

MA 113: Calculus I

Written Assignment 1 (Due on 09/11/24 at 11:59 pm)

NAME:

Instructions: The purpose of this assignment is to develop your ability to formulate and communicate mathematical arguments. Your complete assignment should have your name and section number on each page, be stapled, and be neat and legible. *Unreadable work may receive no credit.*

You should provide well-written, complete answers to each of the questions. We will look for correct mathematical arguments, complete explanations, and correct use of English. Your solution should be formulated in complete sentences. As appropriate, you may want to include diagrams or equations written out on a separate line. You may read your textbook to find examples of how we communicate mathematics. You should also read the page about writing mathematics in the general information section of our Canvas shell.

Students may use word processing software, a writing app on a tablet or pencil and paper to prepare their solutions. It may be simpler to draw graphs and mathematical expressions by hand. The final solution *must* be prepared as a single pdf and uploaded to Canvas. For those that write their solutions on paper, a tablet or phone can be used to produce a pdf of their work. Scanning functionality is built into Google Drive and the Files app on Apple products.

Consider the function f defined as a piecewise function by

$$f(x) = \begin{cases} \sqrt{x-1}, & \text{if } 1 \leq x \leq 5 \\ x-3 & \text{if } 5 < x \leq 8, \end{cases}.$$

1. (a) Sketch the graph of f and give the domain and range of f .
 (b) Sketch the graph of f^{-1} and give the domain and range of f^{-1} . You may draw the graphs of f and f^{-1} on the same axes.
 (c) Give a description of f^{-1} as a piecewise function.
 (d) How are the domains and ranges of f and f^{-1} related?
2. Does the limit of f at 5 exist? Explain why or why not.

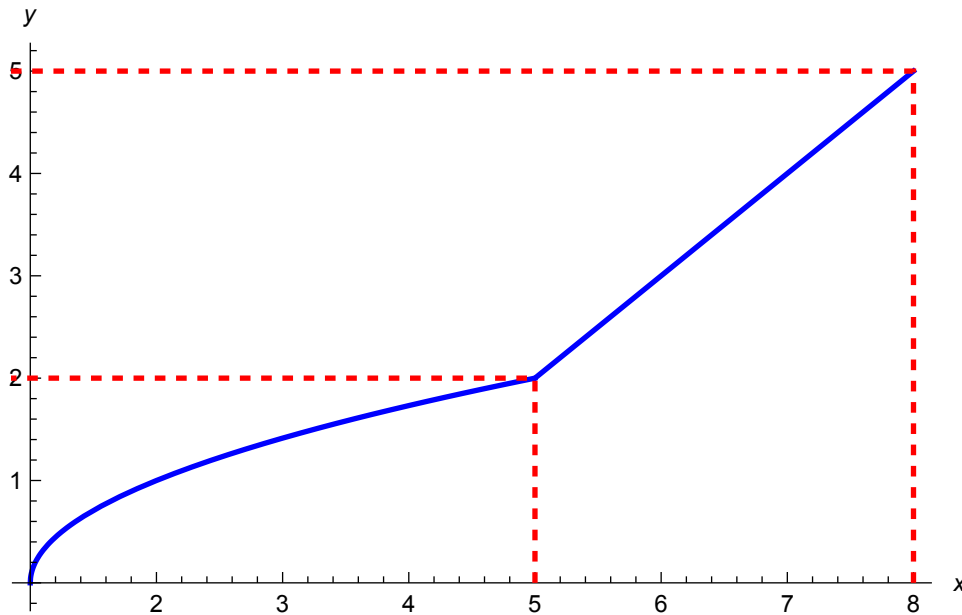
Solution: See §0.6 for Q1 and §1.3 for Q2.

1. a) The graph of f is shown in Figure 1. From the formula defining f , we see that the domain of f is $[1, 8]$ and the range is $[0, 5]$.

b) The graph of f^{-1} is shown in Figure 2. The range of f^{-1} is $[1, 8]$ and the domain is $[0, 5]$.

c) The function f^{-1} is given by

$$f^{-1}(x) = \begin{cases} x^2 + 1, & \text{if } 0 \leq x \leq 2 \\ x + 3 & \text{if } 2 < x \leq 5, \end{cases}.$$

Figure 1: Graph of f (in blue).

d) Since the range of f is domain of f^{-1} and the range of f^{-1} is domain of f , we have

Function	Domain	Range
f	$[1, 8]$	$[0, 5]$
f^{-1}	$[0, 5]$	$[1, 8]$

2. We have $\lim_{x \rightarrow 5^-} f(x) = 2$ and $\lim_{x \rightarrow 5^+} f(x) = 2$ Since the two one-sided limits are equal,

Theorem 1.3.8 implies that the (two-sided) limit of f at 5 does exist and is equal to 2.

Grading:

#1(a)-(d) 1 point for each part (4 total).

#2. 2 points for the answer, 2 points for explanation.

Total 8 points. Be alert to non-standard, but correct solutions and attempt to grade these fairly.

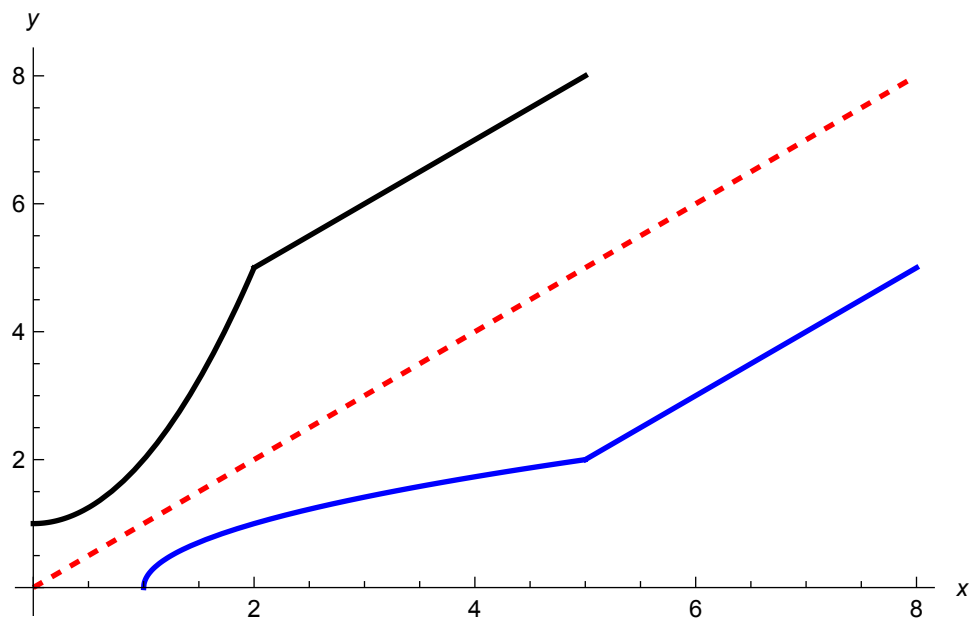


Figure 2: The graph of f^{-1} is shown in black while the graph of f in blue.