Name: ______________________ Section and/or TA: ______________

Do not remove this answer page. You will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or communication capabilities is permitted. You may not use a phone or other communication device during the exam.

The exam consists of 12 multiple choice questions that are worth 5 points each and 4 free response questions that are worth 10 points each. You should work the multiple choice questions on the question page. After you have checked your work carefully, record your answers by completely filling in the circle below that corresponds to your answer. If you must change your answer, make a note on the front of the exam. Be sure to check carefully when you transfer your answers to the cover sheet.

Show all work to receive full credit on the free response problems. You do not need to compute a decimal approximation to your answer. For example, the answer 4π is preferred to 12.57.

Multiple Choice Questions

1 (A) (B) (C) (D) (E) **7** (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E) **8** (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E) **9** (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E) **10** (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E) **11** (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E) **12** (A) (B) (C) (D) (E)

SCORE

Multiple Choice	13	14	15	16	Total Score
60	10	10	10	10	100

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Multiple Choice Questions

1. (5 points) Let $f(x) = x^2e^{-3x}$. Find $f'(x)$.

   A. $xe^{-3x}(2-3x)$

   B. $6xe^{-3x}(1-x)$

   C. $xe^{-3x}(2-x)$

   D. $-6xe^{-3x}$

   E. $3xe^{-3x}(1+x)$

2. (5 points) Find the derivative $f'(x)$ for $f(x) = \dfrac{1}{(x^2+1)^2}$.

   A. $\dfrac{2x}{(2x+1)^3}$

   B. $\dfrac{4x}{(x^2+1)}$

   C. $\dfrac{-4x}{(x^2+1)^3}$

   D. $\dfrac{-2x}{(x^2+1)^3}$

   E. $\dfrac{-4x}{(2x+1)^2}$

3. (5 points) Let $f(x) = \ln(\cos(x))$. Find $f'(x)$.

   A. $\dfrac{1}{\ln(\cos(x))}$

   B. $-\tan(x)$

   C. $\dfrac{-1}{\sin(x)}$

   D. $-\ln(\sin(x))$

   E. $\dfrac{1}{\cos(x)}$

4. (5 points) Which of the expressions is equal to the limit $\lim_{h\to 0} \dfrac{e^{x+h} - e^x}{h}$?

   A. 1

   B. $(\ln(x))' = \dfrac{d}{dx}\{\ln(x)\}$

   C. $(e^x)' = \dfrac{d}{dx}\{e^x\}$

   D. $(e^h)' = \dfrac{d}{dx}\{e^h\}$

   E. The limit does not exist

5. (5 points) Let $F(x) = f(f(x))$ and $G(x) = (F(x))^2$ and suppose that

$$f(5) = 11, \quad f(11) = 23, \quad f'(5) = 2, \quad f'(11) = 2.$$

Find $F'(5)$ and $G'(5)$.

A. $F'(5) = 4$ and $G'(5) = 184$

B. $F'(5) = 4$ and $G'(5) = 22$

C. $F'(5) = 4$ and $G'(5) = 46$

D. $F'(5) = 253$ and $G'(5) = 4$

E. $F'(5) = 2$ and $G'(5) = 184$

6. (5 points) Suppose that the equation of motion for a particle (where s is in meters and t in seconds) is

$$s(t) = \frac{t^3}{3} - 4t^2 + 16t + 6.$$

Find the acceleration after 1 second.

A. -6

B. -4

C. 0

D. 4

E. 1

7. (5 points) Consider the triangle below. Give an expression for the angle u.

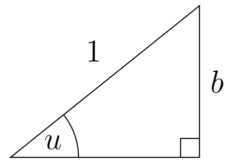
A. $u = \text{arcsec}(b)$

B. $u = \arcsin(b)$

C. $u = \arccos(b)$

D. $u = \arctan(b)$

E. $u = \arccos(1/b)$



8. (5 points) Find the equation of the tangent line to the curve $y = 6x\cos x$ at the point $(\pi, -6\pi)$.

A. $y = 6x - 6$

B. $y = -6x$

C. $y = 6x$

D. $y = -6$

E. $y = -6x - 12$

9. (5 points) Let $h(x) = (x^2 - 5x)f(x)$. Find the derivative $h'(3)$ if $f(3) = 17$ and $f'(3) = 4$.

   A. $h'(3) = 41$
   B. $h'(3) = 21$
   C. $h'(3) = 28$
   D. $h'(3) = 13$
   E. $h'(3) = -7$

10. (5 points) If $t = \cos(u)$ and $u \in (0, \pi)$, then $u = \arccos(t)$. Find the derivative u' as a function of t.

   A. $\dfrac{-\sqrt{1-t^2}}{t}$
   B. $\dfrac{t}{\sqrt{1+t^2}}$
   C. $\dfrac{-1}{\sqrt{1-t^2}}$
   D. $\dfrac{-t}{\sqrt{1-t^2}}$
   E. $\dfrac{\sqrt{1-t^2}}{t}$

11. (5 points) Consider the ellipse defined by the equation $4x^2 + 9y^2 = 36$. Find the slope of the tangent line to the ellipse at the point $(-\sqrt{5}, 4/3)$.

   A. $-\sqrt{5}/3$

   B. 0

   C. $\sqrt{5}/3$

   D. $-2/3$

   E. $2/3$

12. (5 points) Let $f(x) = -5\sin(x)$. Find the 4^{th} derivative $f^{(4)}(x)$.

   A. $-5\cos(x)$

   B. $5\cos(x)$

   C. $5\sin^4(x)$

   D. $-5\sin(x)$

   E. $5\sin(x)$

Free response questions: Show all work clearly using proper notation.

13. (10 points) Consider the curve defined by the equation

$$x^3 + y^3 = 12xy.$$

(a) Find the derivative $\frac{dy}{dx}$ along the curve.

(b) Find the slope of the tangent line at the point $(6, 6)$.

Solution: a) Differentiating we have

$$3x^2 + 3y^2\frac{dy}{dx} = 12(y + x\frac{dy}{dx})$$

Solving for the derivative gives

$$\frac{dy}{dx} = \frac{x^2 - 4y}{4x - y^2}.$$

b) The slope dy/dx is

$$\frac{dy}{dx} = \frac{6^2 - 4 \cdot 6}{4 \cdot 6 - 6^2} = \frac{36 - 24}{24 - 36} = -1.$$

Grading.

a) Differentiate. Product rule (2 point), chain rule (3 point), remaining terms (1 point). Solve for dy/dx (or y') (2 point)

b) Find slope (2 points)

Free response questions: Show all work clearly using proper notation.

14. (10 points)

(a) State the mean value theorem.

(b) Let $f(x) = x^3 - 2x$ on the interval $[-2, 2]$. **Check if the mean value theorem** can be applied to f on $[-2, 2]$. If so, find all values $c \in [-2, 2]$ guaranteed by the mean value theorem.

Solution: a) *Theorem.* If f is continuous on a closed interval $[a, b]$ and differentiable on the open interval (a, b), then there is point c in (a, b) so that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

b) f is continuous on $[-2, 2]$ because it is a polynomial and its derivative $f'(x) = 3x^2 - 2$ exists for all points in $(-2, 2)$. Thus, f is differentiable on $(-2, 2)$. So we can apply the MVT. Thus, by the MVT there is $c \in (-2, 2)$ such that

$$\frac{f(2) - f(-2)}{2 - (-2)} = f'(c).$$

$$\frac{(2^3 - 2(2)) - ((-2)^3 - 2(-2))}{2 + 2} = 3c^2 - 2$$

Thus,

$$c = \pm\frac{2}{\sqrt{3}} = \pm\frac{2\sqrt{3}}{3}.$$

Grading.

a) Continuity, differentiable, intervals (3 points). Existence of c in (a, b) (accept $[a, b]$) (1 point), equation $f'(c) = (f(b) - f(a))/(b - a)$ (1 point).

b) Verify continuity and differentiability hypotheses (1 point), use MVT to obtain $(f(2) - f(-2))/4 = f'(c)$ (2 points), find values of c (2 points).

Free response questions: Show all work clearly using proper notation.

15. (10 points) Suppose that a ball is thrown into the air at time $t = 0$ so that its height above the ground after t seconds is $h(t) = -\frac{9t^2}{2} + 72t$ meters. Include the correct units for each of your answers.

   (a) Find the velocity at the instant the ball is thrown.

   (b) Find the time when the velocity of the ball is zero.

   (c) Find the velocity of the ball as it hits the ground.

Solution: a) $v(t) = h'(t) = -9t + 72$. $v(0) = 72$ m/s.

b) Solve $v(t) = 0$. Then $-9t + 72 = 0$ if $t = 8$ seconds

c) The ball hits the ground when $h(t) = 0$ or $-9t^2/2 + 72t = 0$. Solving $t = 0$ or $t = 16$ seconds. At $t = 16$, the velocity is $h'(16) = -144 + 72 = -72$ meters/second.

Grading.

a) Find $v = h'$ (2 points). Value of $v(0) = 72$ m/s (1 point).

b) Equation $h'(t) = 0$, (1 point). Solution $t = 8$ seconds (1 point)

c) Equation for t, $h(t) = 0$ (1 point), hits ground at $t = 8$ as solution (1 point). Find velocity $v(16)$ (2 points)

Units (1 point) if majority of answers have correct units.

Students may read $v(0) = 72$ directly from $h(t)$ in part a). Award two points for v if they find h' somewhere.

Free response questions: Show all work clearly using proper notation.

16. (10 points) Let $f(x) = \cos(7x)$.

    (a) Find the derivative $f'(x)$.

    (b) Find the equation of the tangent line to the graph of f at $x = 0$.

    (c) Find the equation of the tangent line to the graph of f at $x = \dfrac{\pi}{14}$.

Solution: The derivative of $f(x) = \cos(7x)$ is $f'(x) = -7\sin(7x)$. Then $f'(0) = 0$ and $f'(\frac{\pi}{14}) = -7$.

a) The tangent line at $(0, 1)$ is $y = 0(x - 0) + 1 = 1$.

b) The tangent line at $(\pi/14, 0)$ is $y = -7(x - \pi/14) + 0$ which simplifies to $y = -7x + \frac{\pi}{2}$.

Grading.

a) Derivative (2 points)

b) Line through $(0, 1)$: slope (2 points), equation (2 points)

c) Line through $(\pi/14, 0)$: slope (2 points), equation (2 points)