Name: ______________________ Section and/or TA: ______________

Do not remove this answer page. You will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or communication capabilities is permitted. You may not use a phone or other communication device during the exam.

The exam consists of 12 multiple choice questions that are worth 5 points each and 4 free response questions that are worth 10 points each. You should work the multiple choice questions on the question page. After you have checked your work carefully, record your answers by completely filling in the circle below that corresponds to your answer. If you must change your answer, make a note on the front of the exam. Be sure to check carefully when you transfer your answers to the cover sheet.

Show all work to receive full credit on the free response problems. You do not need to compute a decimal approximation to your answer. For example, the answer $4\pi$ is preferred to 12.57.

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Multiple Choice Questions

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | A | B | C | D | E | **7** | A | B | C | D | E |
| **2** | A | B | C | D | E | **8** | A | B | C | D | E |
| **3** | A | B | C | D | E | **9** | A | B | C | D | E |
| **4** | A | B | C | D | E | **10** | A | B | C | D | E |
| **5** | A | B | C | D | E | **11** | A | B | C | D | E |
| **6** | A | B | C | D | E | **12** | A | B | C | D | E |

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SCORE

| Multiple Choice | 13 | 14 | 15 | 16 | Total Score |
|---|---|---|---|---|---|
| 60 | 10 | 10 | 10 | 10 | 100 |
| | | | | | |

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Multiple Choice Questions

1. (5 points) Give the domain of the function $f(x) = \dfrac{x^2 - 16}{x^2 - 2x}$.

   A. $(-\infty, 0) \cup (0, \infty)$

   B. $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$

   C. $(-\infty, 2) \cup (2, \infty)$

   D. $(-\infty, -4) \cup (0, 2) \cup (4, \infty)$

   *E.* $(-\infty, 0) \cup (0, 2) \cup (2, \infty)$

2. (5 points) Let $f(x) = (x-3)^2$ with the domain $(-\infty, 1]$. Find the inverse function $f^{-1}(x)$.

   A. $f^{-1}(x) = -\sqrt{x} + 3$ with the domain $[0, \infty)$

   B. $f^{-1}(x) = \sqrt{x} - 3$ with the domain $[4, \infty)$

   C. $f^{-1}(x) = \sqrt{x} + 3$ with the domain $(-\infty, 1]$

   D. $f^{-1}(x) = 1/(x-3)^2$ with the domain $(-\infty, 3) \cup (3, \infty)$

   *E.* $f^{-1}(x) = -\sqrt{x} - 3$ *with the domain* $[4, \infty)$

3. (5 points) A ball is dropped from a state of rest at time $t = 0$. The distance traveled after $t$ seconds is given by $s(t) = 4t^2$ meters. Compute the average velocity on the interval $[3, 5]$.

   A. 27.2 meters/second

   B. 40 meters/second

   *C. 32 meters/second*

   D. 68 meters/second

   E. 24 meters/second

4. (5 points) Use the graph at right to find the one-sided limit $\lim_{x\to 1^-} f(x)$.

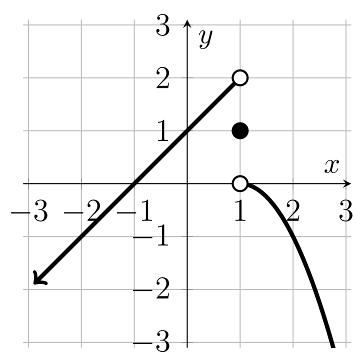
   A. 0

   B. 1

   *C. 2*

   D. $-1$

   E. The limit does not exist



5. (5 points) Suppose $\lim_{x\to 5} f(x) = 8$ and $\lim_{x\to 5} g(x) = 1$. Find

$$\lim_{x\to 5}\left(\frac{(x-1)^2}{f(x)} + (x-3)^2 g(x)\right).$$

A. 97/8

B. 48

*C. 6*

D. 1/8

E. 0

6. (5 points) A function $f$ satisfies $-x^2 + 9x - 25 \leq f(x) \leq x^2 - 15x + 47$ for all real numbers $x$. There is exactly one real number $c$ where we may use the Squeeze Theorem to compute the limit $\lim_{x\to c} f(x) = L$. Find $c$ and $L$.

A. $c = 1$ and $L = -17$

*B. $c = 6$ and $L = -7$*

C. $c = 0$ and $L = -25$

D. $c = 4$ and $L = 7$

E. $c = 3$ and $L = -7$

7. (5 points) Consider the function $f$ defined by

$$f(x) = \begin{cases} \sqrt{x}, & \text{if } 0 \le x \le 4 \\ x + a & \text{if } 4 < x \le 6 \end{cases}.$$

For which value of $a$ is the function $f$ continuous on $[0, 6]$?

*A.* $a = -2$

B. $a = 2$

C. $a = -1$

D. $a = 1$

E. This function is not continuous for any value of $a$.

8. (5 points) Select the statement that best describes the behavior of $f(x) = \dfrac{1}{x^2}$ near $x = 0$.

*A.* $\lim_{x \to 0^+} f(x) = +\infty$ *and* $\lim_{x \to 0^-} f(x) = +\infty$

B. $\lim_{x \to 0^+} f(x) = 0$ and $\lim_{x \to 0^-} f(x) = +\infty$

C. $\lim_{x \to 0^+} f(x) = -\infty$ and $\lim_{x \to 0^-} f(x) = +\infty$

D. $\lim_{x \to 0} f(x) = -\infty$

E. $\lim_{x \to 0^+} f(x) = +\infty$ and $\lim_{x \to 0^-} f(x) = -\infty$

9. (5 points) Find the limit $\lim_{x \to -\infty} \frac{7x^3 - 2x + 11}{4x - 3}$.

   A. 0

   B. $-7/4$

   *C.* $+\infty$

   D. $7/4$

   E. $-\infty$

10. (5 points) Find the equation of the tangent line to the curve $y = x^2$ at the point $(1, 1)$.

   A. $y = \frac{x}{2} - 1$

   *B.* $y = 2x - 1$

   C. $y = 2$

   D. $y = \frac{x}{2} + \frac{1}{2}$

   E. $y = 2x + 1$

11. (5 points) Consider the function $f(x) = \dfrac{1}{x}$. If you evaluate and simplify the expression

$$\frac{f(5+h) - f(5)}{h} \qquad \text{you obtain}$$

*A.* $\dfrac{-1}{5(5+h)}$

B. $\dfrac{1}{5(5-h)}$

C. $5 + h$

D. $\dfrac{5+h}{5}$

E. $\dfrac{1}{5(5+h)}$

12. (5 points) The graph of $f$ is shown below. For which of the following values of $x$ is the derivative $f'(x) > 0$?

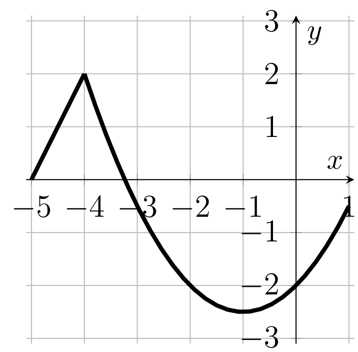
A. $x = -1$

B. $x = -4$

*C.* $x = 0$

D. $x = -2$

E. $x = -3$



*Free response questions: Show all work clearly using proper notation.*

13. (10 points) Let

$$f(x) = \begin{cases} x^2, & 0 \le x \le 2 \\ 10 - x, & 2 < x \le 4 \end{cases}.$$

(a) Give the domain and range of $f$.

(b) Find the formula for the inverse function $f^{-1}$.

(c) Give the domain and range of $f^{-1}$.

**Solution:** a) From the formula defining $f$, we see that the domain of $f$ is $[0, 4]$ and the range is $[0, 4] \cup [6, 8)$.

b) The function $f^{-1}$ is given by $f^{-1}(x) = \begin{cases} \sqrt{x}, & 0 \le x \le 4 \\ 10 - x, & 6 \le x < 8 \end{cases}$

c) The range of $f^{-1}$ is $[0, 4]$ and the domain is $[0, 4] \cup [6, 8)$.

Since the range of $f$ is domain of $f^{-1}$ and the range of $f^{-1}$ is domain of $f$, we have

| Function | Domain | Range |
|---|---|---|
| $f$ | $[0, 4]$ | $[0, 4] \cup [6, 8)$ |
| $f^{-1}$ | $[0, 4] \cup [6, 8)$ | $[0, 4]$ |

Grading: a) 1 point for domain and 1 point for range (2 points total).

b) 2 points for solving for $x$ in terms of $y$ (for each case), 1 point for providing the correct domains (give 5 of 6 points for correct, except minor mistakes and 1 for some progress) (6 total)

c) 1 point for domain and 1 point for range (2 points total).

Follow through–if the formula for $f^{-1}$ is incorrect, but they give domain of the function they found, then award points in parts b) or c).

*Free response questions, show all work, and clearly label your answers*

14. (10 points) For each limit, find the limit or state that it does not exist. Show steps clearly using proper notation.

(a) $\lim_{x\to -2} \frac{2x^2+3x-2}{x^2-4}$

(b) $\lim_{x\to 2} \frac{2x^2+3x-2}{x^2-4}$

(c) $\lim_{x\to \infty} \frac{2x^2+3x-2}{x^2-4}$

**Solution:** 14 a)

$$\lim_{x\to -2} \frac{2x^2+3x-2}{x^2-4} = \lim_{x\to -2} \frac{(2x-1)(x+2)}{(x-2)(x+2)} = \lim_{x\to -2} \frac{(2x-1)}{(x-2)} = \frac{-4-1}{-2-2} = \frac{5}{4}$$

-1 point if drop limit in equalities or keep writing limit after evaluation.

-1 point if one writes

$$\frac{(2x-1)(x+2)}{(x-2)(x+2)} = \frac{(2x-1)}{(x-2)}$$

without specifying $x \neq -2$. (total 4)

14 b) Via the same simplification as part a we have

$$\lim_{x\to 2} \frac{2x^2+3x-2}{x^2-4} \quad = \quad \lim_{x\to 2} \frac{(2x-1)}{(x-2)}$$

The right-hand and left-hand limits differ as $x$ approaches 2, so the limit DNE:

$$\lim_{x\to 2^+} \frac{(2x-1)}{(x-2)} = +\infty$$

$$\lim_{x\to 2^-} \frac{(2x+1)}{(x-2)} = -\infty$$

The easiest way to see this is to write

$$\frac{2x-1}{x-2} = \frac{2x-4+4-1}{x-2} = \frac{2(x-2)}{x-2} + \frac{3}{x-2} = 2 + \frac{5}{x-2}$$

A graph of this function will show the asymptotes at $x = 2$ and $y = 2$. (total 3)

14c) Since the highest powers in the numerator and the denominator as the same, we have

$$\lim_{x\to \infty} \frac{2x^2+3x-2}{x^2-4} = 2$$

Final answer: 1 point. Justification: 2 points.

*Free response questions, show all work, and clearly label your answers*

15. (10 points) (a) State the intermediate value theorem.

    (b) Use the intermediate value theorem to show $e^{-x} - x = 0$ has a solution. Be sure to give the interval on which you are applying the intermediate value theorem.

**Solution:** a) Theorem: If $f$ is continuous on the interval $[a, b]$ and $Y$ lies between $f(a)$ and $f(b)$, then there is a value $c \in [a, b]$ so that $f(c) = Y$.

b) Let $f(x) = e^{-x} - x$. Since $f$ is a difference of two continuous functions, it is continuous everywhere. We try a few values. For example $f(0) = 1$, $f(-1) = 3.71828$, and $f(1) = -0.632121$. Since $f(0) > 0$ and $f(1) < 0$ and $f$ is continuous on the interval $[0, 1]$, then there is a number $c$ in $[0, 1]$ so that $f(c) = 0$.

Grading: a) Hypotheses (2 points), conclusion (2 points). Give full credit if they use the open interval for $c \in (a, b)$. b) Give interval (1 point), show 0 lies between values at endpoints (3 points), observe function is continuous (2 points).

*Free response questions, show all work, and clearly label your answers*

16. (10 points) Let $f(x) = \dfrac{1}{\sqrt{x}}$. Use the **limit definition** of the derivative to find the derivative $f'(4)$.

**Solution:** We write the difference quotient and simplify

$$\begin{aligned}
\frac{f(4+h)-f(4)}{h} &= \frac{1}{h}\cdot\left(\frac{1}{\sqrt{4+h}}-\frac{1}{\sqrt{4}}\right)\\
&= \frac{1}{h}\cdot\left(\frac{\sqrt{4}-\sqrt{4+h}}{\sqrt{4}\sqrt{4+h}}\right)\left(\frac{\sqrt{4}+\sqrt{4+h}}{\sqrt{4}+\sqrt{4+h}}\right) \quad \text{multiply by conjugate}\\
&= \frac{1}{h}\cdot\left(\frac{\sqrt{4}^2-\sqrt{4+h}^2}{2\sqrt{4+h}}\right)\left(\frac{1}{\sqrt{4}+\sqrt{4+h}}\right),\\
&= \frac{1}{h}\cdot\left(\frac{4-4-h}{2\sqrt{4+h}}\right)\left(\frac{1}{2+\sqrt{4+h}}\right),\\
&= \left(\frac{-1}{2\sqrt{4+h}}\right)\left(\frac{1}{2+\sqrt{4+h}}\right), \qquad \text{if } h \neq 0.
\end{aligned}$$

Thus,

$$f'(4) = \lim_{h\to 0}\frac{f(4+h)-f(4)}{h} = \lim_{h\to 0}\left(\frac{-1}{2\sqrt{4+h}}\right)\left(\frac{1}{2+\sqrt{4+h}}\right) = -\frac{1}{16}.$$

Grading: Form difference quotient (3 points), simplify difference quotient to cancel $h$ (3 points), and find limit (4 points).