MA 114 — Calculus II Spring 2015
Exam 3 Apr. 14, 2015

Name: ______________________________

Section: ____________________________

Last 4 digits of student ID #: ________

- No books or notes may be used.
- Turn off all your electronic devices and do not wear ear-plugs during the exam.
- You may use a calculator, but not one which has symbolic manipulation capabilities or a QWERTY keyboard.
- Additional blank sheets for scratch work are available upon request.
- **Multiple Choice Questions:**
  Record your answers on the right of this cover page by marking the box corresponding to the correct answer.
- **Free Response Questions:**
  Show all your work on the page of the problem. Clearly indicate your answer and the reasoning used to arrive at that answer.

Multiple Choice Answers

Question					
1	A	B	~~C~~	D	E
2	A	B	C	D	~~E~~
3	~~A~~	B	C	D	E
4	~~A~~	B	C	D	E
5	A	B	~~C~~	D	E
6	A	B	C	D	~~E~~
7	A	~~B~~	C	D	E

Exam Scores

Question	Score	Total
MC		28
8		15
9		13
10		14
11		15
12		15
Total		100

Unsupported answers for the free response questions may not receive credit!

Record the correct answer to the following problems on the front page of this exam.

1. Which trigonometric substitution is needed to evaluate the integral $\displaystyle\int \frac{1}{(9x^2-1)^{3/2}}\,dx$?

A. $x = 9\sec\theta$.

B. $x = 3\sec\theta$.

C. $x = \dfrac{1}{3}\sec\theta$.

D. $x = 3\tan\theta$.

E. $x = \dfrac{1}{3}\tan\theta$.

$9x^2 - 1 = (3x)^2 - 1^2$

Let $3x = \sec\theta$

OR $x = \frac{1}{3}\sec\theta$.

2. Which of the following is the correct form for the partial fraction decomposition of

$$\frac{6x^2+7x-6}{(x-2)(x+2)^2}?$$

A. $\dfrac{A}{x-2} + \dfrac{B}{x+2}$.

B. $\dfrac{A}{x-2} + \dfrac{B}{(x+2)^2}$.

C. $\dfrac{A}{(x-2)(x+2)} + \dfrac{B}{x+2} + \dfrac{C}{x-2}$.

D. $\dfrac{Ax+B}{(x-2)(x+2)} + \dfrac{C}{x+2}$.

E. $\dfrac{A}{x-2} + \dfrac{B}{x+2} + \dfrac{C}{(x+2)^2}$.

Record the correct answer to the following problems on the front page of this exam.

3. Which of the following integrals represents the arclength of the curve $y = \ln(\sin x)$ over the interval $[\frac{\pi}{6}, \frac{\pi}{4}]$?

A. $\displaystyle\int_{\pi/6}^{\pi/4} \sqrt{1+\cot^2 x}\, dx.$

B. $\displaystyle\int_{\pi/6}^{\pi/4} \sqrt{1+(\ln(\sin x))^2}\, dx$

C. $\displaystyle\int_{\pi/6}^{\pi/4} \frac{1}{2}\pi\sqrt{1+\ln(\sin^2 x)}\, dx.$

D. $\displaystyle\int_{\pi/6}^{\pi/4} \frac{1}{2}\pi\sqrt{1+\tan^2 x}\, dx.$

E. $\displaystyle\int_{\pi/6}^{\pi/4} \pi\sqrt{1-\cot^2 x}\, dx.$

$$S = \int_{\pi/6}^{\pi/4} \sqrt{1+(y')^2}\, dx$$

$$y' = \frac{\cos x}{\sin x} = \cot x.$$

4. What is the surface area of the surface generated by rotating the graph of $y = \sqrt{25 - x^2}$ about the x-axis for $-2 \le x \le 3$?

A. 50π.

B. 25π.

C. 20π.

D. 10π.

E. 5π.

$$y' = \frac{1}{2}(25-x^2)^{-1/2}(-2x)$$

$$S = \int_{-2}^{3} 2\pi y \sqrt{1+(y')^2}\, dx$$

$$= \int_{-2}^{3} 2\pi\sqrt{25-x^2}\sqrt{1+\frac{x^2}{25-x^2}}\, dx$$

$$= \int_{-2}^{3} 2\pi\sqrt{25-x^2+x^2}\, dx$$

$$= \int_{-2}^{3} 10\pi\, dx$$

$$= 50\pi.$$

Record the correct answer to the following problems on the front page of this exam.

5. Which of the following integrals represents the x-moment M_x of a thin plate of constant density $\rho = 4$ covering the region enclosed by the parabola $y = x^2$ and the line $y = 1$?

A. $\displaystyle\int_{-1}^{1} 4(1 - x^2)^2 \, dx.$

B. $\displaystyle\int_{-1}^{1} 4(x^4 - 1) \, dx.$

C. $\displaystyle\int_{-1}^{1} 2(1 - x^4) \, dx.$

D. $\displaystyle\int_{-1}^{1} 4(x^3 - x) \, dx.$

E. $\displaystyle\int_{-1}^{1} 2(x^2 - 1) \, dx.$

Intersection $x^2 = 1 \Rightarrow x = \pm 1.$

$$M_x = \frac{1}{2}\rho \int_{-1}^{1} 1^2 - (x^2)^2 \, dx$$

$$= 2 \int_{-1}^{1} 1 - x^4 \, dx$$

6. Which of the following differential equations are separable?

(I) $xy' - 3y^2 = 0.$ (II) $y' = xy - 3x^2.$ (III) $5y' = 9 - y.$

A. (I) only.

B. (II) only.

C. (III) only.

D. (I) and (II) only.

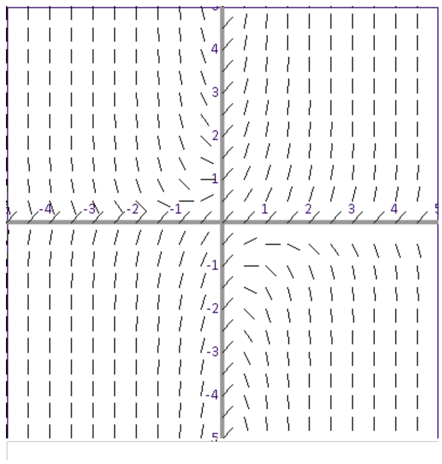
E. (I) and (III) only.

(I) $y' = \frac{1}{x} \cdot 3y^2$

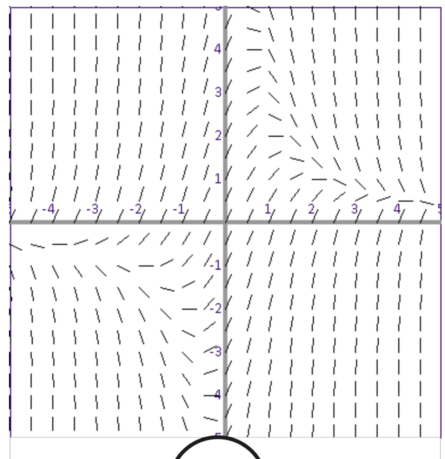
(III) $y' = \frac{1}{5}(9 - y).$

Record the correct answer to the following problems on the front page of this exam.

7. Which of the following is the slope field for $y' = 2 - xy$?



A.



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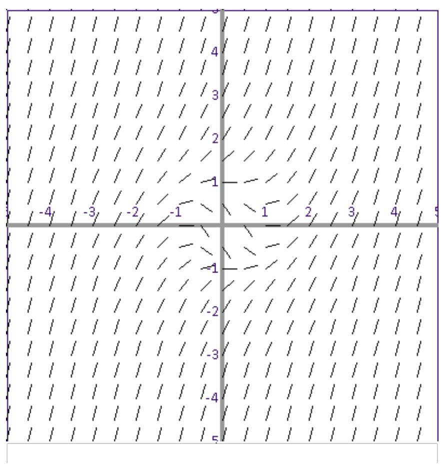

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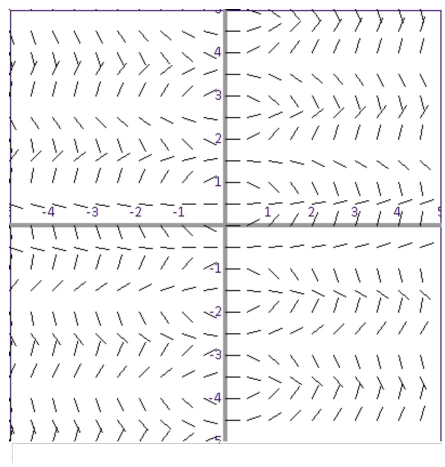
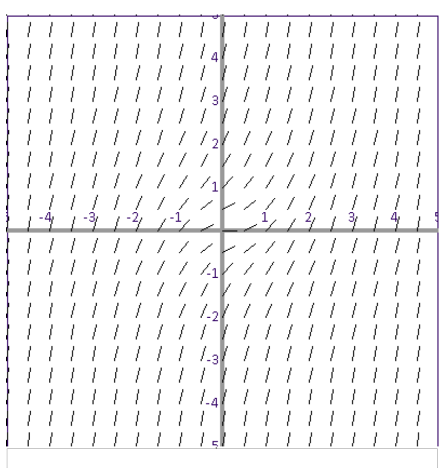


D.



E.

Isoclines $2 - xy = c$

$y = \frac{2-c}{x}, \quad c \in \mathbb{R}$

Free Response Questions: Show your work!

8. Evaluate the integral

$$\int \frac{x^2}{\sqrt{16-x^2}}\,dx.$$

Hint: you may wish to use some of these identities:

$$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta) \qquad \cos^2\theta = \frac{1}{2}(1+\cos 2\theta) \qquad \sin 2\theta = 2\sin\theta\cos\theta.$$

Let $x = 4\sin\theta$

$dx = 4\cos\theta\, d\theta$

$\sqrt{16-x^2} = \sqrt{16-16\sin^2\theta} = \sqrt{16\cos^2\theta} = 4\cos\theta.$

4

$$\int \frac{x^2}{\sqrt{16-x^2}}\,dx = \int \frac{16\sin^2\theta}{4\cos\theta}\,4\cos\theta\,d\theta$$

$$= \int 16\sin^2\theta\,d\theta$$

$$= \int 8(1-\cos 2\theta)\,d\theta$$

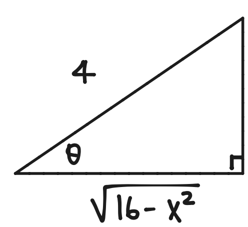
$$= 8\theta - 4\sin 2\theta + C$$

6

$$= 8\arcsin\left(\frac{x}{4}\right) - 8\sin\theta\cos\theta + C$$

$$= 8\arcsin\left(\frac{x}{4}\right) - 8\left(\frac{x}{4}\right)\left(\frac{\sqrt{16-x^2}}{4}\right) + C$$

$$= 8\arcsin\left(\frac{x}{4}\right) - \frac{1}{2}x\sqrt{16-x^2} + C.$$



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Free Response Questions: Show your work!

9. Compute the arclength of the curve $y = \frac{1}{3}x^{3/2}$ over the interval $[0, 4]$.

$$y' = \frac{1}{3}\frac{3}{2}x^{1/2} = \frac{1}{2}x^{1/2}$$

$$S = \int_0^4 \sqrt{1 + \frac{1}{4}x}\, dx$$

6

$$= \int_1^2 4u^{1/2}\, du$$

$$= 4\left(\frac{2}{3}u^{3/2}\right)\Big|_1^2$$

5

Let $u = 1 + \frac{1}{4}x$

$du = \frac{1}{4}dx$

when $x = 0$, $u = 1$

$x = 4$, $u = 2$

$$= \frac{8}{3}\left(2^{3/2} - 1\right)$$

$$= \frac{8}{3}\left(2\sqrt{2} - 1\right).$$

2

10. (a) Find the partial fraction decomposition of the rational function $\dfrac{2x^2 - x + 3}{(x-1)(x^2+1)}$.

$$\frac{2x^2-x+3}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$ 3

$$A(x^2+1) + (Bx+C)(x-1) = 2x^2 - x + 3.$$ 3

$x=1$: $\quad 2A + 0 = 4 \quad \Rightarrow \quad A = 2.$

$x=0$: $\quad A - C = 3 \quad \Rightarrow \quad C = A - 3 = -1.$ 3

$x=2$: $\quad 5A + 2B + C = 9 \quad \Rightarrow \quad 10 + 2B - 1 = 9$

$\quad \Rightarrow \quad B = 0.$

Therefore, $\boxed{\dfrac{2x^2-x+3}{(x-1)(x^2+1)} = \dfrac{2}{x-1} - \dfrac{1}{x^2+1}}$.

(b) Evaluate the integral $\displaystyle\int \frac{3x^2 - 10x + 4}{(x-5)(x^2+4)}\,dx$. You may use the identity

$$\frac{3x^2 - 10x + 4}{(x-5)(x^2+4)} = \frac{1}{x-5} + \frac{2x}{x^2+4}.$$

$$\int \frac{3x^2-10x+4}{(x-5)(x^2+4)}\,dx = \int \frac{1}{x-5} + \frac{2x}{x^2+4}\,dx$$

$$= \ln|x-5| + \ln|x^2+4| + C.$$

2 2 1

Free Response Questions: Show your work!

11. (a) Find the general solution to the differential equation $(1+x^2)y' = xy$.

$$y^{-1}\,dy = \frac{x}{1+x^2}\,dx$$

2

$$\int y^{-1}\,dy = \int \frac{x}{1+x^2}\,dx$$

$$\ln|y| = \frac{1}{2}\ln|1+x^2| + C$$

$$|y| = e^{\frac{1}{2}\ln|1+x^2| + C} = e^{C}\sqrt{1+x^2}$$

$$y = \pm e^{C}\sqrt{1+x^2} = C\sqrt{1+x^2}, \qquad C \neq 0.$$

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Since $y=0$ is a solution ($(1+x)y' = 0 = xy$), then the general solution is $\boxed{y = C\sqrt{1+x^2}, \quad C \in \mathbb{R}.}$

1

(b) Solve the initial value problem $y' = xe^{-y}$, $y(1) = 0$.

$$e^{y}\,dy = x\,dx$$

$$\int e^{y}\,dy = \int x\,dx$$

$$e^{y} = \frac{1}{2}x^2 + C$$

$$y = \ln\left(\frac{1}{2}x^2 + c\right).$$

4

$$y(1) = 0 = \ln\left(\frac{1}{2} + c\right) \Rightarrow c = e^{0} - \frac{1}{2} = \frac{1}{2}.$$

Therefore, $\boxed{y(x) = \ln\left(\frac{1}{2}x^2 + \frac{1}{2}\right).}$

3

12. A bourbon pecan pie is taken out of the oven at 395°F and left to cool in a room of 75°F. Suppose the temperature of the pie fell to 235°F in half an hour.

Let $y(t)$ be the temperature of the pie after t hours. Newton's Law of Cooling states that $y(t)$ satisfies the differential equation $y'(t) = -k(y(t)-T_0)$, where T_0 is the ambient temperature.

(a) Give the general solution to the differential equation, and find the cooling constant k.

General solution is of the form

$$y(t) = T_0 + Ce^{-kt} = 75 + Ce^{-kt}.$$

4

$$y(0) = 395 = 75 + Ce^{0} \Rightarrow C = 320.$$

2

So $y(t) = 75 + 320e^{-kt}$.

$$y(\tfrac{1}{2}) = 235 = 75 + 320e^{-k/2}$$

$$\Rightarrow e^{-k/2} = \frac{160}{320} = \frac{1}{2}$$

$$\Rightarrow k = -2\ln(\tfrac{1}{2}) = \ln(4).$$

4

(b) What is the temperature of the pie t hours after it is taken out of the oven?

$$y(t) = 75 + 320e^{-\ln(4)t}.$$

2

(c) What is the temperature of the pie 2 hours after it is taken out of the oven? Simplify your final answer as much as possible, showing all work.

$$y(2) = 75 + 320e^{-2\ln(4)} = 75 + 320e^{\ln(4^{-2})}$$

$$= 75 + \frac{320}{16}$$

$$= 95^\circ F.$$

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