Contents

Material for Exam 1, Functions and limits

The material for the first three exams may be found in volume 1 of CLP Calculus which may be found at `https://clp.math.uky.edu/clp1`.

§0.4 Functions, domain and ranges,

1. Functions, domain and range
2. Domain and range of rational and algebraic functions
3. Linear and quadratic functions
4. Exam questions:
   (a) Find domain and range of a given function
   (b) Find a function given a description in words
   (c) Evaluate composite functions
   (d) Find a linear function given two points or a point and a slope

§0.6 Inverse functions

1. Definition of composite function
2. Definition of inverse functions
3. Relation between graph of a function and the graph of the inverse function
4. Finding a formula for an inverse function
5. Relation between domain and range for a function and its inverse
6. Exam questions:
   (a) Find inverse and composite functions
   (b) Find domain and range of inverse functions

(c) Graph an inverse function, given the graph of the function

§1.1,1.2 Drawing tangents and a first limit, Another limit and computing velocity

1. Review point slope equation of a line.
2. Slopes of secant lines, Average velocity
3. Algebraic simplification of slope and guessing limiting value.
4. Using numerical estimates to guess slope of tangent line and instantaneous velocity.
5. Distinction between speed and velocity.
6. Exam questions
   (a) Find tangent lines and slopes of secant lines.
   (b) Compute average and instantaneous velocity.

§1.3 Limit of a function

1. Informal definitions of limits and one-sided limits.
2. Estimating limits of rational expressions.
3. Finding limits from the graph of a function.
4. Relation between one and two-sided limits.
5. Limits with infinite values.
6. Exam questions
   (a) Find limits from a graph.
   (b) Use one-sided limits to determine if a (two-sided) limit exists
   (c) Given a piece-wise function with a parameter, choose the parameter to make the function have a limit at the transition point between two pieces

§1.4 Limit laws.

1. Limit of constants and x.
2. Rules for sums, products and quotients.
3. Limits involving powers.
4. Limits requiring simplification
5. Squeeze theorem
6. Exam questions
   (a) Evaluate limits of low degree rational expressions including expressions that require simplification

(b) Evaluating limits of rational functions and powers
(c) Using limit laws to evaluate expressions involving functions with known limits
(d) State the squeeze theorem and use it to evaluate limits.

§1.5 Limits at infinity.

1. Informal definition of a limit at infinity.
2. Limit rules for limits at infinity.
3. Limit rules for limits that are infinite.
4. Examples to illustrate using limit rules for limits that are infinite. (The text does not introduce the term indeterminate form until later.)
5. Exam questions.
   (a) Compute limits at infinity
   (b) Limits of rational functions
   (c) Computing limits with an infinite value

§1.6 Continuity

1. Continuity at a point and on intervals, including closed intervals.
2. Continuity and arithmetic operations.
3. Continuity and composition
4. List of continuous functions including polynomials, rational functions, powers, trigonometric and inverse trigonometric functions, exponential functions and logarithms.
5. The intermediate value theorem.
6. Exam questions
   (a) Find parameters to make a piece-wise function continuous
   (b) Identify sets where a function is continuous
   (c) State the intermediate value theorem
   (d) Use the intermediate value theorem to find solutions of equations
   (e) Limits of elementary functions and compositions

Teaching note: This section may take 1.5 lectures.

§§2.1,2.2 Revisiting tangent lines, The definition of the derivative.

1. Definition of the derivative
2. Finding tangent lines

3. Derivatives of c, x and x^2, $1/x$, $\sqrt{x}$.
4. Continuity of differentiable functions.
5. Exam questions
   (a) State the definition of the derivative.
   (b) Use the definition to compute simple derivatives.
   (c) Writing equation of a tangent line.
   (d) Determining values of $f(a)$ and $f'(a)$ from the tangent line at a.

§2.3 Interpretations of the derivative

1. The derivative as an instantaneous velocity
2. The derivative as the slope of a tangent line
3. Exam questions
   (a) The units for a derivative
   (b) Extracting a function value and instantaneous slope of a function from the tangent line
   (c) Finding instantaneous velocity and the slope of a tangent line

Material for Exam 2, Computing derivatives

§§2.4,2.6 Arithmetic of derivatives

1. Sum, product, reciprocal and quotient rule.
2. Brief discussion of the proofs (see §2.5). These will not be included on exams.
3. The power rule as an application of the product rule and reciprocal rule.
4. The power rule for rational exponents
5. State the power rule for real exponents
6. Examples using low degree polynomials.
7. Examples with polynomials, rational functions and powers
8. Exam questions
   (a) Using the sum, product, reciprocal and quotient rules on expressions which may involve a general function.
   (b) Computing derivatives of polynomials, rational functions and roots. Functions may require simplification before using our rules.
   (c) Finding tangent lines

§2.7 Derivatives of exponential functions.

1. Review of logarithms
2. Definition of e
3. Derivatives of exponential functions, with emphasis on e^x
4. Exam questions.
   (a) Differentiating expressions involving e^x

§2.8 Derivatives of trigonometric functions

1. Basic limits
2. Derivatives of sin and cos
3. Derivative of tan and sec
4. Exam questions
   (a) Tangent lines to trigonometric functions
   (b) Derivatives with respect to an angle of the sides in a right triangle

§2.9 The chain rule

1. Statement of the chain rule

2. Presenting a function as a composition of simpler functions
3. Using the chain rule including examples with composition of more than two functions.
4. Exam questions
   (a) Computing derivatives of composite functions including examples with functions introduced later in this chapter.
   (b) Finding derivatives where we need to apply the chain rule and other rules to compute the derivative.

§2.10 Logarithms

1. The natural logarithm and its properties
2. Derivative of the natural logarithm
3. Writing a^b using log and the exponential function
4. Exam questions
   (a) Derivatives of expressions involving $\log(x)$
   (b) Rewriting a^b as $e^{b\log(a)}$ when $a > 0$ and finding derivative of a^x

Teaching note: The textbook uses the notation $\log = \log_e$ for the natural logarithm. It is common to use ln for this function. We will use ln and log for the natural logarithm in WeBWorK. In the rare instances that we use a logarithm with base other than e, we will include the base explicitly. Note that the `log` key on TI calculators gives the common logarithm or $\log_{10}$ while the `ln` key will give the natural logarithm.

§2.11 Implicit differentiation

1. Implicitly defined curves and graphs of functions
2. Implicit differentiation
3. Derivatives of inverse functions
4. Exam questions
   (a) Computing first derivatives by implicit differentiation
   (b) Finding derivatives of inverse functions by implicit differentiation
   (c) Tangent lines to an implicitly defined curve

§2.12 Inverse trigonometric functions

1. Review of definitions of inverse trig functions arcsin, arccos, arctan and arcsec including domains and ranges
2. Alternate notation of $\arcsin = \sin^{-1}$, $\arccos = \cos^{-1}$, etc..

3. Derivatives of arcsin, arccos, arctan and arcsec
4. Relation between angles and sides in a right triangle.
5. Exam questions
   (a) Derivatives of arcsin, arccos, arctan and arcsec
   (b) Simplifying expressions $g \circ f^{-1}$ and $g^{-1} \circ f$ where g and f are trigonometric functions
   (c) Rates of change for angles in a right triangle

Teaching note: The textbook defines arcsec as the inverse of sec with the domain $[0, \pi/2) \cup (\pi/2, \pi]$. As a consequence the formula for the derivative is slightly different than in our previous textbook which restricted sec to a different domain to define the inverse.

§2.13 The mean value theorem

1. Statement of the mean value theorem
2. Examples illustrating hypotheses
3. Using the derivative to show a function is constant or monotone
4. Using the mean value theorem to estimate functions
5. Exam questions
   (a) State the mean value theorem
   (b) Using the derivative and a value of a function to estimate other values of a function

§2.14 Higher order derivatives

1. Definition of higher order derivatives
2. Leibniz notation d^n/dx^n and the notation $f^{(n)}$
3. Exam questions
   (a) Computing arbitrary derivatives of polynomials, e^x, $\sin(x)$ and $\cos(x)$
   (b) Find a polynomial with specified derivatives at a point
   (c) Computing second or third derivative of a function

§3.1 Velocity and acceleration

1. Velocity and acceleration as derivatives
2. Finding position and velocity given constant acceleration (Example 3.1.2)
3. Exam questions
   (a) Finding velocity and acceleration given position and describing motion

(b) Motion of object moving under constant acceleration

§3.4.1–3 Approximating Functions Near a Specified Point – Taylor Polynomials

1. Linear and quadratic approximations to functions
2. Using linear and quadratic approximations to find approximate values of functions
3. Exam questions
   (a) Finding linear and quadratic approximations at a point
   (b) Approximating values of functions near a given value

Basic facts students should know for Exams 2–4.

Below is a list of derivatives and basic results from trigonometry that students should know for Exam 2 in MA 113 and beyond. Students must learn this material, it will not be provided during exams.

1. The definitions of sin and cos using the unit circle and the definitions of tan, cot, sec and csc in terms of sin and cos.

$$\tan(x) = \frac{\sin(x)}{\cos(x)}, \quad \cot(x) = \frac{\cos(x)}{\sin(x)}, \quad \sec(x) = \frac{1}{\cos(x)}, \quad \csc(x) = \frac{1}{\sin(x)}.$$

2. The definitions of the functions arcsin, arccos, arctan and arcsec as inverses of the trigonometric functions with restricted domains.

3. The trigonometric functions for the special angles 0, $\pi/6$, $\pi/4$, $\pi/3$, and any angle obtained by adding a multiple of $\pi/2$.

θ	$\cos(\theta)$	$\sin(\theta)$	θ	$\cos(\theta)$	$\sin(\theta)$	θ	$\cos(\theta)$	$\sin(\theta)$	θ	$\cos(\theta)$	$\sin(\theta)$
0	1	0	$\frac{\pi}{2}$	0	1	π	-1	0	$\frac{3\pi}{2}$	0	-1
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{2\pi}{3}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{7\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{5\pi}{3}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{3\pi}{4}$	$\frac{-\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{5\pi}{4}$	$\frac{-\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	$\frac{7\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{4\pi}{3}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{11\pi}{6}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$

4. The Pythagorean identity $\sin^2(x)+\cos^2(x) = 1$ and how to derive the identities $1 + \tan^2(x) = \sec^2(x)$ and $1 + \cot^2(x) = \csc^2 x$ from the identity for $\sin(x)$ and $\cos(x)$.

5. The double-angle identities for sine and cosine.

$$\cos(2x) = \cos^2(x)-\sin^2(x) = 2\cos^2(x)-1 = 1-2\sin^2(x), \qquad \sin(2x) = 2\sin(x)\cos(x).$$

6. The properties of the natural logarithm,

$$\begin{aligned} \ln(xy) &= \ln(x) + \ln(y), && x > 0, y > 0 \\ \ln(x/y) &= \ln(x) - \ln(y), && x > 0, y > 0 \\ \ln(x^p) &= p\ln(x), && x > 0,\ p \in (-\infty, \infty) \end{aligned}$$

7. The derivatives of the following functions.

$f(x)$	$f'(x)$	$f(x)$	$f'(x)$
x^n	nx^{n-1}	$\csc(x)$	$-\csc(x)\cot(x)$
e^x	e^x	$\cot(x)$	$-\csc^2(x)$
$\ln(\lvert x\rvert)$	$1/x$	$\arcsin(x)$	$\dfrac{1}{\sqrt{1-x^2}}$
$\sin(x)$	$\cos(x)$	$\arccos(x)$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\cos(x)$	$-\sin(x)$	$\arctan(x)$	$\dfrac{1}{1+x^2}$
$\tan(x)$	$\sec^2(x)$	$\mathrm{arcsec}(x)$	$\dfrac{1}{\lvert x\rvert\sqrt{x^2-1}}$
$\sec(x)$	$\sec(x)\tan(x)$		

Material for Exam 3, Applications of the derivative

§3.2 Related rates

1. Use of triangles in related rate problems
2. Differentiating expressions to find relations between derivatives
3. Exam questions
   (a) Related rates involving sides and angles in a triangle
   (b) Relation between lengths, areas and volumes of geometric objects. Students should know the formula for area of plane figures such as triangles, rectangles and circles. Formulae for three dimensional objects will be provided, if needed on exams.

§3.3 Exponential growth and decay (omit §3.3.3.1).

1. Finding exponential growth and decay models in population growth, radioactive decay and Newton's law of cooling
2. Matching exponential models to given data
3. Half-life and doubling time
4. Exam questions
   (a) Population growth
   (b) Carbon dating
   (c) Newton's law of cooling

§3.5.1–2 Maximum or minimum values

1. Local and global extremes
2. Critical and singular points
3. Using first and second derivatives to find local extremes
4. Finding global extreme values on a closed interval
5. Exam questions
   (a) Find critical points and singular points
   (b) Find local extrema
   (c) Find global extrema on a closed interval

Teaching note: In the CLP book, a critical point is a point where $f'(c)$ exists and equals zero.

§3.6.1–2 Increasing and decreasing functions

1. Domain, range and asymptotes of a graph
2. Identifying intervals of monotonicity
3. Exam questions
   (a) Identifying intervals of monotonicity
   (b) Drawing graphs with specified behavior
   (c) Recognizing behavior of the derivative from the graph of a function

§3.6.3 Concavity

1. Intervals of concavity
2. Inflection points
3. Exam questions
   (a) Identifying intervals of monotonicity and concavity
   (b) Drawing graphs with specified behavior
   (c) Recognizing behavior of the first and second derivatives from the graph of a function

§3.5.3 Optimization–applications

1. Formulating problems given in words as optimization problems
2. Interpreting a solution of an optimization problem as a solution of the original word problem
3. Finding global extreme values on an open interval
4. Explaining how we know we have found a global maximum or minimum
5. Word problems
6. Exam questions
   (a) Optimization problems including using relations to eliminate variables and showing that solution is a global extreme

§3.7 L'Hôpital's rule

1. Statement of l'Hôpital's rule, for limits at a finite point or at infinity
2. Identifying the indeterminate forms $0/0$, ∞/∞, $0 \cdot \infty$ and 1^∞
3. Rewriting limits as an indeterminate form where l'Hôpital's rule applies
4. Evaluating limits
5. Exam questions
   (a) Recognizing limits where l'Hôpital's rule does and does not apply

(b) Evaluating limits for the indeterminate forms $0/0$, ∞/∞, $0 \cdot \infty$, and 1^∞.

§4.1 Introduction to anti-derivatives

1. Definition of anti-derivative and using the definition to verify an anti-derivative
2. Finding anti-derivatives with specified values
3. Finding a function given the second derivative
4. Students should know all entries of the table immediately before Example 4.1.4
5. Exam questions
   (a) Finding a function given its derivative and one value
   (b) Finding a function given its second derivative and two pieces of information

New material for Exam 4

Exam 4 will be cumulative. Approximately half of the questions from Exam 4 will cover the new material below and the remainder will be from the material for the previous exams.

The material below comes from Volume II of CLP Calculus which may be found at `https://clp.math.uky.edu/clp2`.

II-§1.1.1–6 Definition of the integral

1. Computing Riemann sums given sample points.
2. Definition of the definite integral
3. Evaluating integrals using known areas
4. Summation and evaluating area under a parabola
5. Exam questions
   (a) Computing Riemann sums
   (b) Taking limits of Riemann sums
   (c) Expressing certain integrals using known areas of rectangles, triangles or circles
   (d) Finding area under a parabola using limits of Riemann sums

II-§1.2 Properties of the definite integral

1. Linearity of the integral
2. Additivity for intervals
3. Comparison properties (optional §1.2.2)
4. Exam questions
   (a) Using properties of integrals to evaluate new integrals in terms of known integrals
   (b) Estimating integrals with comparison properties

Teaching note: This section may take 1.5 lectures.

II-§1.3 Fundamental theorem of Calculus (2 lectures)

1. Using the definite integral to construct an anti-derivative
2. Using anti-derivatives to evaluate definite integrals
3. Exam questions
   (a) Give precise statement of the fundamental theorem of calculus
   (b) Finding derivatives of functions defined using integrals

   (c) Applying earlier topics to functions defined as an integral
   (d) Evaluating definite integrals using the Fundamental theorem of calculus

II-§1.4 The substitution rule

1. The substitution rule
2. Substitution in definite integrals and handling limits of integration
3. Exam questions
   (a) Evaluating definite and indefinite integrals using the substitution rule
   (b) Changing limits of integration when using the substitution rule

II-§1.5 Area between curves

1. Using definite integral to compute the area between curves
2. Finding areas for curves with multiple intersections
3. Exam questions
   (a) Evaluating areas between curves
   (b) Expressing areas between curves with multiple points of intersection
   (c) Problems which require selecting the axis of integration as part of expressing the area as an integral

January 5, 2024