Exam 1

Name: ________________________ Section: ____________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. If you find you need scratch paper during the exam, please ask. You may not use any of your own notes, paper or anything else not provided. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS). Absolutely no communication device use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show **all work** using proper notation to receive full credit on the free response problems. It will also help you check your answers to show work on multiple choice problems.

Multiple Choice Questions

1 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ **6** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

2 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ **7** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

3 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ **8** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

4 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ **9** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

5 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ **10** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

This page may be used for scratch work.

Multiple Choice Questions

1. (5 points) Find $\int (4x+7)e^x \, dx$.

   A. $(2x^2+7x)e^x + C$
   B. $(2x+7)e^{x+1} + C$
   C. $(4x+7)e^x + C$
   D. $(4x+3)e^x + C$
   E. $(4x+7)e^{x+1} + 4e^{x+1}\ln x + C$

2. (5 points) If $f(2) = 10$, $f(5) = 1$, $f'(2) = 4$ and $f'(5) = 2$, and $f''(x)$ is continuous, what is $\int_2^5 (x+4)f''(x) \, dx$?

   A. $\frac{-125}{2}$
   B. 25
   C. 3
   D. $-2x+1$
   E. 13

3. (5 points) Find $\int \cos^3(7x)\ dx$.

A. $\frac{1}{7}\sin(7x) - \frac{1}{21}\sin^3(7x) + C$

B. $\frac{1}{28}\cos^4(7x) + C$

C. $\frac{1}{7}x - \frac{1}{21}\sin^3(7x) + C$

D. $\frac{7}{2}x + \frac{7}{4}\sin(2x) + C$

E. $\frac{1}{7} - \frac{1}{7}\sin^2(7x) + C$

4. (5 points) Which of the following is equal to the integral

$$\int \frac{dx}{(81 - x^2)^{\frac{3}{2}}}$$

after making the substitution $x = 9\sin(\theta)$?

A. $\int \frac{d\theta}{729\cos^3(\theta)}$

B. $\int \frac{\cos(\theta)d\theta}{81 - 81\sin^3(\theta)}$

C. $\int \frac{1}{81}\sec^2(\theta)\ d\theta$

D. $\int \frac{1}{81}\tan^2(\theta)\ d\theta$

E. $\int \frac{d\theta}{729 - 729\sin^3(\theta)}$

5. (5 points) Which trigonometric substitution would be most helpful in evaluating

$$\int \frac{\sqrt{x^2-36}}{x}\,dx?$$

A. $x = 6\tan\theta$

B. $x = \dfrac{\sin\theta}{6}$

C. $x = 6\sec\theta$

D. $x = \dfrac{\tan\theta}{6}$

E. $x = 6\sin\theta$

6. (5 points) Choose the form of the partial fraction decomposition of

$$\frac{x^3-2}{x^4+5x^2}$$

A. $\dfrac{A}{x^2} + \dfrac{B}{x^2+5}$

B. $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx+D}{x^2+5}$

C. $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x^3} + \dfrac{D}{x^4} + \dfrac{E}{x+5}$

D. $\dfrac{A}{x^2} + \dfrac{Bx+C}{x^2+5}$

E. $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+5} + \dfrac{D}{x^2+5}$

7. (5 points) Find the coefficient B in the partial fraction decomposition

$$\frac{4x-27}{x(x^2+9)} = \frac{A}{x} + \frac{Bx+C}{x^2+9}$$

A. $B = -3$
B. $B = -2$
C. $B = 0$
D. $B = 3$
E. $B = 4$

8. (5 points) Find

$$\int_9^\infty \frac{1}{\sqrt{x}}\, dx$$

A. $\frac{1}{3}$
B. $\frac{2}{3}$
C. 0
D. $\frac{3}{2}$
E. ∞

9. (5 points) Let $f(x)$ be a function that satisfies $|f''(x)| \leq 4$ on the interval $[1, 6]$. Choose the smallest n so that we can be sure that $E_T = |T_n - \int_1^6 f(x)dx| \leq 0.01$, where T_n is the trapezoidal approximation with n intervals.

   A. $n = 65$

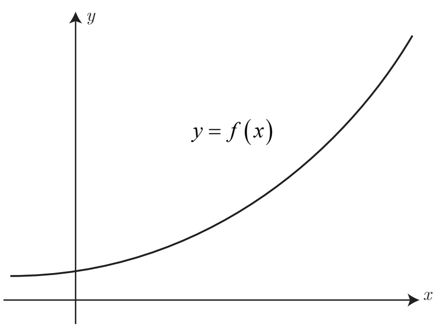
   B. $n = 67$

   C. $n = 80$

   D. $n = 94$

   E. $n = 126$

10. (5 points) For the graph of $y = f(x)$ shown, let I be the value of $\displaystyle\int_0^5 f(x)\,dx$, and let L_n, R_n, M_n and T_n be the approximations using left, right, midpoint and trapezoidal integration. List these in order from smallest to largest:



   A. L_n, T_n, I, M_n, R_n

   B. R_n, T_n, I, M_n, L_n

   C. R_n, M_n, I, T_n, L_n

   D. L_n, M_n, I, T_n, R_n

   E. L_n, T_n, M_n, R_n, I

Free Response Questions: Show all steps clearly to receive full credit.

11. (a) (5 points) Compute $\int \arctan x \, dx$.

Solution: Integrate by parts. Let $u = \arctan x$, so $du = \frac{1}{1+x^2}dx$, and $dv = dx$, so $v = x$. Then

$$\int \arctan x \, dx = x \arctan x - \int \frac{x}{1+x^2} \, dx.$$

Apply u-substitution with $u = 1 + x^2$ so $du = 2xdx$. Then

$$\begin{aligned} x \arctan x - \int \frac{x}{1+x^2} \, dx &= x \arctan x - \int \frac{1}{2} \cdot \frac{1}{u} \, du \\ &= x \arctan x - \frac{1}{2} \ln |u| + C \\ &= x \arctan x - \frac{1}{2} \ln |1 + x^2| + C \end{aligned}$$

(b) (5 points) Compute $\int \frac{x+5}{x^2+9} \, dx$.

Solution:

$$\int \frac{x+5}{x^2+9} \, dx = \int \frac{x}{x^2+9} \, dx + \int \frac{5}{x^2+9} \, dx = \int \frac{x}{x^2+9} \, dx + \frac{5}{3} \arctan\left(\frac{x}{3}\right) + C$$

Now to evaluate $\int \frac{x}{x^2+9} \, dx$ use u-substitution with $u = x^2 + 9$, so $du = 2xdx$. Thus we obtain

$$\int \frac{x}{x^2+9} \, dx = \int \frac{1}{2} \cdot \frac{1}{u} \, du = \frac{1}{2} \ln |u| + C = \frac{1}{2} \ln |x^2 + 9| + C.$$

Then the final answer is

$$\frac{1}{2} \ln |x^2 + 9| + \frac{5}{3} \arctan\left(\frac{x}{3}\right) + C.$$

12. (10 points) Compute $\int \sqrt{49-x^2}\,dx$ using trigonometric substitution. You **must** simplify your answer.

Solution: Let $x = 7\sin\theta$, so $dx = 7\cos\theta d\theta$. The integral becomes

$$\begin{aligned}
\int \sqrt{49-49\sin^2(\theta)}\,7\cos(\theta)\ d\theta &= \int 7\cos(\theta)\,7\cos(\theta)\ d\theta \\
&= \int 49\cos^2(\theta)\ d\theta \\
&= \frac{49}{2}\int 1+\cos(2\theta)\ d\theta \\
&= \frac{49}{2}\left(\theta + \frac{1}{2}\sin(2\theta)\right) + C \\
&= \frac{49}{2}\left(\theta + \frac{1}{2}(2\sin\theta\cos\theta)\right) + C \\
&= \frac{49}{2}\left(\theta + \sin\theta\cos\theta\right) + C
\end{aligned}$$

Since $x = 7\sin\theta$, $\sin\theta = \frac{x}{7}$, so $\theta = \arcsin\frac{x}{7}$ and $\cos\theta = \frac{\sqrt{49-x^2}}{7}$. Therefore,

$$\begin{aligned}
\int \sqrt{49-x^2}\,dx &= \frac{49}{2}\left(\arcsin\left(\frac{x}{7}\right) + \frac{x}{7}\cdot\frac{\sqrt{49-x^2}}{7}\right) + C \\
&= \frac{49}{2}\arcsin\left(\frac{x}{7}\right) + \frac{1}{2}x\sqrt{49-x^2} + C.
\end{aligned}$$

13. (10 points) Compute $\int_0^\infty \frac{x^3}{(1+x^4)^2}\, dx$. Justify your answer by showing your work and using proper notation.

Solution: Apply u substitution with $u = 1 + x^4$ and $du = 4x^3 dx$ to find that

$$\int \frac{x^3}{(1+x^4)^2}\, dx = \frac{-1}{4(1+x^4)} + C.$$

Then we have

$$\lim_{A\to\infty} \int_0^A \frac{x^3}{(1+x^4)^2}\, dx = \lim_{A\to\infty} \frac{-1}{4(1+x^4)}\Big|_0^A = \lim_{A\to\infty} \frac{-1}{4(1+A^4)} - \frac{-1}{4(1+0^4)} = \frac{1}{4}.$$

14. (10 points) Using the method of partial fractions, compute

$$\int \frac{10x^2 - 8x - 6}{x(x+2)(x-3)}\, dx$$

Solution: The partial fraction decomposition is

$$\frac{10x^2 - 8x - 6}{x(x+2)(x-3)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-3}.$$

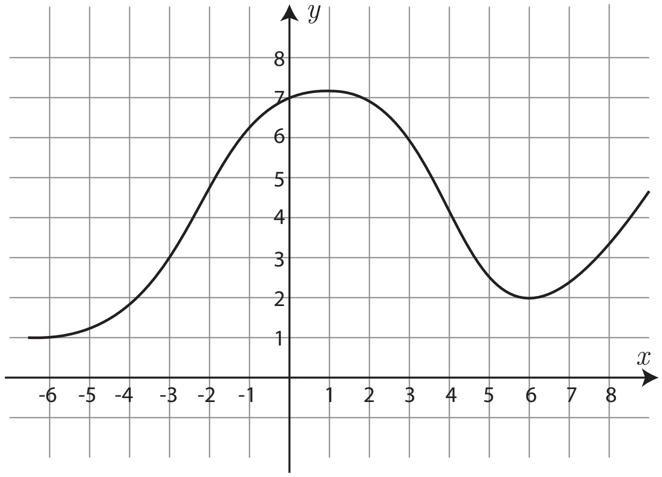
Therefore

$$10x^2 - 8x - 6 = A(x+2)(x-3) + Bx(x-3) + Cx(x+2).$$

Solving, we get $A = 1, B = 5, C = 4$. The integral becomes

$$\int \frac{1}{x} + \frac{5}{x+2} + \frac{4}{x-3}\, dx = \ln|x| + 5\ln|x+2| + 4\ln|x-3| + C.$$

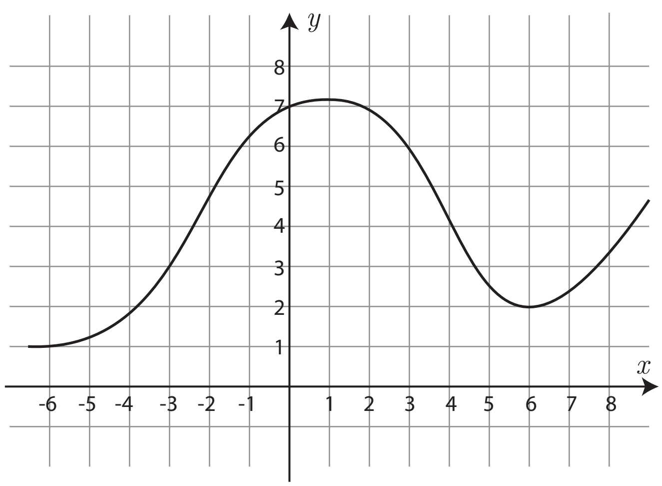
15. (a) (5 points) Apply the **Trapezoidal** rule to estimate the integral $\int_{-6}^{6} f(x)\, dx$ using four intervals (i.e, find T_4), where the graph of $f(x)$ is given below.



Solution: $\Delta x = \frac{6-(-6)}{4} = 3$. Therefore

$$T_4 = \frac{\Delta x}{2}\left(f(-6) + 2f(-3) + 2f(0) + 2f(3) + f(6)\right)$$
$$= \frac{3}{2}\left(1 + 2(3) + 2(7) + 2(6) + 2\right) = \frac{105}{2}.$$

(b) (5 points) Apply **Simpson's** rule to estimate the integral $\int_{-6}^{6} f(x)\, dx$ using four intervals (i.e, find S_4), where the graph of $f(x)$ is given below.



Solution: $\Delta x = \frac{6-(-6)}{4} = 3$. Therefore,

$$S_4 = \frac{\Delta x}{3}(f(-6)+4f(-3)+2f(0)+4f(3)+f(6)) = \frac{3}{3}(1+4(3)+2(7)+4(6)+2) = 53.$$