25 More Trigonometric Identities Worksheet

Concepts:

- Trigonometric Identities
  - Addition and Subtraction Identities
  - Cofunction Identities
  - Double-Angle Identities
  - Half-Angle Identities

(**Sections 7.2 & 7.3**)

1. Find the exact values of the following functions using the addition and subtraction formulas

   (a) $\sin \frac{9\pi}{12}$ (b) $\cos \frac{7\pi}{12}$

2. Write the expression as the sine or cosine of an angle.

   (a) $\sin \frac{\pi}{2} \cos \frac{\pi}{7} + \cos \frac{\pi}{2} \sin \frac{\pi}{7}$

   (b) $\sin 5x \cos x - \cos 5x \sin x$

   (c) $\cos 5x \cos 7x - \sin 5x \sin 7x$

3. Simplify the following expressions as much as possible

   (a) $\tan \left(\frac{9\pi}{2} - x \right)$

   (b) $\sin(x+y) - \sin(x-y)$

   (c) $\cos(x+y) - \cos(x-y)$

   (d) $\dfrac{\sin(x+y) - \sin(x-y)}{\cos(x+y) - \cos(x-y)}$

   (e) $\cos \left(x + \frac{\pi}{3} \right) + \sin \left(x - \frac{\pi}{6} \right)$

4. Verify the following identity:

$$\frac{\cos A - \cos B}{\sin A + \sin B} + \frac{\sin A - \sin B}{\cos A + \cos B} = 0.$$

5. Verify the following identity:

$$\cos(x+y)\cos(x-y) = \cos^2 x - \sin^2 y.$$

6. Use the cofunction identities to evaluate the following expression without using a calculator:

$$\sin^2(21^\circ) + \sin^2(61^\circ) + \sin^2(69^\circ) + \sin^2(29^\circ).$$

7. Find the values of the remaining trigonometric functions of x if

    (a) $\sin x = \dfrac{\sqrt{5}}{3}$ and the terminal point of x is in Quadrant II.

    (b) $\tan x = -\dfrac{\sqrt{11}}{5}$ and $\cos x > 0$.

8. Use an appropriate half-angle formula to find the exact value of each expression.

    (a) $\sin \dfrac{\pi}{12}$

    (b) $\cos \dfrac{\pi}{8}$

    (c) $\tan \dfrac{\pi}{12}$

    (d) $\cos \dfrac{7\pi}{8}$

    (e) $\sin \dfrac{3\pi}{8}$

    (f) $\tan \dfrac{5\pi}{8}$

9. Use an appropriate half-angle formula to simplify $\sqrt{\dfrac{1-\cos 10x}{2}}$.

10. (Question # 90, Section 7.3)To avoid a steep hill, a road is being built in straight segments from P to Q and from Q to R; it makes a turn of t radians at Q, as shown in the figure. The distance from P to S is 40 miles, and the distance from R to S is 10 miles. Use suitable trigonometric functions to express:

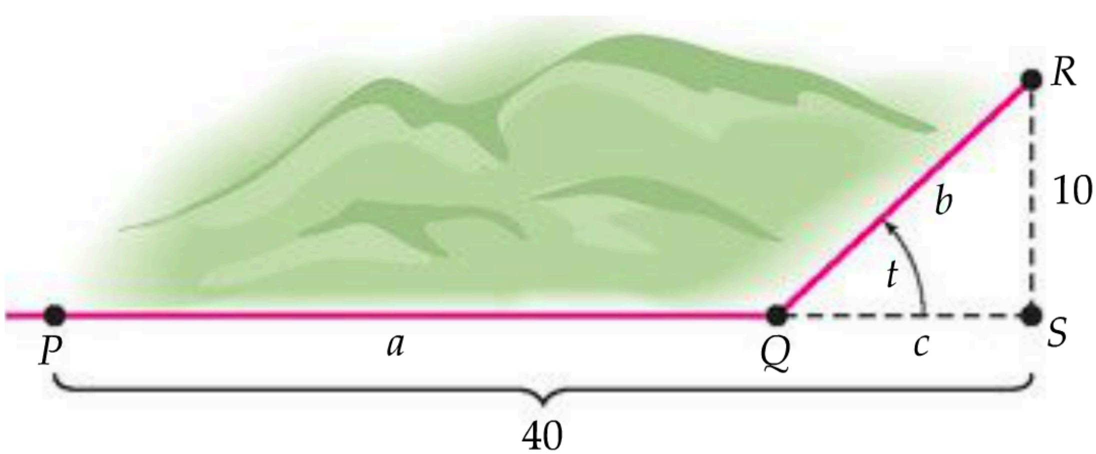
    (a) c in terms of b and t [*Hint*: Place the figure on a coordinate plane with P and Q on the x-axis, with Q at the origin. Then what are the coordinates of R?]

    (b) b in terms of t

    (c) a in terms of t [*Hint*: $a = 40 - c$; use parts (a) and (b).]

    (d) Use parts (b) and (c) and a suitable identity to show that the length $a + b$ of the road is

$$40 + 10\tan\left(\frac{t}{2}\right).$$



26 Inverse Trigonometric Functions

Concepts:

- Domain Restriction
- Inverse Sine, Cosine, and Tangent

(Sections 7.4 & 7.5)

1. Find the exact value for expression or state that it is undefined.

   (a) $\sin^{-1}(\frac{\sqrt{3}}{2})$

   (b) $\arcsin(-\frac{1}{2})$

   (c) $\sin(\sin^{-1}(-1))$

   (d) $\sin^{-1}(\sin(\frac{5\pi}{4}))$

   (e) $\cos^{-1}(-\frac{1}{2})$

   (f) $\arccos(0)$

   (g) $\cos(\cos^{-1}(2.3))$

   (h) $\cos^{-1}(\cos(-\frac{\pi}{6}))$

   (i) $\cos^{-1}(\frac{\sqrt{10}}{2})$

   (j) $\cos(\sin^{-1}(-\frac{4}{5}))$

   (k) $\tan^{-1}(-1)$

   (l) $\cot^{-1}(-1)$

   (m) $\tan^{-1}(\frac{1}{\sqrt{3}})$

   (n) $\cos\left(\cos^{-1}\left(-\frac{2}{5}\right)\right)$

   (o) $\sin^{-1}\left(\sin\left(\frac{11\pi}{6}\right)\right)$

   (p) $\sec^{-1}(2)$

2. Find the exact value for expressions.

   (a) $\sin\left(\cos^{-1}\left(-\dfrac{2}{5}\right)\right)$

   (b) $\tan\left(\sin^{-1}\left(\dfrac{1}{4}\right)\right)$

   (c) $\sec\left(\sin^{-1}\left(\dfrac{2}{3}\right)\right)$

   (d) $\cos\left(\tan^{-1}\left(\dfrac{5}{6}\right)\right)$

   (e) $\cot\left(\tan^{-1}\left(\dfrac{3}{8}\right)\right)$

   (f) $\cos\left(\sec^{-1}\left(\dfrac{7}{3}\right)\right)$

3. Write as an algebraic expression for $\sin(\cos^{-1}(x))$ in terms of x.

4. Write an algebraic expression for $\cos(\tan^{-1}(2x))$ in terms of x.

5. Write an algebraic expression for $\cos\left(\cos^{-1}(x) + \sin^{-1}(x)\right)$ in terms of x.

6. (Question # 21, Section 7.5) Find the exact solutions to $2\sin(x) + 1 = 0$

7. Find all solutions to $\sec^2(x) - 2 = 0$

8. Use an appropriate substitution to find all the solutions to $2\sin(2x) + \sqrt{3} = 0$

9. Find all the solutions to $2\cos(3x) = -1$ in the interval $[0, 2\pi)$.

10. Find all possible solutions of $\cos(2\theta) = -5\cos\theta - 4$ in the interval $[-\pi, 6\pi]$.

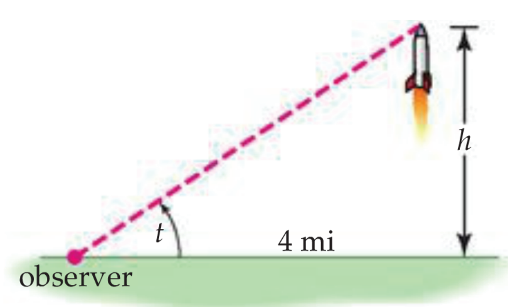
11. Find all possible solutions of $\cos(2\theta) = 4 - 3\cos\theta$.

12. Find all possible solutions of $\cos(2\theta) = 4 - 5\sin\theta$.

13. Find all possible solutions of $7\tan x \sin x = -12\sin x$, round your answers to the nearest tenth of a degree.

14. Let $\cos A = -\dfrac{24}{25}$ and $\sin B = \dfrac{5}{13}$, with $\pi < A < \dfrac{3\pi}{2}$ and $\dfrac{\pi}{2} < B < \pi$. Find the exact value of $\cos(A - B)$.

15. (Question # 67, Section 7.4) A rocket is fired straight up. The line of sight from an observer 4 miles away makes and angle of t radians with the horizontal.
    (a) Express t as a function of the height h of the rocket.
    (b) Find t when the rocket is .25 mile, 1 mile, and 2 miles high respectively.
    (c) When $t = .4$ radian, how high is the rocket?



27 Triangle Trigonometry

Concepts:

- Solving Trigonometric Equations
- Trigonometry for Acute and Obtuse Angles
  - The Law of Cosines

(Sections 7.5 & 8.3)

1. Use factoring, substitution, identities, and/or the quadratic formula to solve.

   (a) $3\sin^2(x) + 2\sin(x) = 5$

   (b) $\tan(x)\cos(x) = \cos(x)$

   (c) $\sin(2x) + \cos(x) = 0$

   (d) $\sin\left(\frac{x}{2}\right) = 1 - \cos(x)$

2. (Question 101, Section 7.5) The number of hours of daylight in Detroit on day t of a non-leap year (with $t = 0$ being January 1) is given by the function

$$d(t) = 3\sin\left[\frac{2\pi}{365}(t - 80)\right] + 12$$

   (a) On what days of the year are there exactly 11 hours of daylight?

   (b) What day has the maximum amount of daylight?

3. (Question 109, Section 7.5) What is wrong with this so-called solution?

$$\sin(x)\tan(x) = \sin(x)$$

$$\tan(x) = 1$$

$$x = \frac{\pi}{4} \qquad \text{or} \qquad \frac{5\pi}{4}$$

4. Solve each triangle.

   (a) Triangle ABC when $b = 4, a = 5.5$, and $C = 90°$.

   (b) Triangle ABC when $a = 20.1, b = 15.6$, and $C = 41°$.

   (c) Triangle ABC when $a = 12, b = 10$, and $c = 20$.

5. Solve the triangle ΔABC.

   (a) $B = 40°,\ a = 12,\ c = 20$

   (b) $a = 8,\ b = 5,\ c = 10$

   (c) $A = 118.2°,\ b = 16.5,\ c = 10.7$

   (d) $a = 6.8,\ b = 12.4,\ c = 15.1$

   (e) $C = 52.5°,\ a = 6.5,\ b = 9$

6. Find the degree measure (rounded to one decimal) of the angles of ΔABC with the following vertices:

   (a) $A\ (0, 0)$; $B\ (3, 7)$; $C\ (2, 8)$.
   (b) $A\ (-3, 4)$; $B\ (5, -2)$; $C\ (1, -4)$.

7. (Question 32, Section 8.3) A plane flies in a staight line at 400 mph for 1 hour and 12 minutes. It makes a 15° turn and flies at 375 mph for 2 hours and 27 minutes. How far is it from its starting point?

8. (Question 34, Section 8.3) A straight tunnel is to be dug through a hill. Two people stand on opposite sides of the hill where the tunnel entrances are to be located. Both can see a stake located 530 meters from the first person and 755 meters from the second. The angle determined by the two people and the stake (vertex) is 77°. How long must the tunnel be?

9. A solider with a range finder determines that at certain time an enemy truck is 300 feet from him. One second later the truck is 350 feet away from him. If the solider had to move his range finder through an angle of 12.83° to make the second measurement, how fast is the truck going?

28 More Triangle Trigonometry

Concepts:

- Trigonometry for Acute and Obtuse Angles
  - The Law of Cosines
  - The Law of Sines
- Applications

(Section 8.4)

1. Solve each triangle.
   (a) Triangle ABC when $a = 4, B = 59.2°$, and $C = 90°$.
   (b) Triangle ABC when $c = 10.3, a = 4.5$, and $C = 90°$.
   (c) Triangle ABC when $A = 37°$, $B = 18.6°$, and $a = 3$.
2. Solve the triangle ΔABC.
   (a) $B = 33°,\ C = 46°,\ b = 4$
   (b) $A = 67°,\ C = 28°, a = 9$
   (c) $b = 30,\ c = 50, C = 60°$
   (d) $a = 30, b = 40, A = 30°$
   (e) $B = 93.5°,\ C = 48.5°,\ b = 7$

NOTE: The remaining problems may use either the *Law of Sines* or the *Law of Cosines* or both.

3. Solve the triangle ΔABC.
   (a) $B = 74°,\ a = 42,\ c = 13.3$
   (b) $C = 33.7°, b = 33.1,\ c = 11.7$
   (c) $a = 48,\ c = 73, C = 43.7°$
   (d) $b = 15.8, c = 19.2, A = 42°$

4. (from Stewart, Redlin, Watson *Precalculus, 4th Ed*) Two tugboats that are 120 ft apart pull a barge together, each having a cable attached to the barge. If the length of one cable is 212 ft and the length of the other is 230 ft, find the angle formed by the two cables.

5. (Question 42, Section 8.4) Each of two observers 400 feet apart measures the angle of elevation to the top of a tree that sits on the straight line between them. These angles are 51° and 65°, respectively. How tall is the tree? How far is the base of its trunk from each observer?

6. (Question 28, Section 8.3) One plane flies west from Cleveland at 350 mph. A second plane leaves Cleveland at the same time and flies southeast at 200 mph. How far apart are the planes after 1 hour and 36 minutes?

7. (from Stewart, Redlin, Watson *Precalculus, 4th Ed*) A satellite orbiting the earth passes directly overhead at observation stations in Phoenix and Los Angeles, 340 mi apart. At an instant when the satellite is between these two station, its angle of elevation is simultaneously observed to be 60° at Phoenix and 75° at Los Angeles. How far is the satellite from Los Angeles?

8. (Question 30, Section 8.3) Two ships leave port, one traveling in a straight course at 22 mph and the other traveling a straight course at 31 mph. Their courses diverge by 38°. How far apart are they after 3 hours?

9. (Example 8, Section 8.4) An airplane A takes off from a carrier B and flies in a straight line for 12 kilometers. At that instant, an observer on a destroyer C, located 5 kilometers from the carrier, notes that the angle determined by the carrier, the destroyer (vertex), and the plane is 37°. How far is the plane from the destroyer?

29 MA 110 Exam 4 Practice Worksheet

CUMULATIVE & Sections 7.2 - 7.5, 8.3 - 8.4

Do not rely solely on this practice exam! Make sure to study exams, homework problems, other work sheets, lecture notes, and the book!!!

1. Solve.
$$2 - 3|2x - 6| > -8.$$
   (a) $(-\infty, 2) \cup (4, \infty)$
   (b) $(-\infty, 4)$
   (c) $(-\infty, \frac{4}{3}) \cup (\frac{14}{3}, \infty)$
   (d) $(2, 4)$
   (e) $(\frac{4}{3}, \frac{14}{3})$

2. Describe the end behavior of the graph of the following polynomial function.
$$Q(x) = -55x^{83} + 15x^{75} - 3$$
   (a) $y \to \infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$
   (b) $y \to \infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$
   (c) $y \to -\infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$
   (d) $y \to -\infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$
   (e) $y \to -55$ as $x \to \infty$ and $y \to -55$ as $x \to -\infty$

3. Solve.
$$\log_7(2) - \log_7(x - 2) = \log_7(x + 1) - \log_7(5)$$
   (a) $x = -3$ and $x = 4$
   (b) $x = 4$
   (c) $x = 2$ and $x = 4$
   (d) $x = 1$
   (e) $x = 7$

4. Find the exact value.
$$\sin\left(\frac{2\pi}{15}\right)\cos\left(\frac{4\pi}{5}\right) - \cos\left(\frac{2\pi}{15}\right)\sin\left(\frac{4\pi}{5}\right)$$
   (a) $\frac{\sqrt{3}}{2}$
   (b) $\frac{1}{2}$
   (c) $-\frac{\sqrt{3}}{2}$
   (d) $-\frac{1}{2}$
   (e) $-\frac{\sqrt{2}}{2}$

5. Find the **exact** value. $\cos^{-1}\left(\cos\left(\frac{11\pi}{6}\right)\right)$

(a) $\frac{11\pi}{6}$

(b) Undefined

(c) $\frac{\sqrt{3}}{2}$

(d) .5236

(e) $\frac{\pi}{6}$

6. Find all the solutions in the interval $[0, \pi)$. $\sin^2(x) - \cos^2(x) = 0$

(a) $x = \frac{\pi}{4}$

(b) $x = -\frac{5\pi}{4}$

(c) There are no solutions to the equation.

(d) $x = \frac{\pi}{4}$ and $x = \frac{3\pi}{4}$

(e) $x = \frac{7\pi}{4}$

7. Solve ΔABC under the given conditions. (Round your answers to one decimal place.)

$$A = 130°,\ b = 11,\ c = 15$$

(a) $a = 24.8,\ B = 15.7°,\ C = 34.3°$

(b) $a = 20.5,\ B = 24.0°,\ C = 26.0°$

(c) $a = 25.9,\ B = 23.2°,\ C = 26.8°$

(d) $a = 23.6,\ B = 20.9°,\ C = 29.1°$

(e) $a = 25.2,\ B = 22.5°,\ C = 27.5°$

8. A vineyard found that each vine produces about 12 lb of grapes when there are about 500 vines planted per acre. Each additional vine planted decreases the the production by 1%. So, if v is the number of additional vines planted, the number of pounds per acre can be modeled by

$$P(v) = (500 + v)(12 - .01v).$$

(a) How many vines should be planted to maximize grape production?

(b) What is the maximum production of grapes?

9. Find an exact value for $\cos\left(\frac{5\pi}{8}\right)$ and simplify the result.

10. Jack and Jill are standing 20 feet apart. Jack's angle of elevation to the top of a nearby tree is 32°, while Jill is closer and measures her angle of elevation at 68°.

(a) How tall is the tree?

(b) How far away is Jill from the base of the tree?