Exam 1

Name: ______________________________ Section: ______________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. You are allowed to use notes on a single piece of 8.5" by 11" paper, front and back, including formulas and theorems. **You are required to turn this notes page in with your exam.** You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS). Absolutely no communication device use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. It will also help you check your answers to show work on multiple choice problems.

Multiple Choice Questions

1 (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E)

7 (A) (B) (C) (D) (E)

8 (A) (B) (C) (D) (E)

9 (A) (B) (C) (D) (E)

10 (A) (B) (C) (D) (E)

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

Multiple Choice Questions

1. (5 points) Find $\int x^3 \ln x \, dx$.

   A. $\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C$

   B. $x^2 + \frac{3}{2}x + C$

   C. $\frac{1}{4}x^4 \ln x + \frac{1}{20}x^5 \ln x + C$

   D. $\frac{1}{4}x^4 + \frac{1}{2}\ln x^2 + C$

   E. $\frac{1}{3}x^3 + \ln x + C$

2. (5 points) If $f(1) = 5$, $f(5) = 2$, $f'(1) = 6$ and $f'(5) = 1$, and $f''(x)$ is continuous, what is $\int_1^5 (3x + 1)f''(x) \, dx$?

   A. $\frac{55}{2}$

   B. 1

   C. -5

   D. 16

   E. -8

3. (5 points) Find $\int \cos^8 x \sin^3 x \, dx$.

   A. $\frac{1}{9}\cos^9 x \cdot \frac{1}{4}\sin^4 x + C$

   B. $\left(\frac{1}{2}x - \frac{1}{4}\sin(2x)\right)^4 + C$

   C. $\frac{1}{9}\cos^9 x - \frac{1}{17}\cos^{17} x + C$

   D. $-\frac{1}{9}\cos^9 x + \frac{1}{11}\cos^{11} x + C$

   E. $-8\cos^7 x + 10\cos^9 x + C$

4. (5 points) Find $\int \cos^2(7x)\ dx$.

A. $\frac{1}{21}\cos^3(7x) + C$

B. $\frac{1}{2}x + \frac{1}{28}\sin(14x) + C$

C. $14\cos(7x)\sin(7x) + C$

D. $\frac{7}{2}x + \frac{7}{4}\sin(2x) + C$

E. $x - \frac{1}{21}\sin^3(7x) + C$

5. (5 points) Which trigonometric substitution would be most helpful in evaluating

$$\int \frac{x^2}{\sqrt{25 - x^2}}\ dx?$$

A. $x = 5\tan\theta$

B. $x = \frac{5}{\sin\theta}$

C. $x = 5\sin\theta$

D. $x = \frac{\tan\theta}{5}$

E. $x = 5\sec\theta$

6. (5 points) Find

$$\int_1^{\infty} \frac{1}{x^7}\,dx$$

A. ∞

B. $\frac{1}{7}$

C. 1

D. $\frac{6}{37}$

E. $\frac{1}{6}$

7. (5 points) What is the form of the partial fraction decomposition of

$$\frac{5x^2 - 7x + 1}{x^3(x^2+7)}?$$

A. $\frac{A}{x^3} + \frac{Bx + C}{x^2 + 7}$

B. $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx + E}{x^2 + 7}$

C. $\frac{Ax + B}{x^3} + \frac{Cx + D}{x^2 + 7}$

D. $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x^2 + 7}$

E. $\frac{A}{x} + \frac{B}{x^3} + \frac{D}{x + 7} + \frac{E}{x^2 + 7}$

8. (5 points) If $\sin(\theta) = 2x$, then what is $\tan(\theta)$?

A. $\frac{1}{\sqrt{1 - 4x^2}}$

B. $\frac{x}{\sqrt{4 - x^2}}$

C. $\frac{\sqrt{1 - 4x^2}}{2x}$

D. $\frac{2x}{\sqrt{1 - 4x^2}}$

E. $\frac{\sqrt{4 - x^2}}{x}$

9. (5 points) Let $f(x)$ be a function that satisfies $|f''(x)| \leq 6$ on the interval $[3,7]$. Choose the smallest n so that we can be sure that $E_M = |M_n - \int_3^7 f(x)dx| \leq 0.002$, where M_n is the midpoint approximation with n intervals.

A. $n = 106$

B. $n = 90$

C. $n = 84$

D. $n = 26$

E. $n = 140$

10. (5 points) Find the Trapezoidal rule estimate of $\displaystyle\int_2^5 f(x)\,dx$ with $n = 6$.

A. $\frac{3}{2}\big(f(2) + 4f(2.5) + 2f(3) + 4f(3.5) + 2f(4) + 4f(4.5) + f(5)\big)$

B. $\frac{1}{2}\big(f(2.25) + f(2.75) + f(3.75) + f(4.25) + f(4.75)\big)$

C. $\frac{1}{6}\big(f(2) + 2f(2.5) + 2f(3) + 2f(3.5) + 2f(4) + 2f(4.5) + f(5)\big)$

D. $\frac{1}{4}\big(f(2) + 2f(2.5) + 2f(3) + 2f(3.5) + 2f(4) + 2f(4.5) + f(5)\big)$

E. $\frac{1}{2}\big(f(2) + f(2.5) + f(3) + f(3.5) + f(4) + f(4.5)\big)$

Free Response Questions: Show all steps clearly to receive full credit.

11. (10 points) Compute $\int x^2 \cos x \, dx$.

Solution: Integrate by parts. Let $u = x^2$, so $du = 2xdx$, and $dv = \cos x dx$, so $v = \sin x$. Then

$$\int x^2 \cos x \, dx = x^2 \sin x - \int 2x \sin x \, dx.$$

Integrate by parts again. Let $u = 2x$, so $du = 2dx$, and $dv = \sin x dx$, $v = -\cos x$. Then

$$\begin{aligned}\int x^2 \cos x \, dx &= x^2 \sin x - \left(-2x \cos x - \int -2 \cos x dx\right) \\ &= x^2 \sin x + 2x \cos x - \int 2 \cos x \, dx \\ &= x^2 \sin x + 2x \cos x - 2 \sin x + C.\end{aligned}$$

12. (10 points) Compute $\int \frac{dx}{\sqrt{x^2+16}}$ using trigonometric substitution. You **must** simplify your answer.

Solution: Let $x = 4\tan\theta$, so $dx = 4\sec^2\theta d\theta$. The integral becomes

$$\int \frac{4\sec^2\theta}{\sqrt{16\tan^2\theta+16}}\,d\theta = \int \frac{4\sec^2\theta}{\sqrt{16\sec^2\theta}}\,d\theta = \int \frac{4\sec^2\theta}{4\sec\theta}\,d\theta = \int \sec\theta\ d\theta$$
$$= \ln|\sec\theta + \tan\theta| + C.$$

Since $x = 4\tan\theta$, $\tan\theta = \frac{x}{4}$, so $\sec\theta = \frac{\sqrt{x^2+16}}{4}$. Therefore,

$$\int \frac{dx}{\sqrt{x^2+16}} = \ln\left|\frac{\sqrt{x^2+16}}{4} + \frac{x}{4}\right| + C$$

13. (10 points) Compute $\int_2^{27} \frac{1}{(x-2)^{\frac{1}{2}}}\,dx$. Justify your answer by showing your work and using proper notation.

Solution: The integral is

$$\lim_{A\to 2^+}\int_A^{27}(x-2)^{-\frac{1}{2}}\,dx = \lim_{A\to 2^+} 2(x-2)^{\frac{1}{2}}\Big|_A^{27} = \lim_{A\to 2^+}(2\sqrt{27-2}-2\sqrt{A-2})$$
$$= 2\sqrt{25}-0 = 10.$$

14. (10 points) Using the method of partial fractions, compute

$$\int \frac{3x^2 - 3x + 8}{x(x-2)^2}\, dx$$

Solution: The partial fraction decomposition is

$$\frac{3x^2 - 3x + 8}{x(x-2)^2} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}.$$

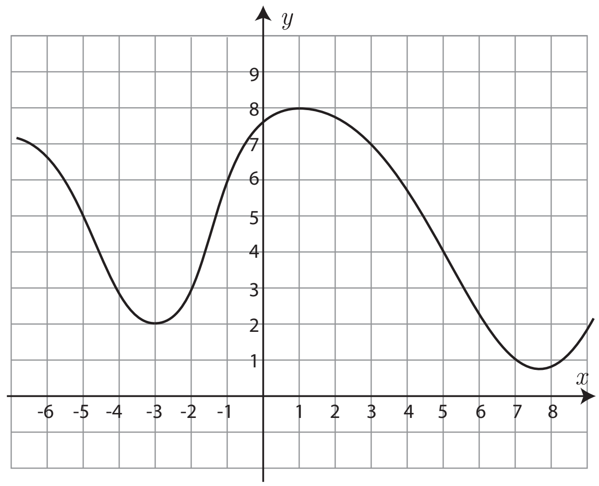
Therefore

$$3x^2 - 3x + 8 = A(x-2)^2 + Bx(x-2) + Cx$$

Solving, we get $A = 2, B = 1, C = 7$. The integral becomes

$$\int \frac{2}{x} + \frac{1}{x-2} + \frac{7}{(x-2)^2}\; dx = \ln|x| + \ln|x-2| - \frac{7}{x-2} + C.$$

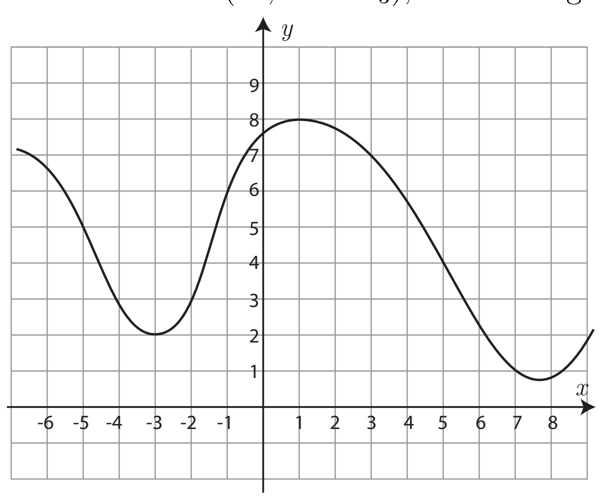
15. (a) (5 points) Apply **Simpson's** rule to estimate the integral $\int_{-5}^{3} f(x)\,dx$ using **four** intervals (i.e, find S_4), where the graph of $f(x)$ is given below.



Solution: $\Delta x = \frac{3-(-5)}{4} = 2$. Therefore

$$S_4 = \frac{\Delta x}{3}\left(f(-5) + 4f(-3) + 2f(-1) + 4f(1) + f(3)\right)$$
$$= \frac{2}{3}\left(5 + 4(2) + 2(6) + 4(8) + 7\right) = \frac{128}{3}.$$

(b) (5 points) Apply the **midpoint** rule to estimate the integral $\int_{-3}^{9} f(x)\,dx$ using **three** intervals (i.e, find M_3), where the graph of $f(x)$ is given below.



Solution: $\Delta x = \frac{9-(-3)}{4} = 4$. The midpoints are $-1, 3, 7$. Therefore,

$$M_3 = \Delta x(f(-1) + f(3) + f(7)) = 4(6 + 7 + 1) = 56.$$