9 Graphing Functions

Concepts:

- Review: Domain of a Function
- Graphs of Functions

You are responsible for graphs of basic functions. You will need to know how to graph some basic functions without the help of your calculator.

- Linear Functions ($f(x) = mx + b$)
- Power Functions ($f(x) = x^n$) where n is a positive integer.
- Square Root Function ($f(x) = \sqrt{x}$)
- Greatest Integer Function ($f(x) = [\![x]\!]$)
- Absolute Value Function ($f(x) = |x|$)
- Piecewise-defined Functions.

(Section 3.3)

1. Find the domain of each of the following functions. Write in interval notation.

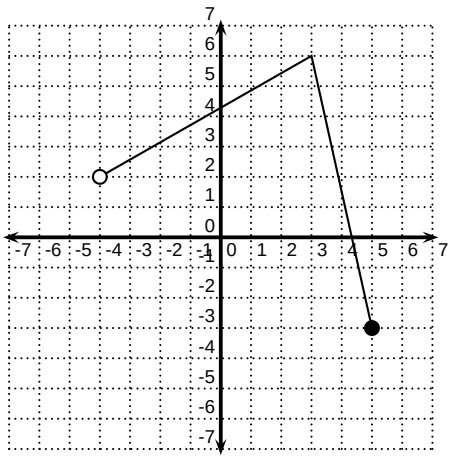
   (a) $a(x) = -2x^8 + 2x^2 - 37$

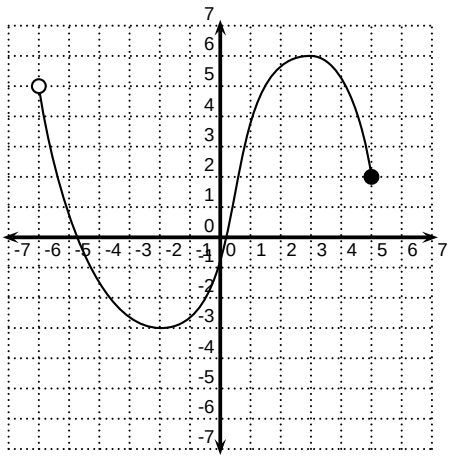
   (b) $b(x) = \dfrac{3x-4}{x+7} + \dfrac{4+x^2}{3x-8}$

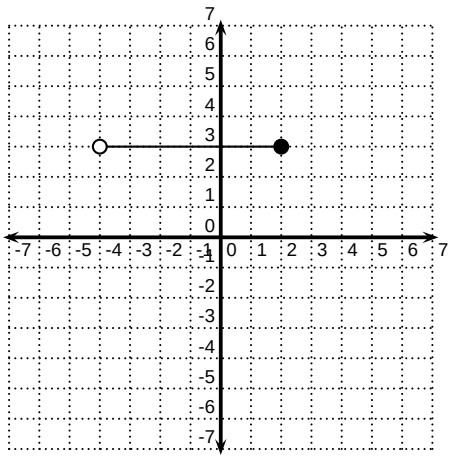
   (c) $f(x) = \dfrac{1}{\sqrt{x+7}}$

   (d) $g(x) = -\dfrac{\sqrt{x+7}}{x^2-7}$

2. Find the domain and range of each of the following functions.





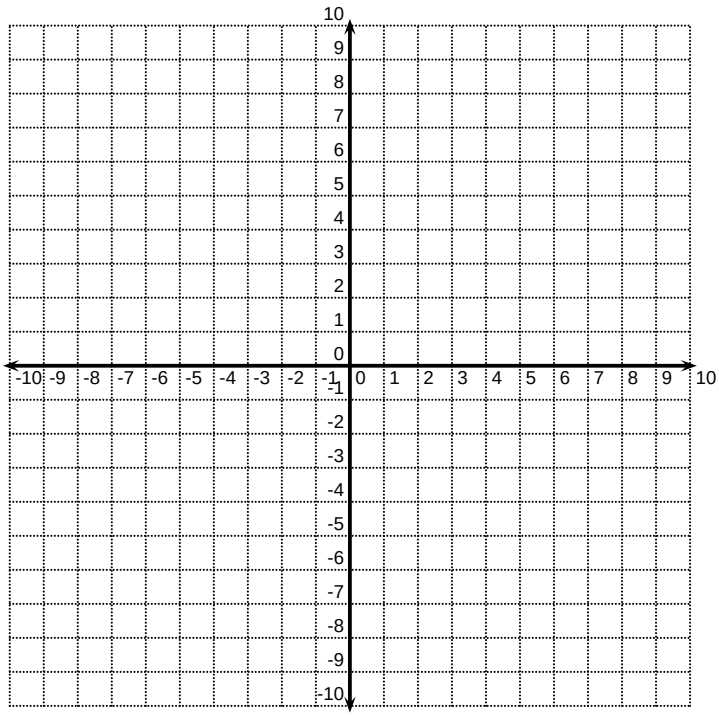


3. For each of the following, sketch a plausible graph of the given function, labeling the axes appropriately and indicating a reasonable domain and range.

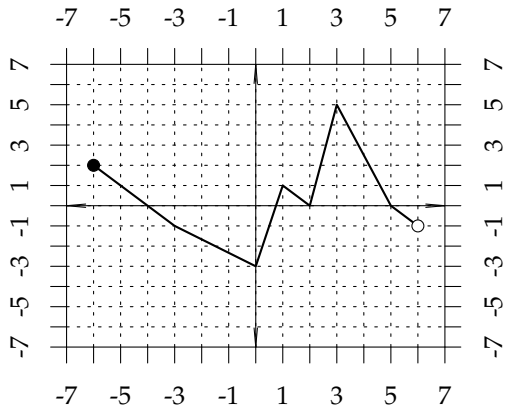
   (a) The cost of mailing a package as a function of the weight of the package in ounces.

   (b) The temperature of the freshly poured coffee in your cup sitting on your desk as a function of the time.

4. Sketch the graph of $f(x) = \begin{cases} 3x + 18 & \text{if } x \leq -3 \\ -x^2 + 4 & \text{if } x > -3 \end{cases}$



5. The graph of $y = f(x)$ is shown below.



(a) For what x values is $f(x) \geq 0$? Write your answer in interval notation.

(b) For what x values is $f(x) < 0$? Write your answer in interval notation.

(c) For what x values is $f(x) \leq -1$? Write your answer in interval notation.

(d) What is $\dfrac{f(3) - f(2)}{2f(-6)}$?

6. (Question 44, Section 3.3) A plane flies from Austin, TX, to Cleveland, OH, a distance of 1200 miles. Let f be the function whose rule is $f(t) =$ distance (in miles) from Austin at time t hours. Draw a plausible graph of f under the given circumstances. Many correct answers may vary.

(a) The flight is nonstop and takes less than 4 hours.

(b) Bad weather forces the plane to land in Dallas (about 200 miles from Austin), remain overnight (for 8 hours), and continue the next day.

(c) The flight is nonstop, but owing to heavy traffic, the plane must fly in a holding pattern over Cincinnati (about 200 miles from Cleveland) for an hour before going on to Cleveland.

10 Transformations and Composition

Concepts:

- How does a graph transformation move a point on a graph?
- Applying transformations to the graph of a function
- Operations on functions
- The domain of a composition of functions.

(Section 3.4 & 3.5)

1. Suppose that the graph of f contains the point $(-4, 7)$. Find a point that must be on the graph of g. Explain how you had to move the point on the original graph f to obtain a point on the new graph g.
   (a) The graph of $g(x) = f(x) + 5$ must contain the point _____.

   (b) The graph of $g(x) = f(x - 5)$ must contain the point _____.

   (c) The graph of $g(x) = \frac{1}{5}f(x)$ must contain the point _____.

   (d) The graph of $g(x) = f\left(\frac{1}{5}x\right)$ must contain the point _____.

   (e) The graph of $g(x) = 3f(7x + 1) + 5$ must contain the point _____.

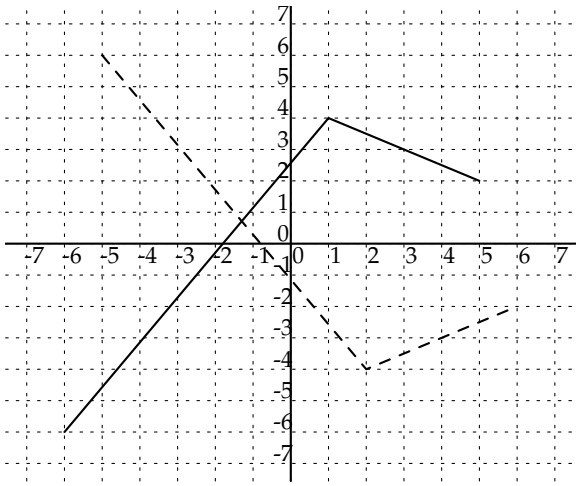
2. Describe the transformations that will transform the graph of $f(x) = x^2$ into the graph of g.
   (a) $g(x) = 2x^2 + 3$
   (b) $g(x) = -2(x + 5)^2 - 7$
   (c) $g(x) = \frac{1}{3}x^2 - 2x + 5$

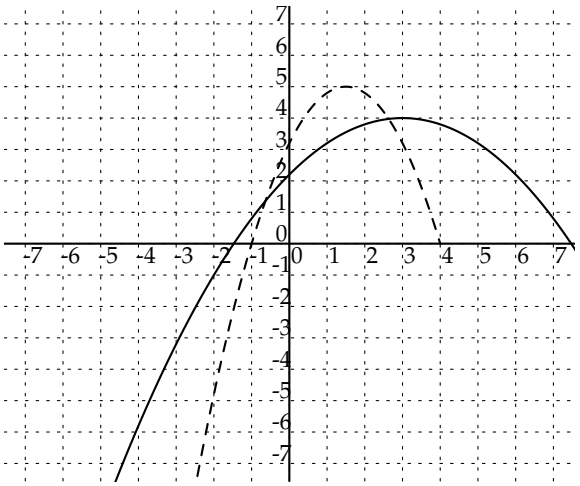
3. Write the rule of a function g whose graph can be obtained from the graph of f by performing the transformations.
   (a) $f(x) = x^3 + 5$; shift the graph horizontally 3 units to the right and then vertically upward 5 units.

   (b) $f(x) = 3x^2 - 2x - 4$; reflect the graph in the x-axis, then shift it vertically downward 7 units.

   (c) $f(x) = \sqrt[4]{x}$; shift the graph horizontally 4 units to the left, stretch it away from the x-axis by a factor of 3, and shift it vertically upward 2 units.

4. For each of the following graphs $f(x)$ is the solid line and $g(x)$ is the dashed line. Describe the transformations that transform the graph of $f(x)$ into the graph of $g(x)$.





5. (Question 47, Section 3.4) A factory has a linear cost function $c(x) = ax + b$, where b represents fixed costs and a represents the variable costs (labor and materials) of making one time, both in thousands of dollars.
   (a) If property taxes (part of the fixed costs) are increased by \$35,000 per year, what effect does this have on the graph of the cost function?
   (b) If the variable costs increase, what effect does this have on the graph of the cost function?

6. Let $f(x) = x^2 + 3$ and $g(x) = 2 - x$.
   (a) Find $f(g(x))$.
   (b) Find $g(f(x))$.
   (c) Find $f(f(x))$.
   (d) Find $g(g(x))$.
   (e) Find $g(g(g(x)))$.

7. Let $f(x) = \dfrac{x}{\sqrt{x+1}}$ and $g(x) = 2x + 5$.
   (a) Find $f(g(x))$.
   (b) Find the domain $f(g(x))$.
   (c) Find $g(f(x))$.
   (d) Find the domain $g(f(x))$.

8. Write $h(x)$ as a composition of three simpler functions. (**HINT:** Think of placing x in a box. What happens first? second? *etc.*? There may be more than one correct answer.)
   (a) $h(x) = \sqrt{x^3 + 5}$
   (b) $h(x) = \dfrac{3}{x^5 - 7}$
   (c) $h(x) = 3(x + 5)^2$
   (d) $h(x) = (3x + 5)^2$

9. Write $g(x) = 2(f(3(x + 1)) - 6)$ as a composition of five simpler functions (Hint:One of these functions should be f.)

11 Inverse Functions and Compositions

Concepts:

- Compose functions
- Write a function as a composite
- Find inverse functions algebraically
- Properties of inverse functions
- Graph inverse functions

(Section 3.5 & 3.7)

1. Let $f(x) = x^2 - 2x - 7$ and $g(x) = x - 3$.
   (a) Find $f(g(x+1))$.
   (b) Find $f(x^2)$.
   (c) Find $(g(x))^2$.
   (d) Find $f(g(\sqrt{x}))$.

2. Write $h(x)$ as a composition of simpler functions.
   (a) $h(x) = \dfrac{\sqrt[3]{x^2-3}+1}{4}$.

   (b) $h(x) = 8[(x-2)^2 - 1] + 3$.

3. You have a 20% off coupon from the manufacturer for the purchase of a new cell phone. Your cell provider is also offering a 10% discount on any new phone. You make two trips to cell phone stores to look at various phones. On your first trip, you speak with Miranda. Miranda tells you that you can take advantage of both the coupon and the discount. She will apply the discount and then apply the coupon to the reduced price. On your second trip, you talk to Ariel. She also says that you can take advantage of both deals, but she tells you that she will apply the coupon and then apply the discount.

   Let x represent the original sticker price of the cell phone.
   (a) Suppose that only the 20% discount applies. Find a function f that models the purchase price of the cell phone as a function of the sticker price x.
   (b) Suppose that only the 10% coupon applies. Find a function g that models the purchase price of the cell phone as a function of the sticker price x.
   (c) If you can take advantage of both deals, then the price you will pay is either $f(g(x))$ or $g(f(x)$, depending on the order in which the coupon and the discount are applied to the price. Find $f(g(x))$ and $g(f(x))$.
   (d) The price that Miranda is offering you is modeled by _____.
   (e) The price that Ariel is offering you is modeled by _____.

4. Find the inverse of the one-to-one functions below. Find the domains and ranges of the function and its inverse.

   (a) $f(x) = \dfrac{2 - x^3}{7}$

   (b) $g(x) = \dfrac{x + 7}{x + 5}$

   (c) $h(x) = \sqrt{x^5 - 2}$

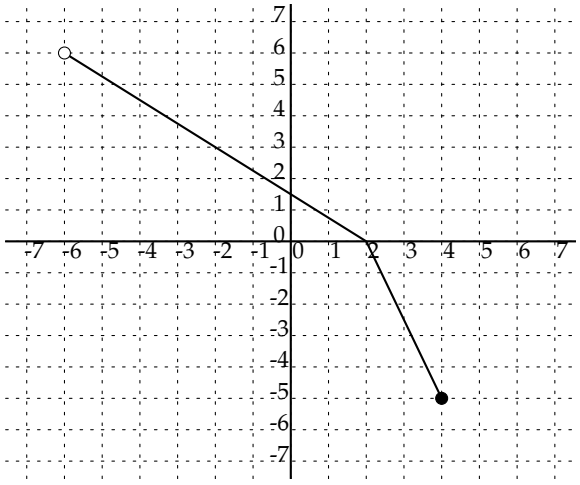
5. Use composition of functions to determine if the pair of functions are inverses of each other.

   (a) $f(x) = \dfrac{1}{x}$ and $g(x) = \dfrac{1}{x}$

   (b) $k(x) = \sqrt[3]{x} - 3$ and $m(x) = x^3 + 27$

   (c) $h(x) = \dfrac{3x + 2}{7}$ and $j(x) = \dfrac{7x - 2}{3}$

6. The graph of a one-to-one function is shown below. (How do you know that this is a one-to-one function?) Sketch the graph of its inverse on the same set of axes.



12 Quadratic & Polynomial Functions Worksheet

Concepts:

- Graphs of Quadratic Functions
- Algebraic Forms of Quadratic Functions
  - Standard Form, $f(x) = ax^2 + bx + c$
  - Vertex Form, $f(x) = a(x - h)^2 + k$
- Understand the Meaning of the Vertex
- Polynomial Functions & Polynomial Division

(Sections 4.1 & 4.2)

1. Find the equation of the unique quadratic function with the following properties and graph the function.
   (a) passes through $(-2, 0)$ and $(5, 0)$ and has a leading coefficient of 5.

   (b) passes through the points $(3, 0)$, $(-2, 0)$ and $(0, 12)$.

   (c) passes through $(0, 0)$, $(1, -1)$, and $(2, 0)$.

   (d) has vertex $(2, 4)$ and passes through the point $(0, -2)$.

2. A golf ball is hit so that its height h in feet after t seconds is $h(t) = -16t^2 + 60t$.
   (a) What is the initial height of the golf ball?

   (b) How high is the golf ball after 1.5 seconds?

   (c) Find the maximum height of the golf ball *algebraically*.

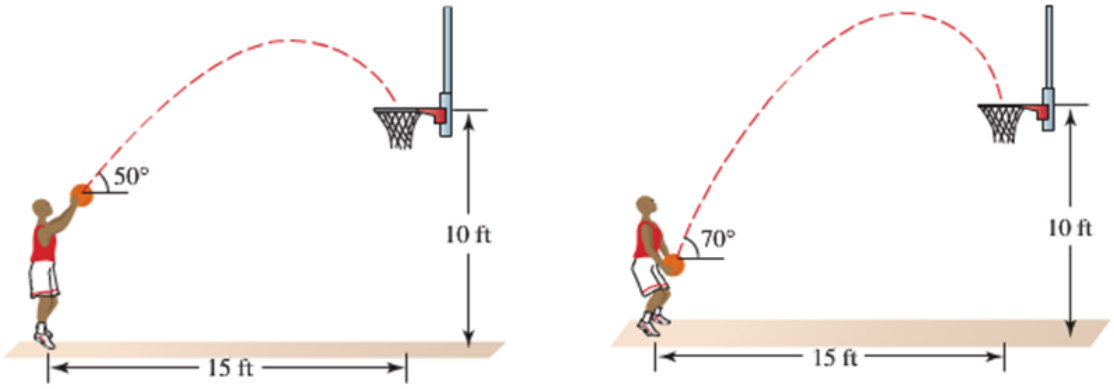
3. **[Challenge]** When a basketball player shoots a foul shot, the ball follows a parabolic arc. This arc depends on both the angle and velocity with which the basketball is released. If a person shoots the basketball overhand from a position 8 feet above the floor, then the path can sometimes be modeled by the parabola $y = \dfrac{-16x^2}{0.434v^2} + 1.15x + 8$, where v is the velocity of the ball in feet per second, as illustrated in the first figure. (Source: C. Rist, *The Physics of Foul Shots*.)

   (a) If the basketball hoop is 10 feet high and located 15 feet away, what initial velocity v should the basketball have?

   (b) Check your answer from part (a) graphically. Plot the point (0, 8) where the ball is released and the point (15, 10) where the basketball hoop is. Does your graph pass through both points?

   (c) What is the maximum height of the basketball?

   (d) If a person releases a basketball underhand from a position 3 feet above the floor, it often has a steeper arc than if it is released overhand and the path sometimes may be modeled by $y = \dfrac{-16x^2}{0.117v^2} + 2.75x + 3$. See the second figure below. Complete parts (a), (b), and (c) from the first part. Then compare the paths for an overhand shot and an underhand shot.



4. Find the maximum value of the function $f(x) = -3x^2 + 10x + 4$.

5. Evaluate $\dfrac{x^3 - 2x^2 + x - 2}{x - 4}$ and express the result in the form $P(x) = D(x)Q(x) + R(x)$.

6. Use the remainder from the above problem to decide if $x - 4$ is a factor of $x^3 - 2x^2 + x - 2$ and to find $P(4)$.

7. Completely factor $f(x) = x^3 - x^2 - 2x + 2$ by finding one root and long division or factoring to find the others. Factors should be exact.

8. Find the zeros of the function $f(x) = 6x^2 - 19x - 36$. Use these zeros to help you factor this function.

9. (Exercise 67, Section 4.2) Use the Factor Theorem to show that for every real number c, $(x - c)$ is **not** a factor of $x^4 + x^2 + 1$.

13 Polynomial Functions and Their Graphs

Concepts:

- How the Leading Term Affects the Shape of the Graph
- End Behavior
- Number of Local Extrema
- Multiplicity of Roots

(Section 4.2 & 4.4)

1. What is the remainder when $f(x) = 2x^{20} - 5x^{17} - 3x^{10} + 2x^3 + 7$ is divided by $x + 1$? (Use the Remainder Theorem)

   Is $x + 1$ a factor of $f(x)$? Why or why not? (Use the Factor Theorem)

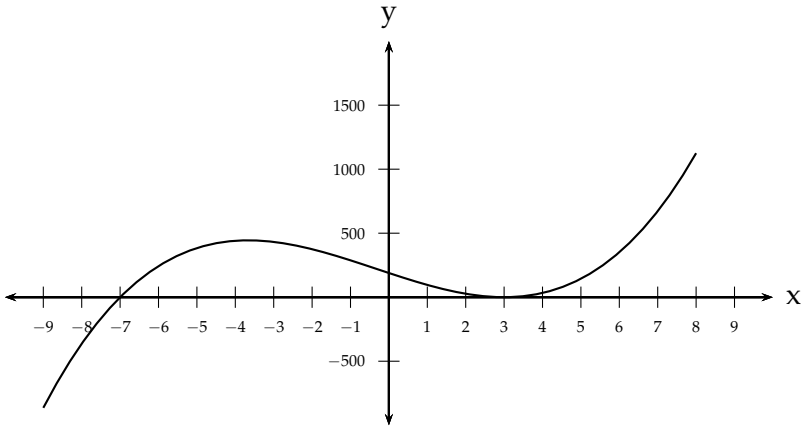
2. What is the maximum number of roots of the polynomial $P(x) = 3x^5 + 2x^7 - 3x^2 + 1$?

3. Describe the end behavior of each polynomial. Use correct mathematical symbols.
   (a) $P(x) = 2x^5 - 3x^2 + 76$

   (b) $Q(x) = -55x^{100} + 15x^{75} - 3$

   (c) $S(x) = (1 - 2x)^{11}(x + 5)^4$

4. The graph of a polynomial $P(x)$ is shown below.



   (a) Is the degree of the polynomial even or odd?

   (b) Is the leading coefficient positive or negative?

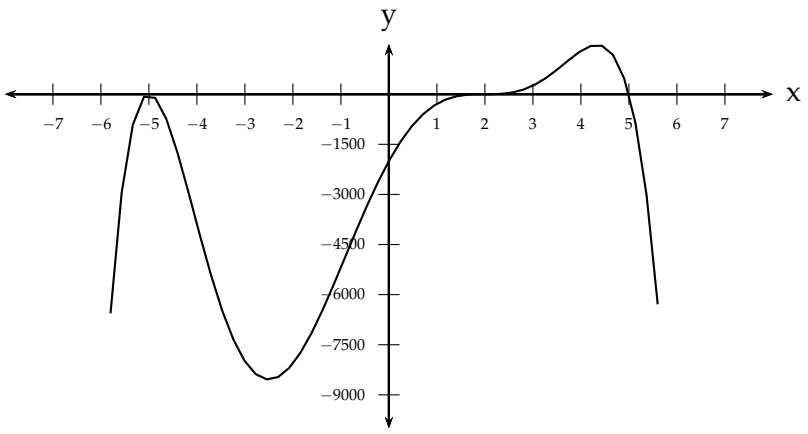
   (c) What can you say about the factors of this polynomial?

   (d) Can you find a formula for the polynomial if you know that the degree of the polynomial is less than or equal to 4 and that $P(1) = 24$

5. Find two polynomials of different degrees that both have only the roots 1, -2 and 5 and that have the same end behavior.

6. Let $f(x) = 3(2x+5)^3(x-9)^2(x+10)(x-4)$.
   (a) What is the degree of $f(x)$?
   (b) What is the end behavior of $f(x)$?
   (c) What are the roots of $f(x)$?
   (d) What are the x-intercepts of $f(x)$?
   (e) Sketch the graph of $f(x)$.

7. The following is not the graph of $y = -2(x+5)(x-2)(x-5)$. Which of the following are clues that it is not the correct graph?
   (a) The graph crosses the x-axis at (2,0), but it should not cross the x-axis at this point.
   (b) The graph displays the wrong end behavior.
   (c) The graph touches the x-axis at (-5,0), but it should cross at this point.
   (d) The graph has the wrong x-intercepts.
   (e) The graph has too many local extreme points to be the graph of a degree 3 polynomial.



14 Rational Functions & Polynomial and Rational Inequalities Worksheet

Concepts:

- Identifying Rational Functions
- Finding the Domain of a Rational Function
- Graphs of a Rational Function
- Vertical and Horizontal Asymptotes
- Holes in the graphs of rational functions
- Equivalent inequalities
- Solving polynomial and rational inequalities

Sections 4.5 & 4.6

1. Give the domain of each function below:

a.) $f(x) = \frac{\sqrt{2x-4}}{x^2-8x+16}$ b.) $g(x) = \frac{2x^2-x-3}{x^2-1}$

2. Give the end behavior (as $x \to -\infty$ and as $x \to \infty$). Then give the equation of any horizontal asymptotes.

a.) $f(x) = \frac{6x-3}{8-3x}$ b.) $g(x) = \frac{3x+1}{2x^2-9}$

c.) $h(x) = \frac{4x^3-3x^2-7x+5}{1-3x}$ d.) $j(x) = \frac{(x+2)^5(1-5x)^3}{(4x+3)(x-4)^4}$

e.) $k(x) = \frac{(x+1)(2x-3)^2}{5-3x-4x^3}$ f.) $m(x) = \frac{(3x+4)(6-4x)^3}{(5-x)^4(2x+8)^2}$

3. Find all the vertical and horizontal asymptotes, holes, x-intercepts and y-intercepts, then graph each function:

a.) $f(x) = \frac{4x+1}{2-8x}$ b.) $g(x) = \frac{x+3}{x^2-x-12}$ c.) $h(x) = \frac{6x^2+11x+3}{x+2}$

4.) How many gallons of a 11% alcohol solution should be added to 10 gallons of an 20% alcohol solution to produce a solution whose alcohol content is 15%?

5.) Write a rational function that would have vertical asymptotes at x = -3 and x = 2 and has a horizontal asymptote at y = 4. (Answers may vary.)

6.) Write a rational function that would have vertical asymptotes at x=0 and x = 4 and has no horizontal asymptote. (Answers may vary)

7.) Solve each inequality algebraically. Give your answers in interval notation:

a.) $4x - 3 > 8x + 7$ b.) $3x + 5 \leq 6x - 11$ c.) $3 < 6x - 8$

8.) Solve each inequality algebraically by factoring then using signs. Give your answer in interval notation:

a.) $x^2 - 8x + 15 \leq 0$ b.) $x^2 + 2x \geq 3$ c.) $4x^2 - 12x + 9 > 0$

9.) Solve each inequality using signs. Give your answer in interval notation:

a.) $(x + 3)(x - 5) > 0$ b.) $x^4 - 4x^2 \geq 5$

c.) $3x^4 - 6x^2 \geq 0$ d.) $\frac{(x-3)(x+2)}{x-1} \leq 0$

e.) $\frac{x^2-10x+25}{x^2-4} > 0$ f.) $\frac{5}{x-3} > \frac{3}{x+1}$

g.) $\frac{2x+5}{x+1} \geq \frac{x+1}{x-1}$ h.) $\frac{3x-5}{x+2} \leq 2$

15 Polynomial and Rational Inequalities Worksheet & Radicals and Rational Exponents

Concepts:

- Equivalent inequalities
- Solving polynomial and rational inequalities
- nth roots
- Rational and Irrational Exponents
- Radicals Applications

Sections 4.6 & 5.1

1. State the domain in interval notation:

   a.) $f(x) = \frac{1}{x^2-49}$ b.) $h(x) = \sqrt{2x-6}$ c.) $g(x) = \sqrt{x^2-2x-3}$

2. Express each of the following as x to a power:

   a.) $(x^3)^4$ b.) $x^3 \cdot x^4$ c.) $\sqrt[3]{x^4}$

   d.) $\sqrt[4]{x^3}$ e.)$(\frac{1}{x^3})^4$ f.) $(\frac{1}{\sqrt[3]{x}})^4$

3. The length of a rectangle is 5 inches longer than its width. What are the possible widths if the area of the rectangle is at least 500 square inches?

4. How many liters of 40% saline solution must be added to 20 liters of 22% saline solution to result in a mixture that is between 30% and 35% saline?

5. Write each expression without radicals:

   a.) $\frac{2}{\sqrt[4]{x}}$ b.) $\sqrt[5]{x^{10}}$ c.) $\sqrt[6]{\sqrt{a^{12}b^4}}$

   d.) $\sqrt{49c^3}\sqrt[5]{c}$ e.) $\sqrt{\frac{x^5}{y^{12}}}$ f.) $\frac{4}{\sqrt[3]{(x-8)}}$

6. Simplify and express answers exactly (no decimals and no negative exponents):

a.) $\sqrt[2]{32x^6}$ b.) $\sqrt{120a^5b^{-4}}$ c.) $\frac{(x^2)^{1/3}(y^2)^{2/3}}{5x^{2/3}y^2}$

7. Multiply then simplify:

a.) $x^{1/2}(x^{2/3} - x^{4/3})$ b.) $(x^{1/2} - y^{1/2})(x^{1/2} + y^{1/2})$

8. Rationalize the denominator and simplify:

a.) $\frac{3}{\sqrt{x}}$ b.) $\frac{1-x}{1-\sqrt{x}}$ c.) $\frac{5}{\sqrt{1-x}}$ d.) $\frac{1+x}{\sqrt{x}-2}$

9. Kroger gas station stores its gasoline in underground tanks that are cylinders lying on their sides. The volume of gasoline in the tank is given by the formula:

$$V = 40h^2\sqrt{\frac{96}{h} - 0.608}$$

where h is the height of the gasoline (in inches) as measured by a depth stick.

a.) If h=12 inches, how many gallons of gas are in the tank (round to nearest tenth of a gallon)?

b.) If h=4 inches, how many gallons of gas are in the tank (round to the nearest tenth of a gallon)?

10. The period T of a swinging pendulum (in seconds) is

$$T = 2\pi\sqrt{\frac{L}{32}}$$

where L is the length of the pendulum (in feet).

a.) Find the period of a pendulum whose length is 2 feet (round to the nearest hundredth of a second)?

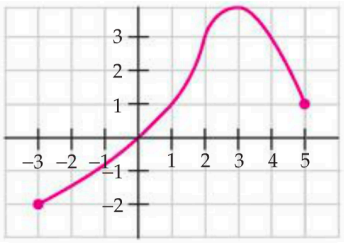
b.) Find the period of a pendulum whose length is 2 yards (round to the nearest hundredth of a second)?

16 MA 110 Exam 2 Practice Worksheet

Sections 3.3 - 3.7, 4.1 - 4.6, 5.1

Do not rely solely on this practice exam! **Make sure to study homework problems, other work sheets, lecture notes, and the book!!!**

1. Given the graph of g to the right, if $t = -3$, find $g(t) + g(6 + t)$.



   (a) 2
   (b) 0
   (c) -1
   (d) 6
   (e) undefined

2. If $f(x) = x^2 + 3$ and $g(x) = 5 - 2x$, find $(f \circ g)(x)$.

   (a) $x^2 - 2x + 8$
   (b) $28 - 4x^2$
   (c) $-2x^3 + 5x^2 - 6x + 15$
   (d) $4x^2 - 20x + 28$
   (e) $-2x^2 - 1$

3. Find a polynomial function g of degree 4 with roots at $x = 0, -1, -3, -2$ and $g(3) = 288$.

   (a) $g(x) = x^4 + 4.8x^3 - 8.8x^2 + 4.8x$
   (b) $g(x) = 0.8x^4 + 4.8x^3 + 8.8x^2 + 4.8x$
   (c) $g(x) = 0.8x^4 + 4.8x^3 + 4.8x^2 + 8.8x$
   (d) $g(x) = x^4 + 4.8x^3 + 8.8x^2 - 0.8x$
   (e) $g(x) = 0.8x^4 - 4.8x^3 + 4.8x^2 + 4.8x$

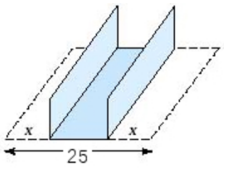
4. For $p(x) = -2(x+3)^3(x-2)^2(x-5)(x+7)$, which of the following is TRUE for the graph of $y = p(x)$.

   (a) The graph will cross the x-axis at $x = 2$
   (b) $y \to \infty$ as $x \to \infty$
   (c) The graph will touch, but not cross, the x-axis at $x = -3$.
   (d) The graph will have exactly 6 local extrema.
   (e) $y \to \infty$ as $x \to -\infty$

5. Write the expression, $\sqrt[7]{t}\sqrt{16t^5}$, without using radicals.

   (a) $4t$
   (b) $4t^{1/37}$
   (c) $4t^{37/14}$
   (d) $4t^{1/14}$
   (e) $4t^{14/37}$

6. (Question 54, Section 4.1) A gutter is to be made by bending up the sides of a 25-inch-wide piece of aluminum. What depth should the gutter be to have the maximum possible cross-sectional area?



   (a) 8.25 inches
   (b) 7.25 inches
   (c) 6.25 inches
   (d) 11.25 inches
   (e) 13.25 inches

7. Solve the inequality, $x^2 - 6x + 8 \leq 0$.

   (a) $(-\infty, 2] \cup [4, \infty)$
   (b) $[2, 4]$
   (c) $[4, 5]$
   (d) $(-\infty, 5] \cup [6, \infty)$
   (e) $[5, 6]$

8. Considering the graph of, $f(x) = \frac{2x^2+4x-6}{x^2-1}$, determine the intercepts, asymptotes, and holes, if any.

9. Describe the transformations that could be applied to the graph of $f(x) = x^2$ in order to obtain the graph of $g(x) = -2x^2 - 16x - 35$

10. Given $g(t) = \frac{t-7}{t-8}$:
   (a) Find $g^{-1}(t)$.
   (b) Find the domain of g and g^{-1}.
   (c) Find the range of g and g^{-1}.