

I. First rewrite the function in the form $y = ax^n$. Then find the derivative.

1. $y = \frac{5}{x^3}$

2. $y = \sqrt[3]{x^{10}}$

3. $y = \frac{1}{5x^3}$

4. $y = \frac{7}{6\sqrt[5]{x^8}}$

II. Rewrite if necessary until you have the sum of a few terms, each of the form ax^n . Then find the derivative. (**Do not** use the product or quotient rule for these.)

5. $y = \frac{x^3 - 3x^2 + 5x + 2}{x^2}$

6. $y = x^2 \left(x^3 + \sqrt{x} - \frac{1}{x^9} + 15 \right)$

III. Find the derivative. You *will* want the product or quotient rule. **Do not simplify** your answer.

7. $y = (3x^2 + 2x - 3)(5x^7 + 4x^3 - 2x + 1)$

8. $y = \frac{8x^4 + 17}{7x^3 + 2x - 1}$

IV. Suppose the functions $f(x)$ and $g(x)$ and their derivatives have the following values at $x = 1$:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	6	2	-7	5

9. Find $h'(1)$ if $h(x) = f(x)g(x)$

10. Find $h'(1)$ if $h(x) = \frac{f(x) + g(x)}{3x + 1}$.

I. Find the derivative of each of the following. Do not simplify your answers.

1. $y = \frac{5}{\sqrt[7]{3x-5}}$ (Rewrite first!)

2. $y = (x^3 + 6)^{23}$

3. $y = \left((x^2 + 1)^4 + 3 \right)^6 + 5x + 10$

II. Suppose f and g and their first derivatives have the following values at $x = 2$ and $x = 4$:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	5	4	7	-3
4	1	-2	9	8

a. Find $h'(2)$ if $h(x) = \sqrt{f(x) + g(x)}$

b. Find $h'(2)$ if $h(x) = f(g(x))$

III. Suppose f and g and their first derivatives have the following values at $x = 1$ and $x = 2$:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	6	1	-7	1/2
2	3	-1	1/2	-4

Find $h'(2)$ if $h(x) = f(x + g(x))$.

Then find the equation of the tangent line to the graph of $y = h(x)$ at $x = 2$.

IV. Find the third derivative of $y = \sqrt{3x+2}$.

I. Find the derivative of each of the following. **Do not simplify** your answers.

1. $y = x^4 + x^e + e^x + e^\pi + \ln x + \ln 7$

2. $y = (3x + \ln x)e^x$

3. $y = \frac{\ln x}{x^3 - 2x}$

4. $y = e^{x^4 + 2x^3 + 7}$

5. $y = \ln(x^3 + 5x - 2)$

6. $y = \sqrt{\ln(8x + 20)}$

7. $y = \ln(\ln(x^2 + e^x))$

8. $y = \ln(x^5 e^x)$ (simplify with logarithm properties before you differentiate.)

II. Find **and simplify** the **second** derivative of $y = e^x(5x + 2)$.

III. Suppose $g(4) = 7$ and $g'(4) = -6$. Find $h'(4)$ if $h(x) = \ln(x^2 + g(x))$.

1. Suppose \$10,000 is invested at an annual interest rate of 5% compounded continuously.
 - a. How long will it take for the investment to double in value?
 - b. How long will it take for the investment to triple in value?
2. A recent college graduate decides he would like to have \$20,000 in five years to make a down payment on a home.
 - a. How much money will he need to invest today in order to have \$20,000 in five years, given that he can invest at an annual interest rate of 4% compounded continuously?
 - b. Suppose instead the best interest rate he can find is only 2.5% (instead of 4%). Now how much will he need to invest?
 - c. Suppose the interest rate is 4% again, but now he would like to have the \$20,000 in only four years. How much does he need to invest?
3. The half-life of caffeine is 5 hours. This means the amount of caffeine in your bloodstream is reduced by 50% every five hours. A grande French Roast has 330 mg of caffeine. Let $Q(t)$ denote the amount of caffeine in your system t hours after drinking your grande French Roast. (For simplicity, assume the entire drink is consumed instantly.)
 - a. How many milligrams of caffeine will be in your system after 5 hours? after 10 hours?
 - b. Let $Q(t) = Q_0 e^{-kt}$. Find Q_0 and k .
 - c. How many milligrams of coffee will be in your system after 2 hours?
4. A bacteria culture triples in size every 7 hours. Three hours from now, the culture will have 8000 bacteria. If $Q(t)$ denotes the number of bacteria at time t , then $Q(t) = Q_0 e^{kt}$. Find Q_0 and k .
5. The graph of $y = f(x)$ passes through the point $(0, 4)$. The slope of f at any point P is three times the y -coordinate of P . Find an expression for $f(x)$, and find $f(2)$.

1. Two trains leave a station at 1:00 p.m. One train travels north at 70 miles per hour. The other train travels east at 50 miles per hour. How fast is the distance between the two trains changing at 4:00 p.m.?
2. Suppose the height of a certain right triangle is always twice its base. Suppose the base of the triangle is expanding at a rate of 5 inches per second. When the base of the triangle is 15 inches long,
 - a. how fast is the height of the triangle increasing?
 - b. how fast is the length of the hypotenuse of the triangle increasing?
 - c. how fast is the area of the triangle increasing?
3. A spherical balloon is being filled at a rate of 100 cubic centimeters per minute. When the radius is 50 cm,
 - a. how fast is the volume increasing?
 - b. how fast is the surface area increasing?
4. The price of a share of stock is increasing at a rate of \$7 per year. An investor is buying stock at a rate of 20 shares per year. Find the rate at which the value of the investor's stock is increasing when the current price of the stock is \$40 per share, and the investor owns 100 shares.
5. Sand falls from a conveyor belt at the rate of $9 \text{ m}^3/\text{min}$ onto the top of a conical pile. The height of the pile is always three-fourths of the radius of the base. How fast is the radius changing when the pile is 4 m high?



(Image by Stephen Farrel, nytimes.com)

Useful Formulas:

Area of a triangle, $A = \frac{1}{2}bh$.

Volume of a sphere: $V = \frac{4}{3}\pi r^3$. Surface area of a sphere: $SA = 4\pi r^2$.

Volume of a cone: $V = \frac{1}{3}\pi r^2 h$.

1. Let $f(x) = x^3 + 3x^2 - 45x + 18$.
 - a. Find the critical numbers (the x -values where $f'(x) = 0$ or $f'(x)$ DNE), if any.
 - b. Use your answer to part (a) find the maximum and minimum values of $f(x)$ on the interval $[2, 5]$.

2. Let $g(x) = \ln(x^2 - 8x + 20)$. Find the critical numbers, if any, and use them to find maximum and minimum values of $f(x)$ on the interval $[0, 10]$.

3. Let $h(x) = \begin{cases} x^2 + 2x + 3 & x \leq 1 \\ x^2 - 4x + 9 & x > 1 \end{cases}$.
 - a. Is $h(x)$ continuous at $x = 1$?
 - b. Is $h(x)$ differentiable at $x = 1$?
 - c. Find the critical numbers of $h(x)$. (Hint: there are three.)
 - d. Find the maximum and minimum values of $h(x)$ on the interval $[0, 2]$.
 - e. Find the maximum and minimum values of $h(x)$ on the interval $[-2, 5]$.

1. On the same graph, plot both $f(x) = x^3 - 3x - 5$ and its derivative on the interval $[-4, 4]$.
What do you notice? In particular, what appears to be true about $f(x)$ when its derivative is zero? What appears to be true about $f(x)$ when its derivative is positive? is negative?
2. Let $g(x) = \frac{x+4}{x+9}$.
 - a. Find the critical numbers of $g(x)$, if any.
 - b. Find the maximum and minimum value of $g(x)$ on the interval $[1, 6]$.
3. Let $h(x) = e^x(x-5)$.
 - a. Find the critical numbers of $h(x)$, if any.
 - b. Find the maximum and minimum value of $h(x)$ on the interval $[0, 6]$.
4. Let $g(x) = x^2 + 3x + 1$. Find a value c in the interval $[3, 9]$ such that $g'(c)$ equals the average rate of change of $g(x)$ on the interval $[3, 9]$.