17 Logarithmic Properties, Solving Exponential & Logarithmic Equations & Models

Concepts:

- Properties of Logarithms
- Simplifying Logarithmic Expressions
- Proving the Quotient Rule for Logarithms
- Using the Change of Base Formula to Find Approximate Values of Logarithms
- Solving Exponential and Logarithmic Equations Algebraically
  - Strategies:
    - Same base exponential expressions that are equal must have equal exponents.
    - Same base logarithmic expressions that are equal must have equal arguments.
    - Isolate exponential expression, rewrite equation in logarithmic form.
    - Isolate logarithmic expression, rewrite equation in exponential form.
- Solving Exponential and Logarithmic Models/Applications

(Section 5.4 - 5.6)

1. Prove the *Quotient Rule for Logarithms.*

$$\log_a\left(\frac{u}{v}\right) = \log_a(u) - \log_a(v)$$

Proof:

2. Write each expression in terms of $\log(x)$, $\log(y)$, and $\log(z)$ if possible. If it is not possible, explain why.

   (a) $\log\left(\frac{x^3y^7}{\sqrt{z}}\right)$

   (b) $\log\left(\frac{x^2+y^2}{z}\right)$

   (c) $\log\left(x^5\sqrt[3]{yz}\right)$

3. Use your calculator to find approximate values for the following.

   (a) $\ln(7)$

   (b) $\log(53)$

   (c) $\log_5(6)$

   (d) $\log_2(21)$

4. Given the magnitude of an earthquake on the Richter scale is given by $R(i) = \log(\frac{i}{i_0})$, where i is the amplitude of the ground motion of the earthquake and i_0 is the amplitude of the ground motion of the "zero" earthquake,

   (a) Find the magnitude on the Richter scale of an earthquake that is 10000 times stronger than the zero quake.

   (b) Find the magnitude on the Richter scale of an earthquake that is 25 times stronger than the zero quake.

5. The half-life of a certain radioactive substance is 2,365 years.

   (a) Find the decay rate constant r.

   (b) How much substance will be left in 100 years if there is currently 500 grams of the substance?

6. Find how long it takes for a deposit to double in value if the annual interest rate is 3.5% and the interest is compounded continuously.

7. The antibiotic clarithromycin is eliminated from the body according to the formula $A(t) = 500e^{-0.1386t}$, where A is the amount remaining in the body (in milligrams) t hours after the drug reaches peak concentration.

   (a) How much time will pass before the amount of drug in the body is reduced to 100 milligrams?

   (b) Find the inverse of $A(t)$ and explain what the inverse function models.

8. You are given models for the population (in millions) of different countries t years after 2005. For each part, determine the year in which the models predict the populations will be equal.

(Source: World Health Organization's 2006 World Health Statistics)

   (a) Rwanda: $R(t) = 9.04(1.05)^t$ and Hungary: $H(t) = 10.1(0.98)^t$.
   (b) Cambodia: $C(t) = 14.07(1.02)^t$ and Kazakhstan: $K(t) = 14.83(0.93)^t$.

9. Find the solution(s) of the following exponential equations. Your answers should be exact.

   (a) $10^{2x^2-3} = 10^{9-x^2}$
   (b) $2^{3x+1} = 3^{x-2}$
   (c) $\dfrac{10}{1+e^{-x}} = 2$
   (d) $3^{4x} - 3^{2x} - 6 = 0$
   (e) $9e^{x-8} = 2$

10. Find the solution(s) of the following logarithmic equations. Your answers should be exact.

   (a) $\log_4(x+2) + \log_4 3 = \log_4 5 + \log_4(2x-3)$
   (b) $\log_3(x+15) - \log_3(x-1) = 2$
   (c) $\log_2(\log_3 x) = 4$
   (d) $\log(x+3) = \log x + \log 3$
   (e) $\log_8(x-5) + \log_8(x+2) = 1$

11. Suppose you're driving your car on a cold winter day (20° F outside) and the engine overheats (at about 220° F). When you park, the engine begins to cool down. The temperature U of the engine t minutes after you park satisfies the equation

$$\ln\left(\frac{U-20}{200}\right) = -0.11t.$$

   (a) Solve the equation for U.
   (b) Use part (a) to find the temperature of the engine after 20 min ($t = 20$).

12. Joni invests \$5000 at an interest rate of 5% per year compounded continuously. How much time will it take for the value of the investment to quadruple.

13. Joni invests \$5000 at an interest rate of 5% per year compounded monthly. How much time will it take for the value of the investment to quadruple.

18 Angles and Their Measurement

Concepts:

- Angles
  - Initial Side and Terminal Side
  - Standard Position
  - Coterminal Angles
- Measuring Angles
  - Radian Measure vs. Degree Measure
  - Radian Measure as a Distance on the Unit Circle
  - Converting between Radian Measure and Degree Measure
  - Finding the Quadrant Associated with the Terminal Side of an Angle
- Identifying the Point on the Unit Circle that Corresponds to an Angle in Standard Position

(**Sections 6.1**)

1. Find the radian measure of each of the following:
   (a) $450°$ angle
   (b) $-50°$ angle
2. Show which of the following points must lie on the unit circle.
   (a) $(0, -1)$
   (b) $(1, -1)$
   (c) $(\frac{3}{5}, -\frac{4}{5})$
   (d) $(\frac{4}{5}, -\frac{3}{5})$
   (e) $(-\frac{\sqrt{5}}{3}, \frac{2}{3})$
   (f) $(\frac{\sqrt{3}}{2}, \frac{1}{2})$
   (g) $(-\frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2})$
   (h) $(\frac{1}{2}, -\frac{\sqrt{3}}{2})$

3. Suppose than an angle of measure θ radians intersects the unit circle at the point $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$.

   (a) What is one possibility for θ ?

   (b) How do you find all the other possibilities?

4. Suppose than an angle of measure θ radians intersects the unit circle at the point $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

   (a) What is one possibility for θ?

   (b) How do you find all the other possibilities?

5. Suppose that an angle of measure θ radians is placed in standard position. Find the location of the terminal side of the angle.
   Possibilities: (A) Quadrant I, (B) Quadrant II, (C) Quadrant III, (D) Quadrant VI, (E) the positive x-axis, (F) the negative x-axis, (G) the positive y-axis, or (H) the negative y-axis.

   (a) $\theta = \frac{74\pi}{3}$

   (b) $\theta = -\frac{74\pi}{3}$

   (c) $\theta = 100\pi$

   (d) $\theta = -100\pi$

   (e) $\theta = 21\pi$

   (f) $\theta = -21\pi$

   (g) $\theta = \frac{102\pi}{7}$

   (h) $\theta = -\frac{102\pi}{7}$

6. Find the terminal point on the unit circle determined by the given value of θ.

   (a) $\theta = 4\pi$

   (b) $\theta = \frac{3\pi}{2}$

   (c) $\theta = -\frac{\pi}{6}$

   (d) $\theta = \frac{7\pi}{6}$

   (e) $\theta = -\frac{7\pi}{4}$

   (f) $\theta = \frac{5\pi}{3}$

   (g) $\theta = -\frac{4\pi}{3}$

   (h) $\theta = \frac{11\pi}{6}$

19 Introduction to Trigonometry Worksheet

Concepts:

- The Trigonometric Functions
  - The Definitions of sin, cos, and tan Based on the Unit Circle
  - Evaluating the Basic Trigonometric Functions at Special Angles
  - The Sign of a Trigonometric Function
- The $\frac{\pi}{4} - \frac{\pi}{4} - \frac{\pi}{2}$ or the $45° - 45° - 90°$ Triangle
- The $\frac{\pi}{6} - \frac{\pi}{3} - \frac{\pi}{2}$ or the $30° - 60° - 90°$ Triangle
- Approximating Values of Trigonometric Functions with Your Calculator
  - Parentheses Are Important
  - Radian Mode vs. Degree Mode
- Understanding Trigonometric Notation
- The Graph of the sin, cos, and tan Functions
- Applying Graph Transformations to the Graphs of the sin, cos, and tan Functions
- Using Graphical Evidence to Make Conjectures about Identities

(**Sections 6.2 & 6.4**)

1. Evaluate the basic trigonometric functions at each of the following angles.
   (a) $\theta = \dfrac{\pi}{3}$
   (b) $\theta = -\dfrac{9\pi}{4}$
   (c) $\theta = 4\pi$
   (d) $\theta = \dfrac{17\pi}{6}$
2. (a) The terminal side of an angle, θ, in standard position contains the point $(-5, 9)$. Evaluate the basic trigonometric functions at θ.
   (b) The terminal side of an angle, θ, in standard position contains the point $(11, 4)$. Evaluate the basic trigonometric functions at θ.
3. Suppose θ is in the fourth quadrant and $\cos(\theta) = \dfrac{1}{5}$. Evaluate the remaining basic trigonometric functions on θ.

4. Suppose θ is in the first quadrant and $\sin(\theta) = \frac{6}{7}$. Find each of the following (give exact answers):

   (a) $\sin(8\pi + \theta)$

   (b) $\tan(-\theta)$

   (c) $\cos(4\pi - \theta)$

5. For each of the following equations, list the transformations that you need to apply to the graph of $y = \sin(x)$, $y = \cos(x)$, or $y = \tan(x)$ to sketch the graph of the equation. Sketch the graph. Be sure that the graph is well-labeled.

   (a) $y = 4\sin(3x)$

   (b) $y = 3\cos\left(\frac{x}{\pi}\right)$

   (c) $y = 2\sin\left(x - \frac{\pi}{3}\right)$

   (d) $y = -\tan\left(x + \frac{\pi}{4}\right)$

   (e) $y = u\cos(vx + w)$. (Assume that u, v, and w are positive.)

6. Use graphical evidence to determine which of the following **MIGHT** be trigonometric identities and which definitely cannot be trigonometric identities.

   (a) $\cos(2x) = \cos^2(x) - \sin^2(x)$

   (b) $\sin(2x) = \sin^2(x) - \cos^2(x)$

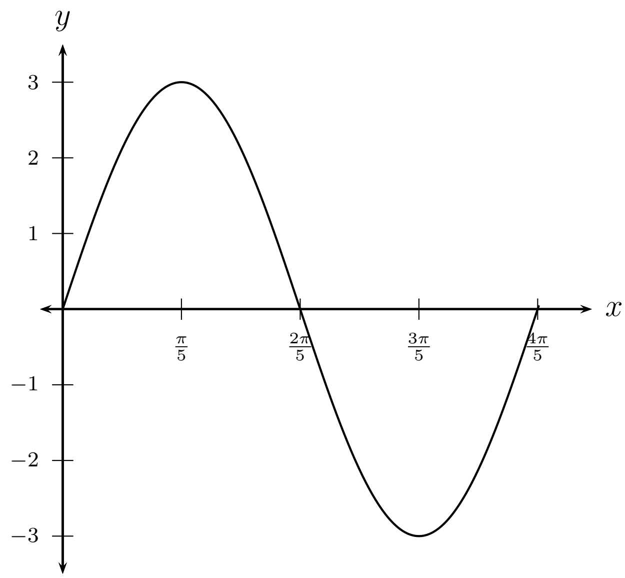
   (c) $\sin(x + y) = \sin(x) + \sin(y)$

   (d) $\sin(x)\cos(y) = \frac{1}{2}(\sin(x + y) + \sin(x - y))$

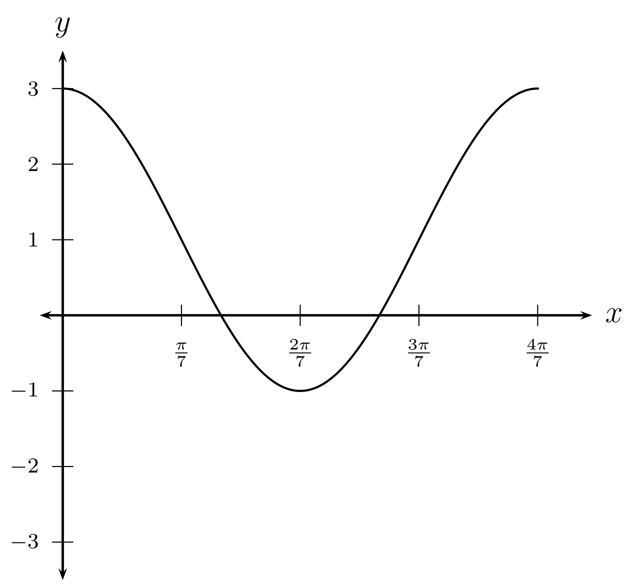
   (e) $\tan(xy) = \tan(x)\tan(y)$

   (f) $\frac{\sin(t^2)}{t} = \sin(t)$

7. The graph of a periodic function is shown below. Find a rule for the function.



8. The graph of a periodic function is shown below. Find a rule for the function.



9. (Question 43 from Section 6.5 of your textbook)

   Burke's blood pressure can be modeled by the function

   $$g(t) = 21\cos(2.5\pi t) + 113,$$

   where t is the time (in seconds) and $f(t)$ is in millimeters of mercury. The highest pressure (systolic) occurs when the heart beats, and the lowest pressure (diastolic) occurs when the heart is at rest between beats. The blood pressure is the ratio systolic/diastolic.

   (a) Graph the blood pressure function over a period of two seconds and determine Burke's blood pressure.

   (b) Find Burke's pulse rate (number of heartbeats per minute).

   (c) According to current guidelines, someone with systolic pressure above 140 or diastolic pressure above 90 has high blood pressure and should see a doctor about it. What would you advise the person in this case?

10. (Question 47 from Section 6.5 of your textbook)

    The current generated by an AM radio transmitter is given by a function of the form $f(t) = A\sin(2000\pi mt)$, where $550 \leq m \leq 1600$ is the location on the broadcast dial and t is measured in seconds. For example, a station at 980 on the AM dial has a function of the form

    $$f(t) = A\sin(2000\pi(980)t) = A\sin(1960000\pi t).$$

    Sound information is added to this signal by varying (modulating) A, that is, by changing the amplitude of the waves being transmitted. (AM means "amplitude modulation.") For a station at 980 on the dial, what is the period of the function f? What is the frequency (number of complete waves per second)?

11. (Question 48 from Section 6.5 of your textbook)

    The number of hours of daylight in Winnipeg, Manitoba, can be approximated by

    $$d(t) = 4.15\sin(.0172t - 1.377) + 12,$$

    where t is measured in days, with $t = 1$ being January 1.

    (a) On what day is there the most daylight? The least? How much daylight is there on these days?

    (b) On which days are there 11 hours or more of daylight? What do you think the period of this function is? Why?

20 Trigonometric Graphs Worksheet

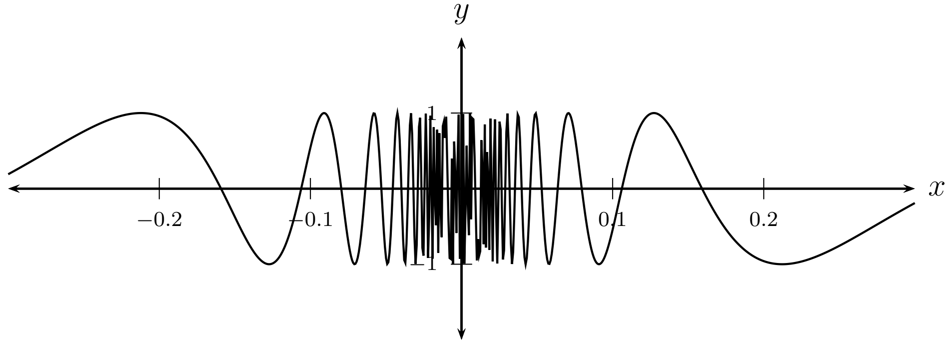
Concepts:

- Period, Amplitude, and Phase Shift
- The csc, sec, and cot Functions
- The Graphs of the csc, sec, and cot Functions

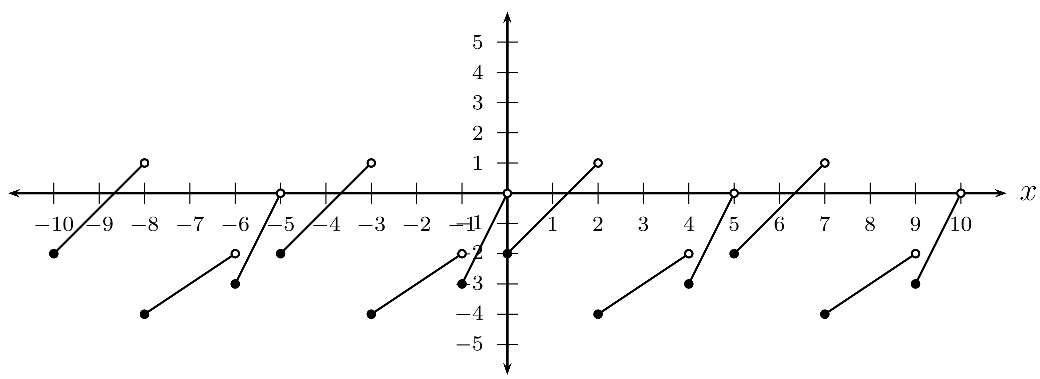
(Sections 6.5 and 6.6)

1. For each graph, (i) find the period if it is defined and (ii) find the amplitude if it is defined.

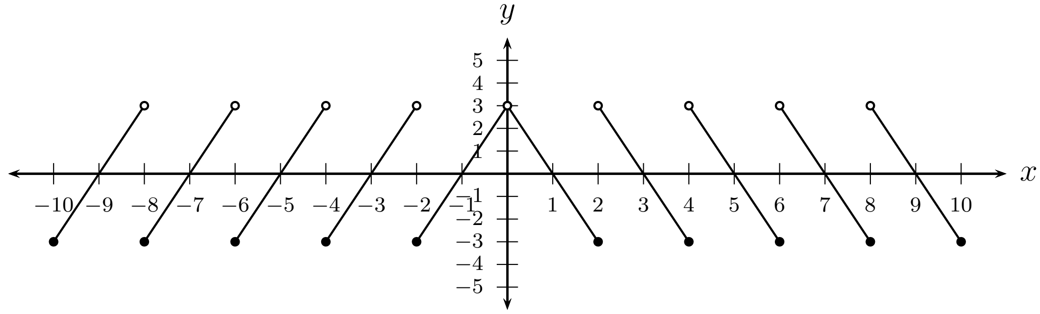
(a)



(b)



(c)



2. For each of the following equations, what is the period, amplitude and phase shift of the graph?

   (a) $y = 4\sin(3x)$
   (b) $y = 3\cos\left(\frac{x}{\pi}\right)$
   (c) $y = 2\sin\left(x - \frac{\pi}{3}\right)$
   (d) $y = -\tan\left(x + \frac{\pi}{4}\right)$
   (e) $y = u\cos(vx + w)$. (Assume that u, v, and w are positive.)

3. Evaluate the csc, sec, and cot functions at each of the following angles.

   (a) $\theta = \frac{\pi}{3}$
   (b) $\theta = -\frac{9\pi}{4}$
   (c) $\theta = 4\pi$
   (d) $\theta = \frac{17\pi}{6}$

4. (a) The terminal side of an angle, θ, in standard position contains the point $(-5, 9)$. Evaluate the csc, sec, and cot functions at θ.
   (b) The terminal side of an angle, θ, in standard position contains the point $(11, 4)$. Evaluate the csc, sec, and cot functions at θ.

5. Suppose θ is in the fourth quadrant and $\cos(\theta) = \frac{1}{5}$. Evaluate the csc, sec, and cot functions on θ.

6. Sketch the graphs of the following equations. Be sure that the graph is well-labeled.

   (a) $y = \csc(x) + 3$
   (b) $y = 5\sec\left(\frac{x}{2}\right) + 3$
   (c) $y = \cot\left(x - \frac{3\pi}{4}\right)$

7. Use algebra and identities to simplify the expression. Assume all denominators are nonzero.

   (a) $\frac{\sin(t)}{\tan(t)}$
   (b) $\frac{1}{\cos(t)} - \sin(t)\tan(t)$

8. Solve each of the following equations.

   (a) $\cos(x)=0$
   (b) $\sin^3 t - \sin t = 0$
   (c) $\cos^2 t - 2\cos t = -1$

9. (Question 43 from Section 6.5 of your textbook)

   Burke's blood pressure can be modeled by the function

   $$g(t) = 21\cos(2.5\pi t) + 113,$$

   where t is the time (in seconds) and $f(t)$ is in millimeters of mercury. The highest pressure (systolic) occurs when the heart beats, and the lowest pressure (diastolic) occurs when the heart is at rest between beats. The blood pressure is the ratio systolic/diastolic.

   (a) Graph the blood pressure function over a period of two seconds and determine Burke's blood pressure.

   (b) Find Burke's pulse rate (number of heartbeats per minute).

   (c) According to current guidelines, someone with systolic pressure above 140 or diastolic pressure above 90 has high blood pressure and should see a doctor about it. What would you advise the person in this case?

10. (Question 47 from Section 6.5 of your textbook)

    The current generated by an AM radio transmitter is given by a function of the form $f(t) = A\sin(2000\pi mt)$, where $550 \leq m \leq 1600$ is the location on the broadcast dial and t is measured in seconds. For example, a station at 980 on the AM dial has a function of the form

    $$f(t) = A\sin(2000\pi(980)t) = A\sin(1960000\pi t).$$

    Sound information is added to this signal by varying (modulating) A, that is, by changing the amplitude of the waves being transmitted. (*AM* means "amplitude modulation.") For a station at 980 on the dial, what is the period of the function f? What is the frequency (number of complete waves per second)?

11. (Question 48 from Section 6.5 of your textbook)

    The number of hours of daylight in Winnipeg, Manitoba, can be approximated by

    $$d(t) = 4.15\sin(.0172t - 1.377) + 12,$$

    where t is measured in days, with $t = 1$ being January 1.

    (a) On what day is there the most daylight? The least? How much daylight is there on these days?

    (b) On which days are there 11 hours or more of daylight? What do you think the period of this function is? Why?

21 Trigonometric Identities Worksheet

Concepts:

- Trigonometric Identities
  - Reciprocal Identities
  - Pythagorean Identities
  - Periodicity Identities
  - Negative Angle Identitites

(**Sections 6.6 & 7.1**)

1. Find the period, phase shift, and sketch the graph of $y = 2\csc(2x - \frac{\pi}{2})$.

2. Find all solutions to the equation $\tan(\theta) = \cot(\theta)$ for $0 < \theta < \pi$

3. Use the Pythagorean identity for each of the following problems.
   (a) Find $\sin(t)$ if $\cos(t) = \dfrac{2}{\sqrt{10}}$ and $\dfrac{3\pi}{2} < t < 2\pi$.
   (b) Find $\cos(t)$ if $\sin(t) = \dfrac{3}{5}$ and $\dfrac{\pi}{2} < t < \pi$.
   (c) Find $\cot(t)$ if $\sin(t) = -\dfrac{2}{5}$ and $\pi < t < \dfrac{3\pi}{2}$.

4. Determine each of the following:
   (a) Find $\cos x$ if $\sin x = -\dfrac{5}{13}$ and $\tan x > 0$.
   (b) Find $\tan x$ if $\cos x = \dfrac{1}{4}$ and $\sin x < 0$.
   (c) Find $\tan x$ if $\sec x = \dfrac{\sqrt{10}}{3}$ and $\sin x > 0$.

5. Use the periodicity and negative angle identities to solve each of the following problems.

   (a) Find $\sin(-t)$ if $\sin(t) = \frac{1}{\sqrt{5}}$.
   (b) Find $\sin(t - 2\pi)$ if $\sin(t) = \frac{1}{\sqrt{2}}$.
   (c) Find $\cos(-t + 4\pi)$ if $\cos(t) = -\frac{\sqrt{6}}{3}$.
   (d) Find $\cot(7\pi - t)$ if $\tan(t) = \sqrt{15}$.

6. Use basic identities to simplify the expression.

   (a) $\cot\theta \sec\theta \sin\theta$
   (b) $\dfrac{\cos^2 x}{\sin^2 x} + \csc x \sin x$
   (c) $\dfrac{1}{\cot^2 x} + \sec x \cos x$
   (d) $\sin^2 x + \tan^2 x + \cos^2 x$
   (e) $\dfrac{1 - \sin^2 x}{\sin x - \csc x}$
   (f) $\dfrac{\cos x}{1 - \sin x} + \dfrac{1 - \sin x}{\cos x}$
   (g) Prove the identity $\sin x\,(\tan x \cos x - \cot x \cos x) = 1 - 2\cos^2 x$

7. State whether or not the equation might be an identity. If it appears to be, prove it.

   (a) $\cot x = \dfrac{\csc x}{\sec x}$
   (b) $\dfrac{\sin(-x)}{\cos(-x)} = -\tan x$
   (c) $1 + \sec^2 x = \tan^2 x$
   (d) $\sin^2 x(\cot x + 1)^2 = \cos^2 x(\tan x + 1)^2$
   (e) $\dfrac{1 + \sin x}{1 - \sin x} = \dfrac{\sec x + \tan x}{\sec x - \tan x}$
   (f) $(1 - \cos^2 x) \csc x = \sin x$
   (g) $\dfrac{\sec x}{\csc x} + \dfrac{\sin x}{\cos x} = 2\tan x$
   (h) $\dfrac{\csc^2 x - 1}{\csc^2 x} = \cos^2 x$
   (i) $\dfrac{\sin x - \cos x}{\tan x} = \dfrac{\tan x}{\sin x + \cos x}$

22 More Trigonometric Identities Worksheet

Concepts:

- Trigonometric Identities
  - Addition and Subtraction Identities
  - Cofunction Identities
  - Double-Angle Identities
  - Power-Reducing Identitites
  - Half-Angle Identities
  - Product-Sum Identities

(**Sections 7.2 & 7.3**)

1. Find the exact values of the following functions using the addition and subtraction formulas
   (a) $\sin \frac{9\pi}{12}$
   (b) $\cos \frac{7\pi}{12}$
2. Write the expression as the sine or cosine of an angle.
   (a) $\sin \frac{\pi}{2} \cos \frac{\pi}{7} + \cos \frac{\pi}{2} \sin \frac{\pi}{7}$
   (b) $\sin 5x \cos x - \cos 5x \sin x$
   (c) $\cos 5x \cos 7x - \sin 5x \sin 7x$
3. Simplify the following expressions as much as possible
   (a) $\tan\left(\frac{9\pi}{2} - x\right)$
   (b) $\sin(x+y) - \sin(x-y)$
   (c) $\cos(x+y) - \cos(x-y)$
   (d) $\frac{\sin(x+y) - \sin(x-y)}{\cos(x+y) - \cos(x-y)}$
   (e) $\cos\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{6}\right)$

4. Verify the following identity:

$$\frac{\cos A - \cos B}{\sin A + \sin B} + \frac{\sin A - \sin B}{\cos A + \cos B} = 0.$$

5. Verify the following identity:

$$\cos(x+y)\cos(x-y) = \cos^2 x - \sin^2 y.$$

6. Use the cofunction identities to evaluate the following expression without using a calculator:

$$\sin^2(21°) + \sin^2(61°) + \sin^2(69°) + \sin^2(29°).$$

7. Find the values of the remaining trigonometric functions of x if

   (a) $\sin x = \frac{\sqrt{5}}{3}$ and the terminal point of x is in Quadrant II.

   (b) $\tan x = -\frac{\sqrt{11}}{5}$ and $\cos x > 0$.

8. Simplify the expression $\dfrac{\sin 14x}{\sin 13x + \sin x}$.

9. Let $f(x) = \sin 4x + \sin 5x$. Verify that $f(x) = 2\cos\frac{x}{2}\sin\frac{9}{2}x$.

10. Use an appropriate half-angle formula to find the exact value of each expression.

    (a) $\sin\frac{\pi}{12}$

    (b) $\cos\frac{\pi}{8}$

    (c) $\tan\frac{\pi}{12}$

    (d) $\cos\frac{7\pi}{8}$

    (e) $\sin\frac{3\pi}{8}$

11. Use an appropriate half-angle formula to simplify $\sqrt{\dfrac{1-\cos 10x}{2}}$.

12. Use an appropriate power-reducing formula to rewrite $cos^4 x sin^2 x$ in terms of the first power of cosine.

13. Write the product $7\cos 6x \cos 7x$ as a sum.

14. Write the sum $\sin 2x - \sin 7x$ as a product.

Ma 110 Exam 3 Review: Sections 5.4-5.6, 6.1-6.2, 6.4-6.6, 7.1-7.3

Do not rely solely on this work sheet! **Make sure to study homework problems, other work sheets, lecture notes, and the book!!!**

1. Section 5.4 (suggested problems from HW23 - #'s 3, 6, 7, 10)
   (a) Write as a single logarithm. $2\log(x-1)+\log(x^2-1)-\log(y-1)$
   (b) Simplify. $e^{x\ln(5)}$
   (c) Express $\ln\left(\frac{2z^3}{2x^4\sqrt{y}}\right)$ in terms of $\ln(x), \ln(y)$ and $\ln(z)$.
2. Section 5.5 (suggested problems from HW24 - #'s 2, 5, 9, 10)
   (a) Solve for x exactly. $\ln(x+5)=3$.
   (b) Solve for x exactly. $e^{x^2-2x-3}=1$
   (c) Section 5.5, question 37: Solve $\log(3x-1)+\log(2)=\log(4)+\log(x+2)$
   (d) How long until \$10,000 doubles in a bank account with a yearly interest rate of $r=7\%$ compounded continuously?
   (e) Section 5.5, question 69: The concentration of carbon dioxide in the atmosphere is 364 parts per million (ppm) and is increasing exponentially at a continuous rate of .4%. How many years will it take for the concentration to reach 500 ppm?
3. Section 5.6 (suggested problems from HW25 - #'s 3, 4, 6)
   (a) A culture starts with 8600 bacteria. After one hour the count is 10,000.
      i. Find a function that models the number of bacteria after t hours.
      ii. Find the number of bacteria after 2 hours.
      iii. How long will it take for the number of bacteria to double?
   (b) The population of California was $10,586,223$ in 1950 and $23,668,562$ in 1980. Assume the population grows exponentially.
      i. Find a function that models the population t years after 1950.
      ii. Find the time required for the population to double.
      iii. In what year was the population $1,000,000$?
   (c) The half-life of radium-226 is 1600 years. Suppose we have a 22-mg sample.
      i. Find a function that models the mass remaining after t years.
      ii. How much of the sample will remain after 4000 years?
      iii. After how long will only 18 mg of the sample remain?

4. Section 6.1 (suggested problems from HW26 - #'s 1, 2, 5, 6, 9)

   (a) Find the radian measure of a $-450°$ angle.

   (b) What quadrant does the angle with measure $\frac{26\pi}{3}$ lie in.

   (c) Suppose that an angle of measure θ radians intersects the unit circle at the point $\left(\frac{-\sqrt{3}}{2}, \frac{-1}{2}\right)$. Give two possibilities for θ.

5. Section 6.2 (suggested problems from HW27 - #'s 3, 5, 6)

   (a) Find the point P on the unit circle corresponding to the angle $\frac{-5\pi}{6}$.

   (b) Evaluate the $\sin, \cos,$ and $\tan$ functions at $t = \frac{5\pi}{2}$.

   (c) Evaluate $\sin, \cos,$ and $\tan$ if the terminal ray of an angle contains the point $(\sqrt{5}, -7)$.

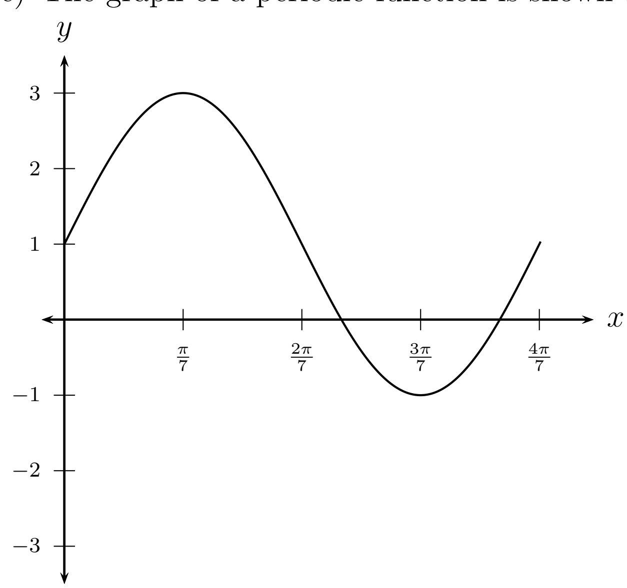
   (d) Approximate $\sin(2.6)$.

6. Section 6.4 (suggested problems from HW28 - #'s 1, 3, 6, 7)

   (a) List the transformations needed to apply to the graph of $y = \sin(x)$ to sketch the graph of $y = 3\sin\left(\frac{x}{\pi}\right)$. Sketch the graph. Be sure that the graph is well-labeled.

   (b) List the transformations needed to apply to the graph of $y = \cos(x)$ to sketch the graph of $y = 2\cos\left(x - \frac{\pi}{3}\right)$. Sketch the graph. Be sure that the graph is well-labeled.

   (c) The graph of a periodic function is shown below. Find a rule for the function.



7. Section 6.5 (suggested problems from HW29 - #'s 2, 3, 4)

   (a) For each state the amplitude, period, phase shift and sketch the graph.

      i. $f(x) = 2\cos\left(3x - \frac{\pi}{2}\right)$

      ii. $f(x) = \tan\left(x + \frac{\pi}{4}\right)$

   (b) How many solutions for x between 0 and 2π does $\sin(x) = -0.2$ have?

8. Section 6.6 (suggested problems from HW30 - #'s 2, 5, 6)

   (a) Evaluate the six trigonometric functions at $t = -\dfrac{7\pi}{3}$.

   (b) Evaluate the six trigonometric functions if the terminal ray of an angle contains the point (.6, .8).

   (c) Answer as True or False.

      i. $\cos^2(t) = 1 + \sin^2(t)$.

      ii. $\cos(t - 2\pi) = \cos(t)$.

      iii. $\csc(t) = \dfrac{1}{\sin(t)}$.

      iv. $\tan(-t) = \tan(t)$.

   (d) Find $\pi < t < 2\pi$ such that $\sec(t) = \sqrt{2}$ exactly.

   (e) Find the other five trigonometric functions exactly if $\cos(t) = \dfrac{1}{3}$ and $\pi < t < 2\pi$.

9. Section 7.1 (suggested problems from HW31 - #'s 4, 8, 9, 10)

   (a) Simplify the following:

      i. $\tan(x)\Big(\sin(x) + \cot(x)\cos(x)\Big)$

      ii. $\Big(\cos(\theta) - \sin(\theta)\Big)^2$

      iii. $\dfrac{\sec(t)}{\sin(t)} - \dfrac{\sin(t)}{\cos(t)}$

   (b) Prove the following:

      i. $\csc^2(x) - \cos^2(x)\csc^2(x) = 1$

      ii. $\big(\sec(\theta) + 1\big)\big(\sec(\theta) - 1\big) = \tan^2(\theta)$

      iii. $\dfrac{\cos(\alpha)}{1 - \sin(\alpha)} = \sec(\alpha) + \tan(\alpha)$

      iv. $\dfrac{\sin(t)}{1 - \cos(t)} + \dfrac{1 - \cos(t)}{\sin(t)} = 2\csc(t)$

      v. $\sec^4(x) - \tan^4(x) = \sec^2(x) + \tan^2(x)$

10. Section 7.2 (suggested problems from HW32 - #'s 5, 6, 8, 10)

   (a) Find the exact value of each of the following expressions:

      i. $\cos\left(\frac{11\pi}{12}\right)$

      ii. $\sin\left(\frac{19\pi}{12}\right)$

      iii. $\tan\left(\frac{17\pi}{12}\right)$

   (b) Write the following expression as a trigonometric function of one number, and find its exact value.

$$\cos\left(\frac{3\pi}{7}\right)\cos\left(\frac{2\pi}{21}\right)+\sin\left(\frac{3\pi}{7}\right)\sin\left(\frac{2\pi}{21}\right)$$

   (c) Prove the cofunction identity: $\sin\left(\frac{\pi}{2}-\theta\right)=\cos(\theta)$

   (d) Section 7.2, Question 45: If x is in the first and y is in the second quadrant, $\sin(x)=\frac{24}{25}$, and $\sin(y)=\frac{4}{5}$, find the exact value of $\sin(x+y)$ and $\tan(x+y)$ and the quadrant in which $x+y$ lies.

11. Section 7.3 (suggested problems from HW33 - #'s 1, 2, 3, 4)

   (a) Find the exact value of each of the following expressions:

      i. $\cos\left(\frac{3\pi}{8}\right)$

      ii. $\sin\left(\frac{5\pi}{8}\right)$

      iii. $\tan\left(\frac{7\pi}{8}\right)$

   (b) Given $\tan(t)=-\frac{4}{3}$ and $\frac{\pi}{2}<t<\pi$, find $\sin(2t), \cos(2t)$, and $\tan(2t)$.

   (c) Given $\tan(x)=1$ and x is in Quadrant III, find $\sin\left(\frac{x}{2}\right), \cos\left(\frac{x}{2}\right)$, and $\tan\left(\frac{x}{2}\right)$.

   (d) Simplify. $\frac{1-\cos(4\theta)}{\sin(4\theta)}$