, 2010. (On October 22, they reduce the price by 1%; on October 23, they reduce the *reduced* price by 1%; etc.)

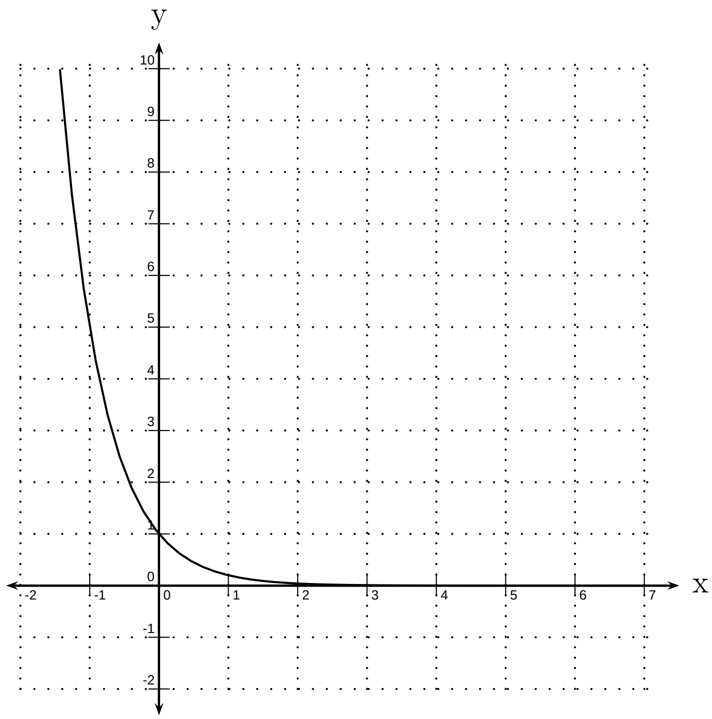
    (a) What will the price of the car be on October 23, 2010?

    (b) Find a function that models the price p of the car t days after October 21, 2010.

    (c) Will the car ever be free?

    (d) On what date does the dealer start to lose money?

11. The graph of g is shown below. Find a formula for $g(x)$.



12. Iodine-131 (I-131) is a radioactive element used in the treatment of thyroid cancer. The half-life of I-131 is about 8.0197 days. A clinical trial is being conducted to determine the best dosage of radioactive iodine for thyroid cancer patients. The patients in group I were given a dose of 1110 MBq, and those in group II were given a dose of 3700 MBq.

   (a) Find a model to represent the amount of radioactive iodine in a patient's body t days after receiving treatment:

      i. for a patient in group I.

      ii. for a patient in group II.

   (b) Suppose that Sue is a patient in group I.

      i. How much I-131 is in Sue's body, 3 days after treatment?

      ii. When will the amount of I-131 in Sue's body reach a level of 100 MBq?

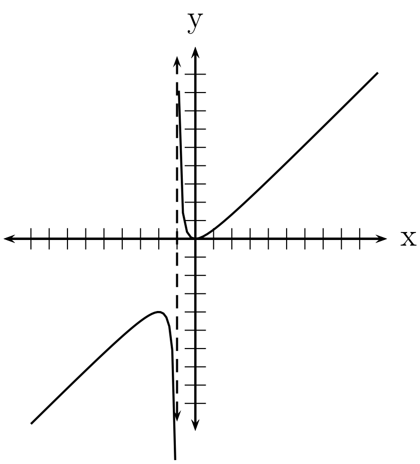
   (c) Suppose that Alice is a patient in group II.

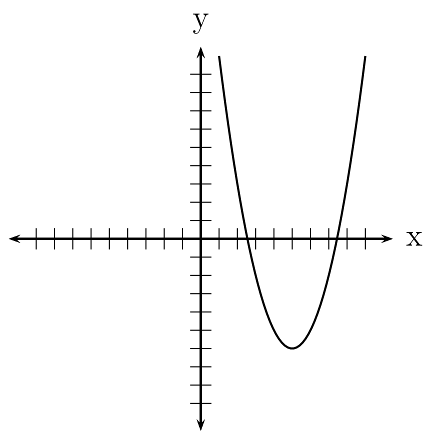
      i. How much I-131 is in Alice's body, 3 days after treatment?

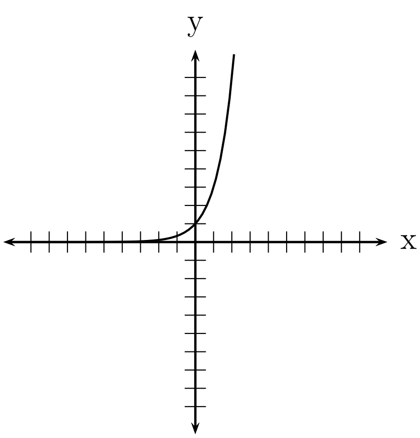
      ii. When will the amount of I-131 in Alice's body reach a level of 1100 MBq?

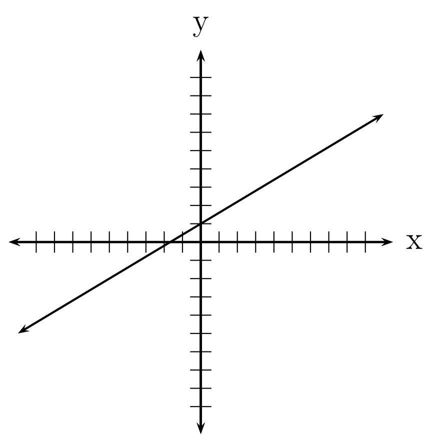
      iii. When will the amount of I-131 in Alice's body reach a level of 100 MBq?

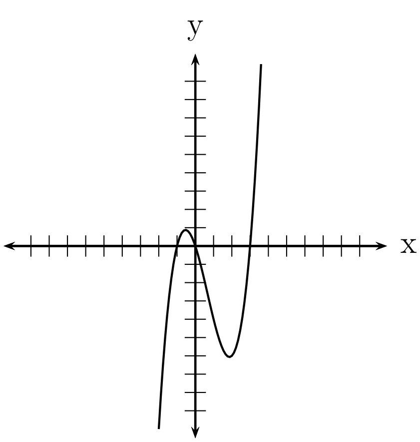
13. Label each of the following graphs with at least one of the following categories: Linear, Quadratic, Polynomial, Rational, Exponential, or None of the Categories Studied So Far. Some graphs may have more than one label.

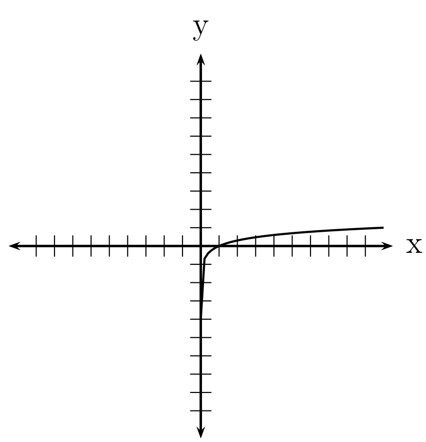












15 Logarithmic Functions

Concepts:

- Logarithms
  - Logarithms as Functions
  - Logarithms as Exponent Pickers
  - Inverse Relationship between Logarithmic and Exponential Functions.
  - The Common Logarithm
    - Definition and Graphs
    - Exponential Notation vs. Logarithmic Notation
    - Evaluating Common Logarithms
  - The Natural Logarithm
    - Definition and Graphs
    - Exponential Notation vs. Logarithmic Notation
    - Evaluating Common Logarithms
  - Logarithms with Different Bases
    - Definition and Graphs
    - Exponential Notation vs. Logarithmic Notation
    - Evaluating Different Base Logarithms

(Section 5.3)

1. Find the exact value of the following logarithms. Do NOT use your calculator.

   (a) $\log_3(27)$

   (b) $\log(\sqrt[3]{100})$

   (c) $\log_5\left(\dfrac{1}{625}\right)$

   (d) $\ln\left(\dfrac{1}{\sqrt[5]{e^3}}\right)$

   (e) $10^{\log(53)}$

   (f) $e^{2\ln(x)}$

2. Using your knowledge of exponents, estimate between which two integer values the following expressions will be. Use your calculator to find approximate values for each. Was your estimation accurate?

   (a) $\log(1008)$

   (b) $\ln(3)$

   (c) $\ln(7)$

   (d) $\log(53)$

3. Convert each exponential statement to an equivalent logarithmic statement.

(a) $8^2 = 64$

(b) $2^x = 16$

(c) $3^4 = y$

(d) $x^{-5} = \left(\frac{1}{32}\right)$

4. Convert each logarithmic statement to an equivalent exponential statement.

(a) $\log_2(32) = 5$

(b) $\ln(x) = 3$

(c) $\log(9) = x$

(d) $\log_x(\frac{1}{64}) = -3$

5. Find all real solutions or state that there are none. Your answers should be exact.

(a) $\ln(x) = 2$

(b) $10^{x+2} = 376$

(c) $9e^{x-8} = 2$

(d) $\log_8(x-5) - \log_8(2x+2) = 0$

6. Find the approximate solutions. (HINT: Try using a graphing approach.)

(a) $\ln(x) = x - 5$

(b) $\log_5(6) = x$

(c) $\log_2(21) = x$

7. Let $f(x) = \ln(3x+7)$. Find $f^{-1}(x)$.

8. Let $f(x) = 2^{5x+3} - 1$. Find $f^{-1}(x)$.

9. Find the domain of $f(x) = \ln(2-3x)$

10. Find the domain of $g(x) = \dfrac{x}{\ln(5x+4)}$

11. Find the domain of $h(x) = \ln(x^2 - 2x - 15)$

12. List the transformations that will change the graph of $f(x) = \ln(x)$ into the graph of the given function.

(a) $g(x) = 3\ln(x)$

(b) $h(x) = \ln(x) - 5$

(c) $k(x) = \ln(x-3)$

(d) $l(x) = \ln(x+2) - 7$

13. Sketch the graph of the function.

    (a) $f(x) = \log(x+4)$

    (b) $g(x) = 2\log(x) - 5$

14. (Question # 75 from Hungerford 5.3 exercises) Show that $g(x) = ln\left(\dfrac{x}{1-x}\right)$ is the inverse function of $f(x) = \dfrac{1}{1+e^{-1}}$

15. (Question # 77 from Hungerford 5.3 exercises) Suppose $f(x) = A\ln(x) + B$, where A and B are constants. If $f(1) = 10$ and $f(e) = 1$, what are A and B?

16. (Question # 78 from Hungerford 5.3 exercises) $f(x) = A\ln(x) + B$, where A and B are constants. If $f(e) = 5$ and $f(e^2) = 8$, what are A and B?

17. (Question # 49 from Hungerford 5.3 exercises) Do the graphs of $f(x) = \log(x^2)$ and $g(x) = 2\log(x)$ appear to be the same? How do they differ?

18. (Question # 50 from Hungerford 5.3 exercises) Do the graphs of $h(x) = \log(x^3)$ and $k(x) = 3\log(x)$ appear to be the same? How do they differ?

19. (Question # 82 from Hungerford 5.3 exercises) Students in a precalculus class were given a final exam. Each month thereafter, they took an equivalent exam. The class average on the exam taken after t months is given by:

$$F(t) = 82 - 8\ln(t+1).$$

    (a) What was the class average after six months?

    (b) After a year?

    (c) When did the class average drop below 55?

20. (Question # 83 from Hungerford 5.3 exercises) One person with a flu virus visited the campus. The number T of days it took for the virus to infect x people was given by:

$$T = -.93\ln\left[\frac{7000-x}{6999x}\right].$$

    (a) How many days did it take for 6000 people to become infected?

    (b) After two weeks, how many people were infected?

    (c) How large was this campus population? (HINT: Think about the domain!)

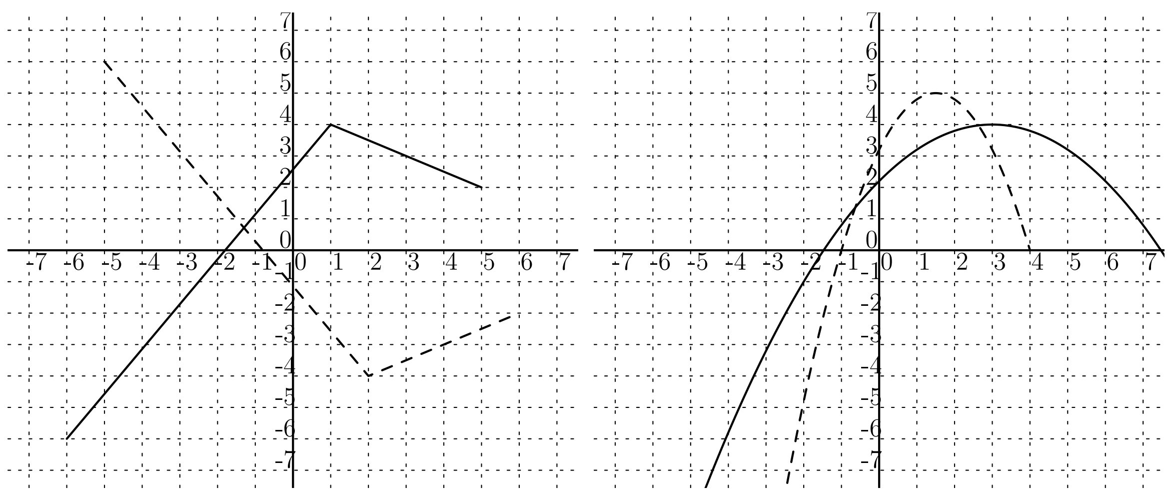
Ma 110 Exam 2 Review: Sections 3.4-3.5, 3.7, 4.1-4.2, 4.4-4.7, 5.1-5.3

Do not rely solely on this work sheet! **Make sure to study homework problems, other work sheets, lecture notes, and the book!!!**

1. Section 3.4

   (a) Describe the transformations that transform the graph of $y = x^3$ to the graph of $y = -2(x-5)^3 - 2$

   (b) For each of the following graphs $f(x)$ is the solid line and $g(x)$ is the dashed line. Describe the transformations that transform the graph of $f(x)$ into the graph of $g(x)$.



2. Section 3.5

   (a) Find the composition $f(g(x))$ for the functions $f(x) = \sqrt{x+1}$ and $g(x) = x^2 - 1$.

   (b) For the functions $g(x) = \dfrac{1}{x}$ and $f(x) = \dfrac{1}{x}$, find the composition $g(f(x))$ and simplify. State the domain of the composition.

   (c) Car A leaves a point at 8:00am traveling due north at 50mi/hr. Car B leaves the same point at the same time traveling due east at 60mi/hr. Find the distance between the cars at time as a function of time t.

3. Section 3.7

   (a) Determine which of the functions $f(x) = x^2 - 3x + 1$, $f(x) = x^3 - 5$, and $f(x) = \sqrt{x+3}$ are one-to-one. (A graphing calculator may be useful.)

   (b) Find the inverse function $f^{-1}(x)$ for $f(x) = \dfrac{x-1}{x+2}$.

   (c) Find the inverse of $g(x) = 2x^3 + 5$.

   (d) Use composition to show that $f(x) = 2x - 3$ and $g(x) = \dfrac{x+3}{2}$ are inverses of each other.

4. Section 4.1

   (a) Find the maximum value of the function $f(x) = -3x^2 + 10x + 4$.

   (b) A farmer has 1600 feet of fence to build a rectangular pen. What dimensions should he make the pen to maximize the area enclosed by the pen?

   (c) Describe the transformations that could be applied to the graph of $f(x) = x^2$ to obtain the graph of $g(x) = -4x^2 - 8x + 3$

   (d) Find the equation of the unique quadratic function that has a vertex at the point $(-2, 5)$ and an x-intercept of -1.

5. Section 4.2

   (a) Evaluate $\dfrac{x^3 - 2x^2 + x - 2}{x - 4}$ and express the result in the form $P(x) = D(x)Q(x) + R(x)$.

   (b) Use the remainder from the above problem to decide if $x - 4$ is a factor of $x^3 - 2x + x - 2$ and to find $P(4)$.

   (c) What is the remainder when $f(x) = 2x^{90} - 5x^{70} - 3x^{15} + 7$ is divided by $x + 1$?

   (d) Completely factor $f(x) = x^3 - x^2 - 2x + 2$ by using a calculator to find one root and long division to find the others. Factors should be exact.

   (e) What is the maximum number of roots of the polynomial $P(x) = 5x^3 + 4x^5 - 3x + 1.2$?

6. Section 4.4

   (a) Determine the end behavior of $f(x) = -3x^5 + 2x^2 - 5$.

   (b) Determine the end behavior of $f(x) = 3x^6 + 2x^2 - 5$.

   (c) Sketch the graph of $f(x) = (x-1)^2(x+3)(x-5)$.

   (d) Which one of the following statements is false

      i. The graphs of all polynomials are continuous.

      ii. The graphs of all polynomials are smooth.

      iii. The graph of a polynomial may have a vertical asymptote.

      iv. The graph of a polynomial never contains a sharp corner.

      v. The domain of any polynomial is $(-\infty, \infty)$.

   (e) Use a graphing calculator to find the local extrema of the function $f(x) = 3x^4 - 8x^3 - 6x^2 + 24x + 1$

7. Section 4.5

   (a) Describe the end behavior of the graph of $f(x) = \dfrac{3x^3 - 4x^2 - 5}{2x^3 - 5x + 1}$

   (b) Sketch the graph of the function $\dfrac{x-1}{x^2+5x-6}$. Label the vertical and horizontal asymptotes, holes, x-intercepts, y-intercepts and describe the end behavior.

8. Section 4.6

   (a) Find the solutions to the inequality. Express you answer in interval notation.

$$(x-1)(x+2)(x-4) \leq 0$$

   (b) Find the solutions to the inequality. Express you answer in interval notation.

$$\frac{2}{x-3} \geq \frac{3}{x-1}$$

   (c) Verify the solution to the previous problem graphically.

9. Section 4.7

   (a) Perform the operation and express answer in standard complex form $(4+i)(2-3i)$

   (b) Perform the operation and express answer in standard complex form $(2-3i)^2$

   (c) Solve the quadratic equation $x^2 + 3x + 8 = 0$ and express answer in standard complex form.

   (d) Write the number $\dfrac{1}{2-i}$ in standard complex form (rationalize the denominator).

10. Section 5.1

    (a) Simplify, and write the exact answer (do not approximate): $\sqrt{150} + \sqrt{24}$

    (b) Simplify the expression $\dfrac{(b^x)^{x-1}}{b^{-x}}$

    (c) Perform the operation and simplify $(\sqrt{x} + y)^2$

    (d) Simplify the expression $\sqrt{\sqrt[3]{\sqrt{a^3b^4}}}$ without using radicals

    (e) Rationalize the numerator of the expression $\dfrac{\sqrt{y} - 5}{10}$.

11. Section 5.2

    (a) A population of bacteria doubles every two hours. If there is initially 1000 bacteria present, write a function that expresses the total number of bacteria P, after t hours.

    (b) If a certain radioactive substance decays with decay constant r=0.0015, how much of 100 grams of the substance will be left after two years?

    (c) If \$5,000 is deposited in a bank account which has a yearly interest rate of $r = 2.5\%$ compounded continuously, find how much is in the account after 2.5 years.

    (d) How long until \$10,000 doubles in a bank account with a yearly interest rate of $r = 7\%$ compounded continuously?

12. Section 5.3

    (a) Convert $x^{-3} = \left(\dfrac{1}{64}\right)$ to an equivalent logarithmic statement.

    (b) Convert $\log_2(\dfrac{1}{32}) = x$to an equivalent exponential statement.

    (c) Find the domain of $f(x) = \ln(x^2 + 3x + 2)$

    (d) Solve for x exactly.

    $$\log_2(x - 1) = 3$$

    (e) Solve for x exactly.

    $$e^{2x-3} = 4$$

    (f) Use a graphing calculator to solve the equation for x. Express answer to three decimal places.

    $$\ln(x) + 4 = 5^x$$

    .