17   Exponential Functions

Concepts:

- Exponential Functions

    - Graphing Exponential Functions
    - Exponential Growth and Exponential Decay

- The Irrational Number e and Continuously Compounded Interest

(Section 5.2 & 5.2A)

1. Sketch a complete graph of each of the following:

    (a) 3^x

    (b) 2^{-x}

    (c) e^x
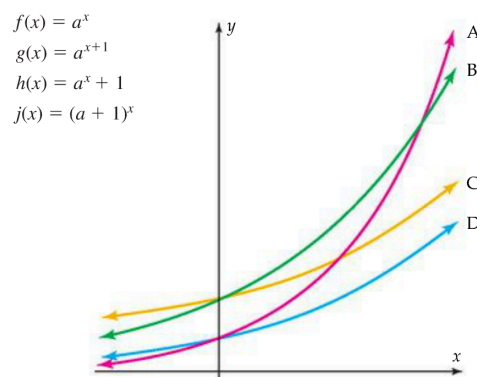
2. (Question 17, Section 5.2) Match the functions to the graphs. Assume $a > 1$.

    (a) graph $A =$

    (b) graph $B =$

    (c) graph $C =$

    (d) graph $D =$

$f(x) = a^x$
$g(x) = a^{x+1}$
$h(x) = a^x + 1$
$j(x) = (a + 1)^x$

3. Describe the transformations necessary for producing each graph from the graph of $f(x) = e^x$.

    (a) $g(x) = e^x - 5$

    (b) $h(x) = e^{x-5}$

    (c) $k(x) = -5e^{-x}$

4. If \$10,000 is invested at an interest rate of 3% per year, find the amounts in the account after 3 years if interest is compounded

    (a) quarterly

    (b) monthly

    (c) daily

5. If \$10,000 is invested at an interest rate of 4% per year, compounded quarterly, find the value of the investment after the given number of years.

   (a) 5 years   (c) 15 years

   (b) 10 years   (d) 20 years

6. Joni invests \$5000 at an interest rate of 5% per year compounded continuously. How much time will it take for the value of the investment to quadruple.

7. Between 1790 and 1860, the population y of the United States (in millions) in the year x was given by $y = 3.9572(1.0299^x)$, where x is the number of years since 1790. Find the population in the year 1858. (The question similar to this on Webassign expects your answer to be expressed as an actual number, not in units of 1,000,000.)

8. According to the *Kelly Blue Book*, the factory invoice price of a 2010 Buick LaCrosse 4-door CX Sedan is \$25,945.21 and the MSRP is \$26,995.00. (The factory invoice price is the price the dealer pays to the factory. The MSRP is the Manufacturer's Suggested Retail Price.) A certain dealership is charging the MSRP for the LaCrosse, but they need to clear their inventory to make room for the 2011 models. They decide to reduce the price of the car by 1% every day after October 21, 2010. (On October 22, they reduce the price by 1%; on October 23, they reduce the *reduced* price by 1%; etc.)

   (a) What will the price of the car be on October 23, 2010?

   (b) Find a function that models the price p of the car t days after October 21, 2010.

   (c) Will the car ever be free?

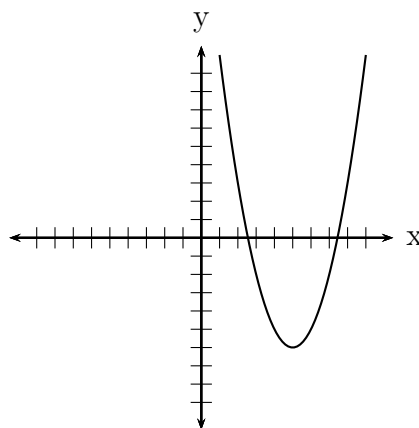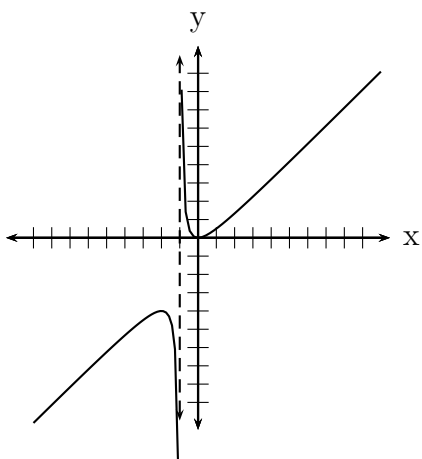   (d) On what date does the dealer start to lose money?

9. The graph of g is shown below. Find a formula for $g(x)$.

10. Label each of the following graphs with at least one of the following categories: Linear, Quadratic, Polynomial, Rational, Exponential, or None of the Categories Studied So Far. Some graphs may have more than one label.
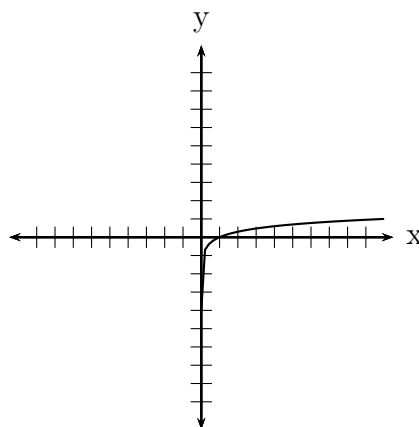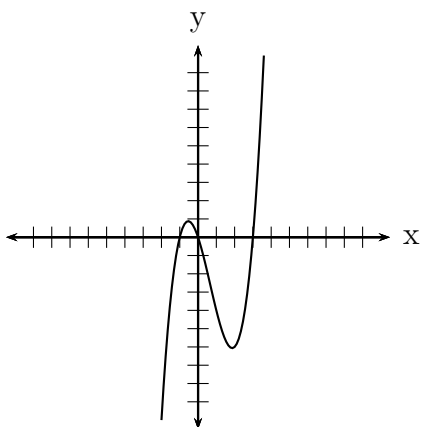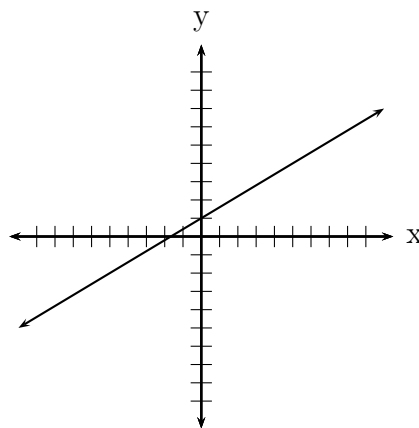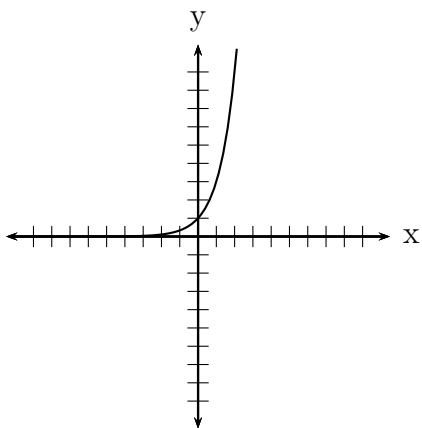
18   Logarithmic Functions

Concepts:

- Logarithms as Functions

- Logarithms as Exponent Pickers

- Inverse Relationship between Logarithmic and Exponential Functions.

- Properties of Logarithms

(Sections 5.3 & 5.4)

1. Find the exact value of the following logarithms. Do NOT use your calculator.

   (a) $\log_3(27)$

   (b) $\log(\sqrt[3]{100})$

   (c) $10^{\log(53)}$

   (d) $e^{2\ln(x)}$

   (e) $\ln\left(\dfrac{1}{\sqrt[5]{e^3}}\right)$

2. Using your knowledge of exponents, estimate between which two integer values the following expressions will be. Use your calculator to find approximate values for each. Was your estimation accurate?

   (a) $\log(1008)$

   (b) $\ln(3)$

   (c) $\ln(7)$

   (d) $\log(53)$

3. Convert each exponential statement to an equivalent logarithmic statement.

   (a) $8^2 = 64$

   (b) $2^x = 16$

   (c) $3^4 = y$

   (d) $x^{-5} = \left(\dfrac{1}{32}\right)$

4. Convert each logarithmic statement to an equivalent exponential statement.

   (a) $\log_2(32) = 5$

   (b) $\ln(x) = 3$

   (c) $\log(9) = x$

   (d) $\log_x\left(\dfrac{1}{64}\right) = -3$

5. (Questions 49 & 50, Section 5.3) Do the graphs of following functions appear to be the same? How do they differ?

   (a) $f(x) = \log(x^2)$ and $g(x) = 2\log(x)$

   (b) $h(x) = \log(x^3)$ and $k(x) = 3\log(x)$

6. Find all real solutions or state that there are none. Your answers should be exact.

   (a) $\ln(x) = 2$

   (b) $10^{x+2} = 376$

   (c) $9e^{x-8} = 2$

   (d) $\log_8(x-5) - \log_8(2x+2) = 0$

7. Given the following functions, find $f^{-1}(x)$.

    (a) $f(x) = \ln(3x + 7)$ (b) $f(x) = 2^{5x+3} - 1$

8. Given the following functions, find the *domain*.

    (a) $f(x) = \ln(2 - 3x)$ (b) $g(x) = \dfrac{x}{\ln(5x + 4)}$ (c) $h(x) = \ln(x^2 - 2x - 15)$

9. What transformations are needed to find the graph of each function from the graph of $f(x) = \ln(x)$?

    (a) $g(x) = 3\ln(x)$ (b) $h(x) = \ln(x) - 5$ (c) $k(x) = \ln(x - 3)$ (d) $l(x) = \ln(x+2)-7$

10. (Question # 78, Section 5.3) $f(x) = A\ln(x) + B$, where A and B are constants. If $f(e) = 5$ and $f(e^2) = 8$, what are A and B?

11. (Question # 82, Section 5.3) Students in a precalculus class were given a final exam. Each month thereafter, they took an equivalent exam. The class average on the exam taken after t months is given by:
$$F(t) = 82 - 8\ln(t + 1).$$

    (a) What was the class average after six months?

    (b) After a year?

    (c) When did the class average drop below 55?

12. Write each expression as a single logarithm. If it is not possible, explain why.

    (a) $3\left(\ln(x)\right) - 4\left(\ln(x^2) + \ln(x)\right)$

    (b) $-5\log(x - y) + 3\left(\ln(x^2) - \ln(y)\right)$

    (c) $13\left(\ln(e^2 - e)\right) - 13$

13. Write each expression in terms of $\log(x)$, $\log(y)$, and $\log(z)$. If it is not possible, explain why.

    (a) $\log\left(\dfrac{x^3 y^7}{\sqrt{z}}\right)$ (b) $\log\left(\dfrac{x^2 + y^2}{z}\right)$ (c) $\log\left(x^5 \sqrt[3]{yz}\right)$

14. Prove the *Quotient Rule for Logarithms*, $\log_a\left(\dfrac{u}{v}\right) = \log_a(u) - \log_a(v)$.

19 Logarithmic Properties & Equations

Concepts:

- Using the Change of Base Formula to Find Approximate Values of Logarithms

- Solving Exponential and Logarithmic Equations Algebraically

  - Strategies:
    * Same base exponential expressions that are equal must have equal exponents.
    * Same base logarithmic expressions that are equal must have equal arguments.
    * Isolate exponential expression, rewrite equation in logarithmic form.
    * Isolate logarithmic expression, rewrite equation in exponential form.

- Solving Exponential and Logarithmic Models/Applications

(Sections 5.4 - 5.5)

1. Use your calculator to find approximate values for the following.

   (a) $\ln(7)$
   (b) $\log(53)$
   (c) $\log_5(6)$
   (d) $\log_2(21)$

2. Given the magnitude of an earthquake on the Richter scale is given by $R(i) = \log(\frac{i}{i_0})$, where i is the amplitude of the ground motion of the earthquake and i_0 is the amplitude of the ground motion of the "zero" earthquake,

   (a) Find the magnitude on the Richter scale of an earthquake that is 10000 times stronger than the zero quake.

   (b) Find the magnitude on the Richter scale of an earthquake that is 25 times stronger than the zero quake.

3. The half-life of a certain radioactive substance is 2,365 years.

   (a) Find the decay rate constant r.

   (b) How much substance will be left in 100 years if there is currently 500 grams of the substance?

4. The antibiotic clarithromycin is eliminated from the body according to the formula $A(t) = 500e^{-0.1386t}$, where A is the amount remaining in the body (in milligrams) t hours after the drug reaches peak concentration.

   (a) How much time will pass before the amount of drug in the body is reduced to 100 milligrams?

   (b) Find the inverse of $A(t)$ and explain what the inverse function models.

5. You are given models for the population (in millions) of different countries t years after 2005. For each part, determine the year in which the models predict the populations will be equal.

(Source: World Health Organization's 2006 World Health Statistics)

(a) Rwanda: $R(t) = 9.04(1.05)^t$ and Hungary: $H(t) = 10.1(0.98)^t$.

(b) Cambodia: $C(t) = 14.07(1.02)^t$ and Kazakhstan: $K(t) = 14.83(0.93)^t$.

6. Find the solution(s) of the following exponential equations. Your answers should be exact.

(a) $10^{2x^2-3} = 10^{9-x^2}$

(b) $2^{3x+1} = 3^{x-2}$

(c) $\dfrac{10}{1+e^{-x}} = 2$

(d) $3^{4x} - 3^{2x} - 6 = 0$

(e) $9e^{x-8} = 2$

7. Find the solution(s) of the following logarithmic equations. Your answers should be exact.

(a) $\log_4(x+2) + \log_4 3 = \log_4 5 + \log_4(2x-3)$

(b) $\log_3(x+15) - \log_3(x-1) = 2$

(c) $\log_2(\log_3 x) = 4$

(d) $\log(x+3) = \log x + \log 3$

(e) $\log_8(x-5) + \log_8(x+2) = 1$

8. Suppose you're driving your car on a cold winter day ($20°$ F outside) and the engine overheats (at about $220°$ F). When you park, the engine begins to cool down. The temperature U of the engine t minutes after you park satisfies the equation

$$\ln\left(\frac{U-20}{200}\right) = -0.11t.$$

(a) Solve the equation for U.

(b) Use part (a) to find the temperature of the engine after 20 min ($t = 20$).

9. Find how long it takes for a deposit to double in value if the annual interest rate is 3.5% and the interest is compounded continuously.

10. Joni invests \$5000 at an interest rate of 5% per year compounded continuously. How much time will it take for the value of the investment to quadruple?

11. Joni invests \$5000 at an interest rate of 5% per year compounded monthly. How much time will it take for the value of the investment to quadruple?

20 Angles, Their Measurement, and Basic Trig Functions

Concepts:

- Angles

  - Initial Side, Terminal Side, Standard Position & Coterminal Angles

- Measuring Angles

  - Radian Measure vs. Degree Measure
  - Radian Measure as a Distance on the Unit Circle
  - Converting between Radian Measure and Degree Measure
  - Finding the Quadrant Associated with the Terminal Side of an Angle

- Identifying the Point on the Unit Circle that Corresponds to an Angle in Standard Position

- The Trigonometric Functions

  - The Definitions of sin, cos, and tan Based on the Unit Circle
  - Evaluating the Basic Trigonometric Functions at Special Angles
  - The Sign of a Trigonometric Function

- The $\frac{\pi}{4} - \frac{\pi}{4} - \frac{\pi}{2}$ or the $45° - 45° - 90°$ Triangle

- The $\frac{\pi}{6} - \frac{\pi}{3} - \frac{\pi}{2}$ or the $30° - 60° - 90°$ Triangle

(**Sections 6.1 & 6.2**)

1. Find the radian measure of each of the following:

   (a) $450°$ angle
   (b) $-50°$ angle

2. Show which of the following points must lie on the unit circle.

   (a) $(0, -1)$
   (b) $(1, -1)$
   (c) $\left(\frac{3}{5}, -\frac{4}{5}\right)$
   (d) $\left(\frac{4}{5}, -\frac{3}{5}\right)$

   (e) $\left(-\frac{\sqrt{5}}{3}, \frac{2}{3}\right)$
   (f) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
   (g) $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}\right)$
   (h) $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

3. Suppose than an angle of measure θ radians intersects the unit circle at the point $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$.

   (a) What is one possibility for θ ?
   (b) How do you find all the other possibilities?

4. Suppose than an angle of measure θ radians intersects the unit circle at the point $\left(\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$.

    (a) What is one possibility for θ?

    (b) How do you find all the other possibilities?

5. Suppose that an angle of measure θ radians is placed in standard position. Find the location of the terminal side of the angle.
Possibilities: (A) Quadrant I, (B) Quadrant II, (C) Quadrant III, (D) Quadrant VI, (E) the positive x-axis, (F) the negative x-axis, (G) the positive y-axis, or (H) the negative y-axis.

    (a) $\theta = \frac{74\pi}{3}$

    (b) $\theta = -\frac{74\pi}{3}$

    (c) $\theta = 100\pi$

    (d) $\theta = -100\pi$

    (e) $\theta = 21\pi$

    (f) $\theta = -21\pi$

    (g) $\theta = \frac{102\pi}{7}$

    (h) $\theta = -\frac{102\pi}{7}$

6. Find the terminal point on the unit circle determined by the given value of θ.

    (a) $\theta = 4\pi$

    (b) $\theta = \frac{3\pi}{2}$

    (c) $\theta = -\frac{\pi}{6}$

    (d) $\theta = \frac{7\pi}{6}$

    (e) $\theta = -\frac{7\pi}{4}$

    (f) $\theta = \frac{5\pi}{3}$

    (g) $\theta = -\frac{4\pi}{3}$

    (h) $\theta = \frac{11\pi}{6}$

7. Evaluate the basic trigonometric functions at each of the following angles.

    (a) $\theta = \dfrac{\pi}{3}$

    (b) $\theta = -\dfrac{9\pi}{4}$

    (c) $\theta = 4\pi$

    (d) $\theta = \dfrac{17\pi}{6}$

8.   (a) The terminal side of an angle, θ, in standard position contains the point $(-5, 9)$. Evaluate the basic trigonometric functions at θ.

    (b) The terminal side of an angle, θ, in standard position contains the point $(11, 4)$. Evaluate the basic trigonometric functions at θ.

9. Suppose θ is in the fourth quadrant and $\cos(\theta) = \dfrac{1}{5}$. Evaluate the remaining basic trigonometric functions on θ.

10. Suppose θ is in the first quadrant and $\sin(\theta) = \dfrac{6}{7}$. Find each of the following (give exact answers):

    (a) $\sin(8\pi + \theta)$

    (b) $\tan(-\theta)$

    (c) $\cos(4\pi - \theta)$

21 Introduction to Trigonometric Graphs Worksheet

Concepts:

- Understanding Trigonometric Notation

- The Graph of the sin, cos, and tan Functions

- Applying Graph Transformations to the Graphs of the sin, cos, and tan Functions

- Using Graphical Evidence to Make Conjectures about Identities

(**Sections 6.2 & 6.4**)

1. (Question 43, Section 6.2) When a plane flies faster than the speed of sound, the sound waves it generates trail the plane in a cone shape, as shown in the figure in the Section 6.2 exercises of your textbook. When the bottom part of the cone hits the ground you hear a sonic boom. The equation that describes this situation is
$$\sin\left(\frac{t}{2}\right) = \frac{w}{p}$$
where t is the radian measure of the angle of the cone, w is the speed of the the sound wave, p is the speed of the plane, and $p > w$.

   (a) Find the speed of the sound wave when the plane flies at 1200 mph and $t = .8$.

   (b) Find the speed of the plane if the sound wave travels at 500 mph and $t = .7$.

2. (Question 58, Section 6.2) Given that the terminal side of an angle of t radians in standard position lies in Quadrant III on the line $2y - 5x = 0$, find $\sin(t), \cos(t), \tan(t)$. HINT: Find a point on the terminal side of the angle.

3. Find all the solutions to the following equations:

   (a) $\cos(t) = -1$
   (b) $\tan(t) = 0$
   (c) $\sin(t) = 1$

4. For each of the following equations, list the transformations that you need to apply to the graph of $y = \sin(x)$, $y = \cos(x)$, or $y = \tan(x)$ to sketch the graph of the equation. Sketch the graph. Be sure that the graph is well-labeled.

   (a) $y = 4\sin(3x)$
   (b) $y = 3\cos\left(\frac{x}{\pi}\right)$
   (c) $y = 2\sin\left(x - \frac{\pi}{3}\right)$
   (d) $y = -\tan\left(x + \frac{\pi}{4}\right)$
   (e) $y = u\cos(vx + w)$. (Assume that u, v, and w are positive.)

5. Use graphical evidence to determine which of the following **MIGHT** be trigonometric identities and which definitely cannot be trigonometric identities.

   (a) $\cos(2x) = \cos^2(x) - \sin^2(x)$
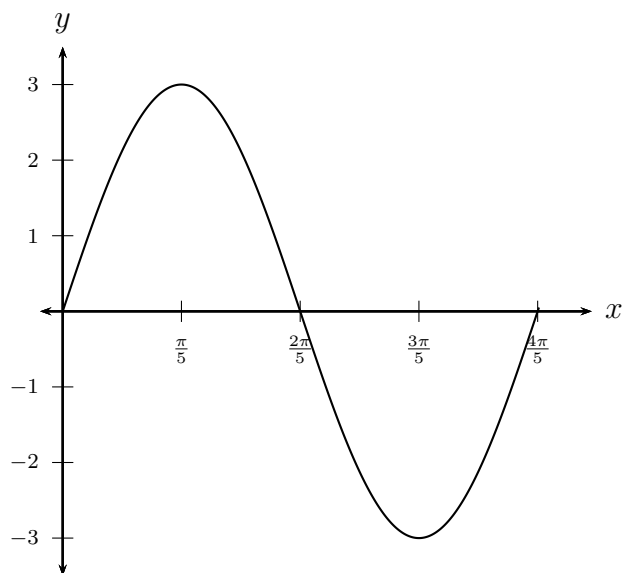
   (b) $\sin(2x) = \sin^2(x) - \cos^2(x)$

   (c) $\sin(2 + x) = \sin(2) + \sin(x)$

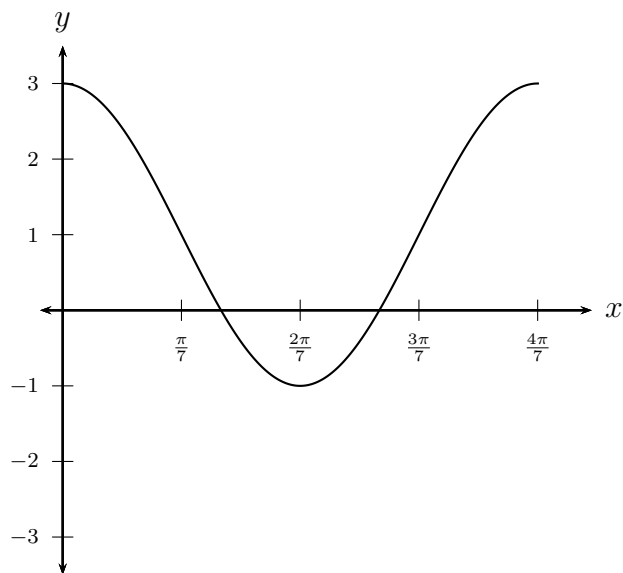   (d) $\sin(3)\cos(x) = \dfrac{1}{2}(\sin(3 + x) + \sin(3 - x))$

   (e) $\tan(5\theta) = \tan(5)\tan(\theta)$

   (f) $\dfrac{\sin(t^2)}{t} = \sin(t)$

6. The graph of a trigonometric function is shown below. Find a rule for the function.

7. The graph of a trigonometric function is shown below. Find a rule for the function.
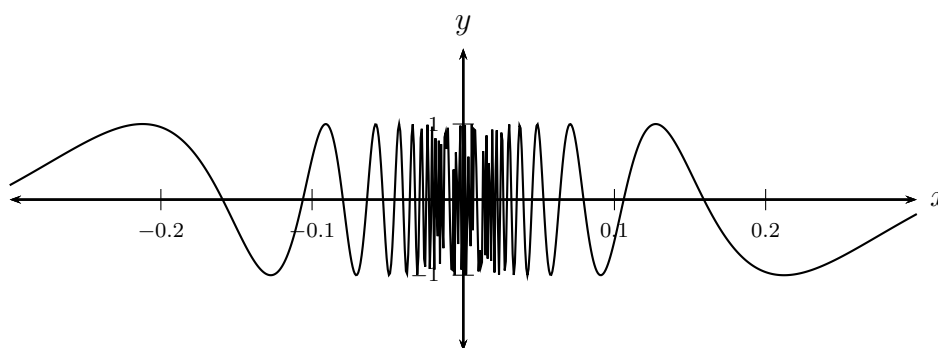
22 More Trigonometric Graphs Worksheet

Concepts:

- Period, Amplitude, and Phase Shift

- The csc, sec, and cot Functions

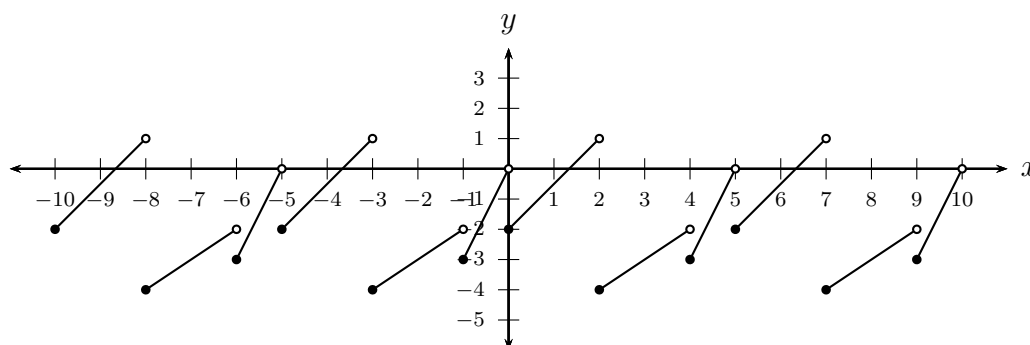- The Graphs of the csc, sec, and cot Functions

(Sections 6.5 and 6.6)

1. For each graph, (i) find the period if it is defined and (ii) find the amplitude if it is defined.

   (a)

   (b)

2. For each of the following equations, what is the period, amplitude and phase shift of the graph? (Assume that u, v, and w are positive.)

   (a) $y = 3\cos\left(\dfrac{x}{\pi}\right)$

   (b) $y = 2\sin\left(x - \dfrac{\pi}{3}\right)$

   (c) $y = -\tan\left(x + \dfrac{\pi}{4}\right)$

   (d) $y = u\cos(vx + w)$.

3. (Question 48 from Section 6.5 of your textbook)

The number of hours of daylight in Winnipeg, Manitoba, can be approximated by

$$d(t) = 4.15\sin(.0172t - 1.377) + 12,$$

where t is measured in days, with $t = 1$ being January 1.

(a) On what day is there the most daylight? The least? How much daylight is there on these days?

(b) On which days are there 11 hours or more of daylight? What do you think the period of this function is? Why?

4. Evaluate the csc, sec, and cot functions at each of the following angles.

(a) $\theta = \dfrac{\pi}{3}$

(b) $\theta = -\dfrac{9\pi}{4}$

(c) $\theta = 4\pi$

(d) $\theta = \dfrac{17\pi}{6}$

5. (a) The terminal side of an angle, θ, in standard position contains the point $(-5, 9)$. Evaluate the csc, sec, and cot functions at θ.

(b) The terminal side of an angle, θ, in standard position contains the point $(11, 4)$. Evaluate the csc, sec, and cot functions at θ.

6. Suppose θ is in Quad IV and $\cos(\theta) = \dfrac{1}{5}$. Find $\csc(\theta)$, $\sec(\theta)$, and $\cot(\theta)$.

7. Sketch the graphs of the following equations. Be sure that the graph is well-labeled.

(a) $y = \csc(x) + 3$

(b) $y = 5\sec\left(\dfrac{x}{2}\right) + 3$

(c) $y = \cot\left(x - \dfrac{3\pi}{4}\right)$

8. Use algebra and identities to simplify the expression. Assume all denominators are nonzero.

(a) $\dfrac{\sin(t)}{\tan(t)}$

(b) $\dfrac{1}{\cos(t)} - \sin(t)\tan(t)$

9. Solve each of the following equations.

(a) $\cos(x)=0$

(b) $\sin^3(t) - \sin(t) = 0$

(c) $\cos^2(t) - 2\cos(t) = -1$

23 Trigonometric Identities Worksheet

Concepts:

- Trigonometric Identities
    - Reciprocal Identities
    - Pythagorean Identities
    - Periodicity Identities
    - Negative Angle Identitites

(**Sections 6.6 & 7.1**)

1. Find the period, phase shift, and sketch the graph of $y = 2\csc(2x - \frac{\pi}{2})$.

2. Find all solutions to the equation $\tan(\theta) = \cot(\theta)$ for $0 < \theta < \pi$

3. Use the Pythagorean identity for each of the following problems.

    (a) Find $\sin(t)$ if $\cos(t) = \dfrac{2}{\sqrt{10}}$ and $\dfrac{3\pi}{2} < t < 2\pi$.

    (b) Find $\cos(t)$ if $\sin(t) = \dfrac{3}{5}$ and $\dfrac{\pi}{2} < t < \pi$.

    (c) Find $\cot(t)$ if $\sin(t) = -\dfrac{2}{5}$ and $\pi < t < \dfrac{3\pi}{2}$.

4. Determine each of the following:

    (a) Find $\cos x$ if $\sin x = -\dfrac{5}{13}$ and $\tan x > 0$.

    (b) Find $\tan x$ if $\cos x = \dfrac{1}{4}$ and $\sin x < 0$.

    (c) Find $\tan x$ if $\sec x = \dfrac{\sqrt{10}}{3}$ and $\sin x > 0$.

5. Use the periodicity and negative angle identities to solve each of the following problems.

   (a) Find $\sin(-t)$ if $\sin(t) = \frac{1}{\sqrt{5}}$.

   (b) Find $\sin(t - 2\pi)$ if $\sin(t) = \frac{1}{\sqrt{2}}$.

   (c) Find $\cos(-t + 4\pi)$ if $\cos(t) = -\frac{\sqrt{6}}{3}$.

   (d) Find $\cot(7\pi - t)$ if $\tan(t) = \sqrt{15}$.

6. Use basic identities to simplify the expression.

   (a) $\cot\theta \sec\theta \sin\theta$

   (b) $\dfrac{\cos^2 x}{\sin^2 x} + \csc x \sin x$

   (c) $\dfrac{1}{\cot^2 x} + \sec x \cos x$

   (d) $\sin^2 x + \tan^2 x + \cos^2 x$

   (e) $\dfrac{1 - \sin^2 x}{\sin x - \csc x}$

   (f) $\dfrac{\cos x}{1 - \sin x} + \dfrac{1 - \sin x}{\cos x}$

   (g) Prove the identity $\sin x \,(\tan x \cos x - \cot x \cos x) = 1 - 2\cos^2 x$

7. State whether or not the equation might be an identity. If it appears to be, prove it.

   (a) $\cot x = \dfrac{\csc x}{\sec x}$

   (b) $\dfrac{\sin(-x)}{\cos(-x)} = -\tan x$

   (c) $1 + \sec^2 x = \tan^2 x$

   (d) $\sin^2 x(\cot x + 1)^2 = \cos^2 x(\tan x + 1)^2$

   (e) $\dfrac{1 + \sin x}{1 - \sin x} = \dfrac{\sec x + \tan x}{\sec x - \tan x}$

   (f) $(1 - \cos^2 x)\csc x = \sin x$

   (g) $\dfrac{\sec x}{\csc x} + \dfrac{\sin x}{\cos x} = 2\tan x$

   (h) $\dfrac{\csc^2 x - 1}{\csc^2 x} = \cos^2 x$

   (i) $\dfrac{\sin x - \cos x}{\tan x} = \dfrac{\tan x}{\sin x + \cos x}$

24 MA 110 Exam 3 Practice Worksheet

1. If \$5,000 is deposited in a bank account which has a yearly interest rate of r = 2.5% compounded continuously, find how much is in the account after 2 years and 7 months.

   (a) \$5333.13

   (b) \$5333.57

   (c) \$9537.88

   (d) \$5349.15

   (e) \$9820.16

2. Find the domain of $f(x) = \ln(x^2 + 3x + 2)$.

   (a) $(-\infty, -2) \cup (-1, \infty)$

   (b) $(-\infty, \infty)$

   (c) $[0, \infty)$

   (d) $(-2, -1)$

   (e) $(0, \infty)$

3. Rewrite the following expression as a single logarithm. $3 \log_a x - \frac{5}{3} \log_a y + \frac{1}{6} \log_a w - 5 \log_a z$.

   (a) $\log_a \dfrac{x^3 z^5}{w^{1/6} y^{5/3}}$

   (b) $\log_a \dfrac{x^3 w^{1/6}}{y^{5/3} z^5}$

   (c) $\log_a \dfrac{x^3 y^{5/3}}{w^{1/6} z^5}$

   (d) $\log_a \dfrac{x^3}{w^{1/6} y^{5/3} z^5}$

   (e) $\log_a \left(3x - \frac{5}{3}y + \frac{1}{6}w - 5z\right)$

4. Find the radian measure of a *standard position* angle between 0 and 2π that is coterminal with $-\frac{5\pi}{6}$.

   (a) $\dfrac{5\pi}{6}$

   (b) $-\dfrac{7\pi}{6}$

   (c) $\dfrac{\pi}{6}$

   (d) $\dfrac{7\pi}{6}$

   (e) $-\dfrac{5\pi}{6}$

5. Find the point on the unit circle corresponding to the angle $-\frac{27\pi}{6}$ radians.

   (a) $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

   (b) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

   (c) $(0, -1)$

   (d) $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

   (e) $(1, 0)$

6. Determine the number of solutions between 0 and 6π of the following equation. $\qquad \cos(t) = -\frac{1}{2}$

   (a) 1

   (b) 2

   (c) 3

   (d) 4

   (e) 6

7. Find the value of $\cot(t)$ given that $\sin(t) = -\frac{7}{8}$ and $\sec(t) < 0$.

   (a) $\frac{7}{\sqrt{15}}$

   (b) $-\frac{\sqrt{15}}{8}$

   (c) $-\frac{8}{7}$

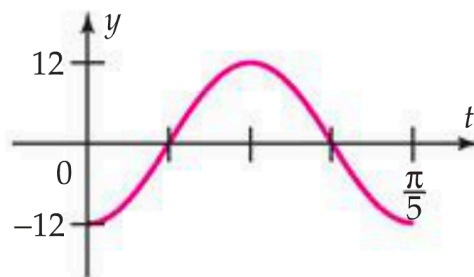   (d) $\frac{\sqrt{15}}{7}$

   (e) $-\frac{8}{\sqrt{15}}$

8. Solve the equation for x.
$$\log_3(x) = \log_3(x - 6) + 3$$

9. Consider the following graph of the form $f(t) = A\sin(bt + c)$.

   (a) Find the amplitude, period, and phase shift.

   (b) Find A, b, and c and state the rule of the function.

10. Prove the following identity:
$$\frac{\sin(t)}{1 - \cos(t)} + \frac{1 - \cos(t)}{\sin(t)} = 2\csc(t)$$

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