Exam 1

Multiple Choice Questions

1. Find $\int \sin(x)\cos(\cos(x))\,dx$

   A. $\cos(\sin(x)) + C$

   B. $-\sin(\cos(x)) + C$

   C. $-\cos(x)\sin(\sin(x)) + C$

   D. $\sin(x) + \cos(x) + C$

   E. $\sin^2(x) + \cos^2(x) + C$

2. $\int x\cos(2x)\,dx =$.

   A. $-\frac{x}{2}\sin(2x) + \frac{1}{4}\cos(2x) + C$

   B. $\frac{x}{2}\sin(2x) - \frac{1}{4}\cos(2x) + C$

   C. $\frac{x}{2}\sin(2x) + \frac{1}{4}\cos(2x) + C$

   D. $-2x\sin(2x) + \cos(2x) + C$

   E. $-2x\sin(2x) - 4\cos(2x) + C$

3. Which of the following is the correct form of the partial fraction expansion of

$$\frac{x^2+3x-10}{(x^2+4x+6)(x^2-1)(x+1)}\,?$$

A. $\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+1}+\frac{Dx+E}{x^2+4x+6}$.

B. $\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{x-1}+\frac{Dx+E}{x^2+4x+6}$.

C. $\frac{B}{(x+1)^2}+\frac{C}{x-1}+\frac{Dx+E}{x^2+4x+6}$.

D. $\frac{A}{x+1}+\frac{Bx+C}{x^2-1}+\frac{Dx+E}{x^2+4x+6}$.

E. $\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{x-1}+\frac{D}{x^2+4x+6}$.

4. Use the fact that

$$\frac{13x^2+6x-24}{(3x-1)(x^2+4)}=\frac{6x+4}{x^2+4}-\frac{5}{3x-1}$$

to evaluate the integral

$$\int \frac{13x^2+6x-24}{(3x-1)(x^2+4)}\,dx.$$

A. $6\ln|x^2+4|-\frac{5}{3}\ln|3x-1|+C$

B. $12\arctan\left(\frac{x}{2}\right)-\frac{5}{3}\ln|3x-1|+C$

C. $\frac{8}{x+2}-\frac{5}{3}\ln|3x-1|+6\ln|x+2|+C$

D. $3\ln|x^2+4|-\frac{5}{3}\ln|3x-1|+2\arctan\left(\frac{x}{2}\right)+C$

E. $4\ln|x-2|+2\ln|x+2|-\frac{5}{3}\ln|3x-1|+C$

5. What is the area of the region enclosed by the curves $y^2 = x$ and $y = x$?

   A. $1/2$

   B. $1/3$

   C. 1

   D. $1/6$

   E. $3/2$

6. The table below gives values of f, f', g, and g' for selected values of x. If

$$\int_0^1 f'(x)g(x)dx = 5,$$

then $\int_0^1 f(x)g'(x)dx =$

x	0	1
$f(x)$	2	4
$f'(x)$	6	-3
$g(x)$	-4	3
$g'(x)$	2	-1

   A. -14

   B. -13

   C. -2

   D. 7

   E. 15

7. Which trigonometric integral is obtained after trigonometric substitution for

$$\int \frac{\sqrt{4-x^2}}{x}\,dx.$$

A. $\int \tan(\theta)d\theta$

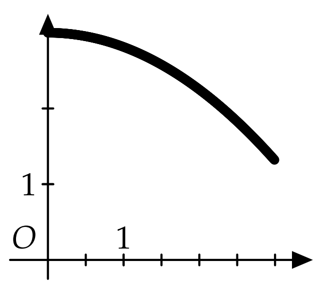
B. $\frac{1}{2}\int \frac{\sin^2(\theta)}{\cos(\theta)}\,d\theta$

C. $2\int \frac{\cos^2(\theta)}{\sin(\theta)}\,d\theta$

D. $\int \sin(\theta)d\theta$

E. $\int \cos(\theta)d\theta$

8. The graph of the function f is shown below for $0 \le x \le 3$. Of the following, which has the smallest value?



A. $\int_1^3 f(x)\,dx$

B. Left Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 6 subintervals of equal length.

C. Right Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 6 subintervals of equal length.

D. Midpoint sum approximation of $\int_1^3 f(x)\,dx$ with 6 subintervals of equal length.

E. Trapezoidal sum approximation of $\int_1^3 f(x)\,dx$ with 6 subintervals of equal length.

9. The function f is continuous on the closed interval $[2, 14]$ and has values as shown the table below. Using three subintervals, what is the approximation of $\int_2^{14} f(x)dx$ found by using the Trapezoid rule?

x	2	6	10	14
$f(x)$	12	28	34	30

A. 249

B. 296

C. 332

D. 368

E. $387.\overline{33}$

10. What are all the values of p for which $\int_1^{\infty} \frac{1}{x^{2p}}\, dx$ converges

A. $p < -1$

B. $p > 0$

C. $p > \frac{1}{2}$

D. $p > 1$

E. There are no values of p for which this integral converges.

Free Response Questions

11. (9 points) Find the following definite integral

$$\int_0^{\sqrt[3]{\pi/2}} v^2 \cos(v^3)\, dv.$$

Solution:

Method I: Do the antiderivative using the substitution $u = v^3$, $du = 3v^2\, dv$.

$$\begin{aligned} \int v^2 \cos(v^3)\, dv &= \frac{1}{3} \int \cos(u)\, du \\ &= \frac{1}{3} \sin(u) + C \\ &= \frac{1}{3} \sin(v^3) + C \end{aligned}$$

Then,

$$\int_0^{\sqrt[3]{\pi/2}} v^2 \cos(v^3)\, dv = \frac{1}{3} \sin(v^3) + C \bigg|_0^{\sqrt[3]{\pi/2}} = \frac{1}{3}.$$

Method II: Let $u = v^3$, then $du = 3v^2\, dv$. When $v = 0$, $u = 0$ and when $v = \sqrt[3]{\pi/2}$, $u = \pi/2$, so

$$\int_0^{\sqrt[3]{\pi/2}} v^2 \cos(v^3)\, dv = \frac{1}{3} \int_0^{\pi/2} \cos(u)\, du = \frac{1}{3} \sin(u) \bigg|_0^{\pi/2} = \frac{1}{3}.$$

12. Consider the two curves $y = x^2 + 2$ and $y = x + 1$.

(a) (5 points) Find the area of the region enclosed by these two curves and the vertical lines $x = 0$ and $x = 3$.

Solution:

$$A = \int_0^3 (x^2+2-(x+1))\,dx = \int_0^3 \left(x^2 - x + 1\right)\,dx = \frac{1}{3}x^3 - \frac{1}{2}x^2 + x\Big|_0^3 = \frac{15}{2}.$$

(b) (1 point) There is a vertical line $x = a$ for $0 < a < 3$ so that the area between the two curves between $x = 0$ and $x = a$ is exactly $\frac{3}{2}$. Write down the cubic equation that you need to solve in order to find a.

Solution:

$$\frac{3}{2} = \int_0^a \left(x^2 - x + 1\right)\,dx$$
$$\frac{3}{2} = \frac{1}{3}a^3 - \frac{1}{2}a^2 + a$$
$$2a^3 - 3a^2 + 6a - 9 = 0$$

13. Find the following antiderivatives:

(a) (7 points) $\int x^2 \sin(x)\, dx$

Solution: Let

$$u = x^2 \qquad dv = \sin(x)\, dx$$
$$du = 2x\, dx \qquad v = -cos(x)$$

$$\int x^2 \sin(x)\, dx = -x^2\cos(x) - \int (-2x\cos(x))\, dx = -x^2\cos(x) + 2\int x\cos(x)\, dx$$

Let

$$u = x \qquad dv = \cos(x)\, dx$$
$$du = dx \qquad v = \sin(x)$$

$$\begin{aligned}\int x^2 \sin(x)\, dx &= -x^2\cos(x) + 2\int x\cos(x)\, dx \\ &= -x^2\cos(x) + 2x\sin(x) - 2\int \sin(x)\, dx \\ &= -x^2\cos(x) + 2x\sin(x) + 2\cos(x) + C\end{aligned}$$

(b) (6 points) $\int x^5 \sqrt[3]{x^3+1}\, dx$

Solution: Let $u = x^3 + 1$, then $du = 3x^2\, dx$.

$$\begin{aligned}\int x^5 \sqrt[3]{x^3+1}\, dx &= \int x^3 \sqrt[3]{x^3+1} x^2\, dx \\ &= \frac{1}{3}\int (u-1)u^{1/3}\, du \\ &= \frac{1}{3}\int (u^{4/3} - u^{1/3})\, du \\ &= \frac{1}{7}u^{7/3} - \frac{1}{4}u^{4/3} + C \\ &= \frac{1}{7}(x^3+1)^{7/3} - \frac{1}{4}(x^3+1)^{4/3} + C\end{aligned}$$

14. Find the following integrals

(a) (7 points) $\int_1^\infty \frac{x}{(1+x^2)^2}\,dx.$

Solution: Let $u = 1 + x^2$, then $du = 2x\,dx$ and

$$\begin{aligned}\int_1^\infty \frac{x}{(1+x^2)^2}\,dx &= \frac{1}{2}\int_2^\infty \frac{1}{u^2}du = \frac{1}{2}\lim_{M\to\infty}\int_2^M \frac{1}{u^2}du \\ &= \frac{1}{2}\lim_{M\to\infty}\left(-\frac{1}{u}\right|_2^M \\ &= \frac{1}{2}\lim_{M\to\infty}\left(\frac{1}{2}-\frac{1}{M}\right) \\ &= \frac{1}{4}\end{aligned}$$

(b) (5 points) $\int_0^8 x^{-2/3}\,dx.$

Solution:

$$\begin{aligned}\int_0^8 x^{-2/3}\,dx &= \lim_{a\to 0^+}\int_a^8 x^{-2/3}\,dx \\ &= \lim_{a\to 0^+}\left(3x^{1/3}\right|_a^8 \\ &= \lim_{a\to 0^+}\left(6-3a^{1/3}\right) \\ &= 6\end{aligned}$$

15. (5 points) Compute the integral

$$\int \sin^3(x)dx.$$

Hint: Note that $\sin^3(x) = \sin^2(x)\sin(x)$ and use a trigonometric identity followed by substitution.

Solution:

$$\int \sin^3(x)\,dx = \int \sin^2(x)\sin(x)\,dx = \int \Big(1-\cos^2(x)\Big)\sin(x)\,dx$$

Letting $u = \cos(x)$, we have $du = -\sin(x)\,dx$ and

$$\begin{aligned} &= \int (1-u^2)(-du) = \int (u^2-1)\,du \\ &= \frac{1}{3}u^3 - u + C \\ &= \frac{1}{3}\cos^3(x) - \cos(x) + C \end{aligned}$$

16. (5 points) A table of values for a continuous function f is shown below. If four equal subintervals of $[0,2]$ are used, what is the Simpson's rule approximation for $\int_0^2 f(x)\,dx$.

x	0.0	0.5	1.0	1.5	2.0
$f(x)$	2	8	6	12	10

Solution: The partition gives $\Delta x = \frac{1}{2}$, so by Simpson's Rule the integral is approximated by

$$\frac{1/2}{3}\left(1\times 2 + 4\times 8 + 2\times 6 + 4\times 12 + 10\right) = \frac{52}{3} = 17.\overline{33}.$$