# 1 Basics Review Worksheet

#### **Concepts:**

- Scientific notation.
- Square roots and principal square roots.
- Negation.
- Absolute Value.

### (Section 1.1)

- 1. Express the number in normal decimal notation.
  - (a) There are  $6.02 \times 10^{23}$  molecules in each mole.
  - (b) The mass of an electron is  $9.10938188 \times 10^{-31}$  kg.
- 2. 1 mile = \_\_\_\_\_ inches. Write your answer in scientific notation. (**HINT:** There are 5280 feet in one mile.)
- 3. 1 second = \_\_\_\_\_ years. Write your answer in normal decimal notation. (Assume that there are 365 days in a year.)

#### 4. TRUE or FALSE

(a) \_\_\_\_\_ 11 is the only square root of 121.

(b) \_\_\_\_\_ 
$$\sqrt{121} = \pm 11$$

- (c) \_\_\_\_\_  $\sqrt{3^2 + 4^2} = \sqrt{3 + 4}$
- 5. Simplify.
  - (a)  $\sqrt{75}\sqrt{12}$  (b)  $\sqrt{2535} \sqrt{135}$ .
- 6. Express the given statement in symbols.
  - (a) x is nonnegative. (b) d is not greater than 7.
- 7. For each arithmetic statement, write a corresponding geometric statement.
  - (a)  $a \ge b$
  - (b) a + 5 = b
  - (c) a + c > b, (c > 0)

- 8. For each geometric statement, write a corresponding arithmetic statement.
  - (a) a lies 6 units to the right of b on a horizontal number line.
  - (b) a lies at least 4 units below b on a vertical number line.
- 9. Given real numbers b, c, d such that b < 0, c > 0, and d < 0. Determine which of the expressions are positive?
  - (a) b c
  - (b) bc bd
  - (c)  $b^2 c c^2 d$
- 10. Find the exact value of the expression. You may not use parentheses in your answer. Which of the expressions are positive?
  - (a)  $-(\sqrt{245} 13)$ (b) -(x - 6) if x > 6(c) -(x - 6) if x < 6(d)  $-((\pi - 3) - 1)$
- 11. Simplify, and write the given number without using absolute values.
  - (a) 3 |2 5|(b)  $|\sqrt{2} - 2|$ (c)  $|3 - \pi| + 3$
- 12. Write the given number without using absolute values.
  - (a) |a-5| if a < 5
  - (b) |c-d| if  $c \ge d$
- 13. Translate the given algebraic statement into a geometric statement about distance.
  - (a) |x-3| < 2
  - (b)  $|x+7| \le 3$
- 14. Draw a graph representing each of the following algebraic statements.
  - (a) |x 17| > 7
  - (b)  $|x 17| \le 7$
- 15. Use a geometric approach to solve the given equation or inequality.
  - (a) |x-2| = 1 (b)  $|x+2| \ge 3$

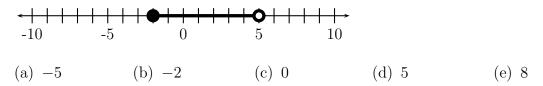
# 2 Solving Equations Worksheet

#### **Concepts:**

- Number Lines
- The Definition of Absolute Value
- Solving Equations The Algebraic Approach
  - One Variable Type; Variables in Denominator; Power Equations
  - Quadratic & Quadratic Type Equations
    - \* Zero Product Property; Quadratic Formula; Completing the Square

(Sections 1.1-1.2)

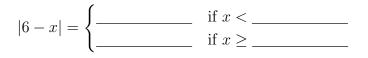
1. Which of the following numbers is included in the graph?



2. Which of the following numbers are included in the interval  $(-\infty, 7) \cup [20, 35)$ ?

(a) $-2,000$	(c) $7.0000001$	(e) $19.99999$	(g) 24	(i) 35
(b) 7	(d) 15	(f) $20.00001$	(h) $34.99999$	(j) 2,000,000

- - (a) Write a distance sentence that corresponds to this number line.
  - (b) Write an algebraic equation or inequality that corresponds to this number line.
- 4. Find the exact value of  $|\pi 6|$ . Your answer may not include absolute value symbols.
- 5. Complete the definition of |6 x|.



6. Solve. (Think about how to undo the operations that are being applied to the variable.)

(a) 
$$\frac{8-2s}{5} = 13$$
 (b)  $-5[14 - (3x+1)^3] = 11$ 

7. Solve for  $d, V = \frac{\pi d^2 h}{4}$ . (The formula is for volume. Does your solution simplify?)

8. Solve.

(a) 
$$\frac{x}{x+2} = \frac{1}{x-5}$$
 (b)  $\frac{3y^2 - 2y + 14}{y^2 + y - 2} = \frac{5}{y-1}$  (c)  $\frac{x}{x+2} = \frac{5}{x} + 1$ 

9. How many solutions does each have? (I)  $x^3 + 5 = 0$  (II)  $x^4 = -4$ 10. Use the Zero Product Property to solve the quadratic equation.

(a) 
$$x^2 - 14 = 3x + 14$$
 (b)  $3x^2 + 16x + 5 = 0$ 

11. Solve the quadratic equation by completing the square.

- (a)  $x^2 2x = 12$  (b)  $9x^2 = 12x + 1$
- 12. Solve the quadratic equation by a method of your choice.

(a) 
$$20x + 35 = 3x^2 + 4x$$
 (b)  $7x^2 + x + 1 = 0$ 

- 13. Find a number k such that the equation has only one real solution.  $x^2 + kx + 25 = 0$
- 14. Solve.
  - (a)  $2x^6 = 9x^3 + 5$  (b)  $3x^8 + x^4 10 = 0$  (c)  $(y-2)^2 + 5(y-2) = 3$

# 3 Absolute Value Equations & Inequalities Worksheet

### **Concepts:**

- Solving Equations Algebraically
- Absolute Value Equations and Inequalities

(Sections 1.2 - 1.2A/4.6A)

1. We have studied the following techniques for solving equations in class: unwrapping a variable, multiplying by a common denominator, taking roots of both sides of an equation, using the zero product property, completing the square, simplifying, using the quadratic formula, using geometry, and substituting in a quadratic type equation.

For each of the following equations, determine which technique you could use to solve the equation. There may be more than one or zero techniques.

- (a)  $3 x + 2x^2 = 5 + x$ (b)  $3x^5 - 7 = 2$ (c)  $x^5 + 3\sqrt{x} = 7$ (d)  $\frac{5}{x+2} - \frac{5+x}{2x} = \frac{7x}{x+2}$ (e) -4x + 3[5(x+7) - 3x + 2] = 7(x+5)(f)  $\frac{1}{x+2} = 5x$ (g)  $x^4 + 2x^2 - 1 = 0$ (h)  $x^4 + 2x - 1 = 0$
- 2. For which values of k does the equation have exactly two real solutions.

$$kx^2 + 8x + 1 = 0$$

3. (Question 86, Section 1.2) Data from the U.S. Department of Health and Human Services indicates that the cumulative number N of reported cases of AIDS in the United States in year x can be approximated by the equation

$$N = 3362.1x^2 - 17,270.3x + 24,043,$$

where x = 0 corresponds to 1980. In what year did the total reach 550,000?

4. Complete the definition of |x - 6|.

$$|x-6| = \begin{cases} & \text{if } x < \\ & \text{if } x \ge \\ & & \text{if } x \ge \\ \end{cases}$$

- 5. Solve each equation or inequality algebraically. As you solve the equation or inequality, discuss the geometry (i.e., the number line) behind each step.
  - (a) |x-3| = 4 (b) |x+5| > 2 (c)  $|4-x| \le 6$
- 6. Solve each equation or inequality or algebraically.
  - (a) 3|4x+1| = 5 (b) 3|4-x|+6 = 2 (c) |3x-2| = 5x+4
  - (d)  $2|x-1| + 4 \le 8$  (d) |5x+7| + 4 > 10 (d) |5x+7| + 4 > 1
- 7. (Question 13, Section 1.2A) In statistical quality control, one needs to find the proportion of the product that is not acceptable. The upper and lower control limits are found by solving the following equation (in which  $\overline{p}$  is the mean percent defective and n is the sample size) for CL.

$$|CL - \overline{p}| = 3\sqrt{\frac{\overline{p}(1 - \overline{p})}{n}}$$

Find the control limits when  $\overline{p} = .02$  and n = 200.

- 8. (Question 33, Section 4.6A) A factory manufactures iron rods. The customer specifies the length of the rod, and the factory produces the desired item. Obviously, they aren't going to be able to make the length *exact*, but they guarantee that the manufactured rod will be within 1 millimeter of the requested length.
  - (a) If a customer orders a rod of length 3 meters, what is the range of acceptable lengths of rod for the factory to produce?
  - (b) If a customer orders a rod of length  $\delta$  meters, what is the acceptable range? Write your answer as a single absolute value inequality, and label all variables you use.

## 4 The Cartesian Coordinate System Worksheet

### **Concepts:**

- The Cartesian Coordinate System
- Graphs of Equations in Two Variables
- *x*-intercepts and *y*-intercepts
- Distance in Two Dimensions and the Pythagorean Theorem
- Equations of Circles
- Midpoints
- Lines

(Sections 1.3-1.4)

- 1. Which of the following points is on the graph of  $x^2 y^3 = 1$ ?
  - (a) (3,2) (b) (0,1) (c) (1,0) (d) (0,-1)
- 2. Find the intercepts of the graph of  $x^2 y^3 = 1$ .
- 3. Find the point on the y-axis that is equidistant to (2,5) and (-1,3).
- 4. Find the perimeter & area of the triangle with vertices A(-2, -5), B(-2, 7), and C(10, 10).
- 5. What are the center and radius of the circle defined by  $(x+3)^2 + y^2 = 9$ ? Sketch the graph.
- 6. Is the graph of any of the following a circle? If so, find the center and radius.
  - (a)  $x^2 + 6x + y^2 10y + 26 = 0$
  - (b)  $4x^2 8x + 4y^2 + 4y 23 = 0$
  - (c)  $x^2 2x + y^2 + 8y + 26 = 0$
- 7. Describe the graph of  $x^2 + 4x + y^2 + 10y + 29 = 0$ .

- 8. A diameter of a circle has endpoints (1, -2) and (3, 6). Find an equation for the circle.
- 9. The center of a circle is  $(5, \frac{1}{4})$ , and circle passes through the point (-2, 3). Find an equation for the circle.
- 10. For each point, determine if it is inside, outside, or on the circle,  $(x+5)^2 + (y-3)^2 = 36$ .
  - (a) (4,2) (b) (-5,0) (c) (1,2)
- 11. Which of the following are equations for the line through the points (1, 5) and (2, -3)?

(a) $y+3 = -8(x-2)$	(d) $y+3 = \frac{1}{8}(x-2)$	(g) $y - 5 = -8(x - 1)$
(b) $y = -8(x-1) + 5$	(e) $y-5 = \frac{-1}{8}(x-1)$	(h) $y - 5 = -8x - 1$
(c) $y+3 = \frac{-1}{8}(x-2)$	(f) $y - 5 = \frac{1}{8}(x - 1)$	(i) $y - 5 = \frac{-1}{8}x - 1$

- 12. **TRUE or FALSE:** The line through the points (0, -1) and (-1, 4) is perpendicular to the line through the points (2, -8) and (7, -7).
- 13. **TRUE or FALSE:** The line through the points (-5, -7) and (-8, -5) is parallel to the line through the points (-7, 0) and (-10, 2).
- 14. Find an equation for the line that is parallel to  $y = \frac{5}{6}x + 7$  and contains the point (3,21).
- 15. Find an equation for the line that is perpendicular to  $y = \frac{5}{6}x + 4$  and contains the point (0,14).
- 16. How many intersection points could:
  - (a) a circle and a line have? (b) two lines have?
- 17. Find the points of intersection between the graphs of y = 4 and  $y = x^2 3x$ . Sketch the graphs.
- 18. Find the points of intersection between the graphs of 3x + 5y = 1 and 2x 10y = 7. Sketch the graphs.

# 5 Linear Models and Circles & Ellipses Worksheet

### Concepts:

- Solving Applied Problems Involving Linear Models
- Circles & Ellipses:
  - sketch a graph from an equation
  - $-\,$  find an equation given information about a graph
- Solving Applied Problems Involving Ellipses

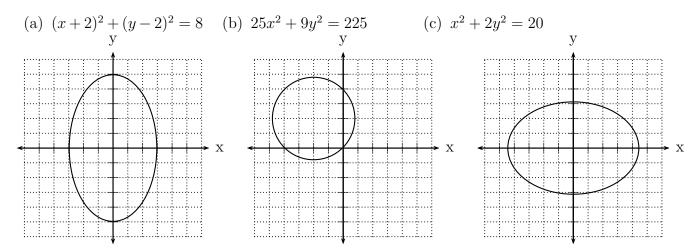
(Sections 1.4 & 10.1)

1. This problem was taken from the Precalculus textbook by David H. Collingwood and K. David Prince. It is available at http://www.math.washington.edu/~m120/.

Two planes flying opposite directions (North and South) pass each other 80 miles apart at the same altitude. The northbound plane is flying 200mph (miles per hour) and the southbound plane is flying 150 mph. How far apart are the planes in 20 minutes? When are the planes 300 miles apart?

- 2. (Question 78, Section 1.4) According to the Center of Science in the Public Interest, the maximum healthy weight for a person who is 5 feet 5 inches tall is 150 pounds, and the maximum weight for someone 6 feet 3 inches tall is 200 pounds. The relationship between weight and height here is linear.
  - (a) Find a linear equation that gives the maximum healthy weight y for a person whose height is x inches over 4 feet 10 inches. (Thus x = 0 corresponds to 4 feet 10 inches, x = 2 to 5 feet, etc.)
  - (b) What is the maximum healthy weight for a person whose height is 5 feet? 6 feet?
  - (c) How tall is a person who is at a maximum healthy weight of 220 pounds?

3. Match each of the following equations with the appropriate graph.



- 4. For each of the following, identify which conic section is represented by the equation. If it is a circle, give its center and radius. If it is an ellipse, give its center, vertices, and foci. Sketch the graph of each.
  - (a)  $5x^2 + 5y^2 + 20x + 20 = 30y$ .

(b) 
$$\frac{(x-1)^2}{8} + \frac{(y+3)^2}{8} = 1$$

(c) 
$$\frac{(x-1)^2}{36} + \frac{(y-6)^2}{8} = 1.$$

(d) 
$$x^2 + 5y^2 - 2x + 30y + 26 = 0.$$

- 5. Find the equation of the ellipse that satisfies the given conditions. HINT: there may be more than one equation.
  - (a) Center (-4, 5); endpoints of major and minor axes: (0, 5), (-4, 16), (-8, 5), (-4, -6).
  - (b) Center (-3, -9); foci on the line y = -9; major axis of length 11; minor axis of length 5.
- 6. (Question 65, Section 10.1) The bottom of a bridge is shaped like half an ellipse and is 20 feet above a 100-foot wide river at its center. Find the height of the bridge bottom over a point on the river 25 feet from the center of the river.
- 7. (Question 61, Section 10.1) The orbit of the moon around the earth is an ellipse with the earth as one focus. If the length of the major axis of the orbit is 477,736 miles and the length of the minor axis is 477,078 miles, find the minimum and maximum distances from the earth to the moon.

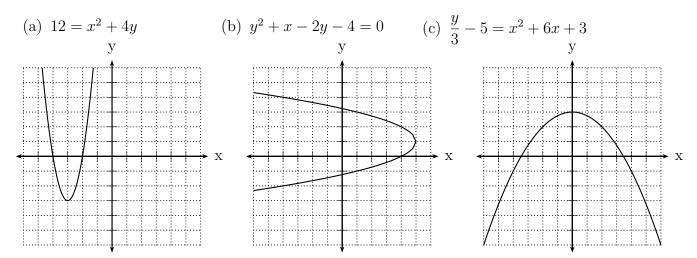
## 6 Parabolas and Functions Worksheet

#### **Concepts:**

- Parabolas:
  - sketch a graph from an equation
  - find an equation given information about a graph
- Solving Applied Problems Involving Parabolas
- Definition of a Function
- Domain of a Function

(Section 10.3 & 3.1)

1. Match each of the following equations with the appropriate graph.



- 2. For each of the following, determine the vertex, focus, and directrix of the parabola *without graphing* and state whether it opens upward, downward, left, or right. Sketch the graph of each.
  - (a)  $y + (x+8)^2 = 3$ .
  - (b)  $x = 5(y 3)^2$ .
  - (c)  $2y^2 = x 4y 4$ .
  - (d)  $4x^2 40x 2y + 111 = 0.$

- 3. Find the equation of the parabola satisfying the given conditions.
  - (a) Vertex (-2, -5); (-4, -4) and (-4, -6) on graph.
  - (b) Vertex (0,7); axis x = 0; (2,-5) on graph.
- 4. (Question 75, Section 10.3) A parabolic satellite dish is 4 feet in diameter and 1.5 feet deep. How far from the vertex should the receiver be placed to catch all the signals that hit the dish? [HINT: See Example 11 in Section 10.3 of your textbook.]
- 5. (Question 81, Section 10.3) The cables of a suspension bridge are shaped like parabolas. The cables are attached to the towers 100 feet from the bridge surface, and the towers are 420 feet apart. The cables touch the bridge surface at the center (midway between the towers). At a point on the bridge 100 feet from one of the towers, how far is the cable from the bridge surface?
- 6. The amount of postage required to mail a first-class letter is determined by its weight. In this situation, is weight a function of postage? Or vice versa? Or both?
- 7. In the following identify the independent (input) and the dependent (output) variables.
  - (a) The amount of property tax you owe is a function of the assessed value of your home in dollars.
  - (b) The length of your fingernails is a function of the amount of time that has passed since your last manicure.
  - (c) The cost of mailing a letter is a function of the weight of the package in ounces.
  - (d) The amount of water required for your lawn (in gallons) is a function of the temperature (in degrees).
  - (e) A person's blood alcohol level is a function of the number of alcoholic drinks consumed in a 2-hour period.
- 8. Find the domain of each of the following functions. Write the domain in interval notation.

(a) 
$$a(x) = x^5 + 2x^2 - 6$$
  
(b)  $b(x) = \frac{x+1}{x-5} + \frac{x+4}{2x+1}$   
(c)  $c(x) = \sqrt[3]{x+7}$   
(d)  $d(x) = \sqrt{x+7}$   
(e)  $e(x) = \frac{1}{\sqrt[3]{10-x}}$   
(f)  $f(x) = \frac{1}{\sqrt[4]{10-x}}$   
(g)  $g(x) = \sqrt{x+7} - \frac{1}{x^2-5}$   
(h)  $h(x) = \begin{cases} \frac{1}{x} & \text{if } x \le -2\\ \frac{1}{x+3} & \text{if } x > -2 \end{cases}$ 

# 7 Functions and Functional Notation Worksheet

#### **Concepts:**

- Review
- Function Notation
- Piecewise-defined Functions

## (Sections 3.1-3.2)

- 1. Solve:
  - (a)  $|x-5| \le 2$  (b)  $\frac{3}{x} + \frac{5}{x+2} = 2$  (c) |x+2| > 2
- 2. If (1,1) is on a circle with center (2,-3), what is the radius of the circle? Find the equation of the circle.
- 3. The number of recreational visits to the National Parks of the United States is displayed in the table. The number of visits to the national parks, *p*, is a function of the year, *t*.

	-			
Year	Recreational Visits to US			
	National Parks			
	(millions of people)			
1990	258.7			
1995	269.6			
1999	287.1			
2000	285.9			
2001	279.9			
2002	277.3			
2003	266.1			
2004	276.4			
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Source: www.census.gov

- (a) Solve p(t) = 277.3 for *t* and explain the meaning of the solution.
- (b) Evaluate p(2000) and write a sentence explaining what the numerical value you find means in its real-world context.
- (c) Estimate p(2010) and discuss the accuracy of your prediction.
- (d) Estimate the solution to p(t) = 300 and discuss the accuracy of your approximation.

4. Evaluate the given function at the given values:

(a) 
$$g(t) = \frac{t+2}{t-2}$$
;  $g(-2)$ ,  $g(2)$ ,  $g(0)$ ,  $g(a)$ ,  $g(a^2-2)$ ,  $g(a+1)$   
(b)  $h(u) = 2|u-1|$ ;  $h(-2)$ ,  $h(0)$ ,  $h(\frac{1}{2})$ ,  $h(2)$ ,  $h(x+1)$ ,  $h(x^2+2)$ 

5. Let  $g(x) = x^2 + x$ .

(a) What is 
$$g(x^2)$$
?(c) What is  $\frac{g(2x)}{2g(x)}$ ?(e) What is  $g(x-1)$ ?(b) What is  $(g(x))^2$ ?(d) What is  $g(a+b)$ ?(f) What is  $\frac{g(x+h)-g(x)}{h}$ ?

6. Let

$$h(x) = \begin{cases} 10 & \text{if } x < -4 \\ x^2 + 10 & \text{if } -4 \le x \le 6 \\ x + 15 & \text{if } x > 6 \end{cases}$$
(a) Find  $h(5)$ .
(b) Find  $h(-4)$ .
(c) Find  $h(-6)$ .
(d) Find  $h(10)$ .

7. An epidemiological study of the spread of malaria in a rural area finds that the total number P of people who contracted malaria t days into an outbreak is modeled by the function

$$P(t) = -\frac{1}{4}t^2 + 7t + 180, \qquad 1 \le t \le 14.$$

- (a) How many people have contracted malaria 14 days into the outbreak?
- (b) How many people have contracted malaria 6 days into the outbreak?
- 8. According to http://revenue.ky.gov/, the tax brackets for the 2015 Kentucky state taxes are described below.

more than	but not more than	then your tax is	plus:
\$0	\$3,000	2.00% of your taxable income	\$0
\$3,001	\$4,000	3.00% of the amount over \$3,000	\$60
\$4,001	\$5,000	4.00% of the amount over \$4,000	\$90
\$5,001	\$8,000	5.00% of the amount over \$5,000	\$130
\$8,001	\$75,000	5.80% of the amount over \$8,000	\$280
\$75,001		6.00% of the amount over \$75,000	\$4,160

If your taxable income on Form 740, line 11 is:

They give the following example.

Taxable income 6,800. Tax = (6,800-5,000) × .05(5%) + 130 = 220.

Use this tax table to write a piecewise-defined function KYTax(I) where *I* is the adjusted gross income on Form 740 line 11 of the Kentucky tax form 740, and KYTax(I) is the amount of tax owed by a resident of Kentucky.

# 8 MA 110 Exam 1 Practice Worksheet

Sections 1.1 - 1.4, 4.6A, 10.1, 10.3, 3.1 - 3.2

Do not rely solely on this practice exam! Make sure to study homework problems, other work sheets, lecture notes, and the book!!!

- 1. Which of the following statements is equivalent to the algebraic statement,  $|x+5| \leq 4$ ?
  - (a) The distance between x and 5 is greater than or equal to 4 units.
  - (b) The distance between x and 5 is less than or equal to 4 units.
  - (c) The distance between x and -5 is less than 4 units.
  - (d) The distance between x and -5 is less than or equal to 4 units.
  - (e) The distance between x and -5 is greater than or equal to 4 units.
- 2. Solve the equation,  $A = \frac{h}{2}(d+c)$ , for d.

(a) 
$$d = \frac{2A+h}{ch}$$
  
(b) 
$$d = \frac{2A-ch}{h}$$
  
(c) 
$$d = \frac{2A-ch}{ch}$$
  
(d) 
$$d = \frac{2hA-ch}{2}$$
  
(e) 
$$d = \frac{A+2ch}{h}$$

- 3. Solve the inequality,  $|4x + 3| \le 3$ .
  - (a)  $-6 \le x \le 0$ (b)  $-1/4 \le x \le 0$ (c)  $-3/2 \le x \le 0$ (d)  $0 \le x$ (e)  $0 \le x \le 3$

4. Find the x- and y-intercepts of the graph of the equation,  $x^2 - 5xy + 7y^2 = 1$ .

(a) x = -1, 1 and  $= -\sqrt{\frac{1}{5}}, \sqrt{\frac{1}{5}}$ (b) x = -1, 1 and  $= -\frac{1}{5}, \frac{1}{5}$ (c) x = -1, 1 and  $= -\frac{1}{12}, \frac{1}{12}$ (d) x = -1, 1 and  $= -\frac{1}{7}, \frac{1}{7}$ (e) x = -1, 1 and  $= -\sqrt{\frac{1}{7}}, \sqrt{\frac{1}{7}}$ 

5. Find a real number k such that the line, 7x - ky + 6 = 0, has a y-intercept of -7.

(a) k = 7(b)  $k = \frac{6}{13}$ (c)  $k = -\frac{7}{6}$ (d)  $k = -\frac{6}{7}$ (e)  $k = \frac{7}{36}$ 

- 6. Find the equation of the ellipse with a center at (0,0); vertices at (5,0) and (-5,0); and a minor axis of length 6.
  - (a)  $\frac{x^2}{25} + \frac{y^2}{9} = 1$ (b)  $\frac{x^2}{25} + \frac{y^2}{4} = 1$ (c)  $\frac{x^2}{16} + \frac{y^2}{4} = 1$ (d)  $\frac{x^2}{9} + \frac{y^2}{25} = 1$ (e)  $\frac{x^2}{16} + \frac{y^2}{9} = 1$
- 7. Given  $g(t) = t^2 5$ , find g(5 r).
  - (a)  $g(5-r) = r^2 + 10r$
  - (b)  $g(5-r) = r^2 10r 20$
  - (c)  $g(5-r) = r^2 10r$
  - (d)  $g(5-r) = r^2 + 10r + 20$
  - (e)  $g(5-r) = r^2 10r + 20$
- 8. Find all real solutions of the equation,  $6z^4 7z^2 + 2 = 0$ , exactly.

9. Determine the vertex, focus, and directrix of the parabola,  $x = y^2 + y + 7$ , WITHOUT graphing. State whether it opens up, down, left or right. SHOW YOUR WORK!

- 10. Given the function,  $g(t) = \sqrt{(t-5)^2}$ , find:
  - (a) g(-2) and g(2)
  - (b) the domain of g