

1. Find the limits:

a. $\lim_{n \rightarrow \infty} \frac{n(3n+1)^2}{5n^3 + 23n^2 + 10n + 4}$

b. $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \frac{k+5}{n}$

2. Compute the integral $\int_0^5 3x^2 dx$ via the following steps:

- Write down a sum which approximates the integral. The sum should use n rectangles of equal width, and the height of each rectangle should be determined by the right endpoint of the rectangle. (Your answer should be a summation involving the variables k and n .)
- Use the summation formulas to express the summation in part (a) in simpler terms. (This should be a variable expression only involving n .)
- Compute the limit as the number of rectangles increases to infinity.

3. Write down a sum which approximates the integral $\int_1^4 (5x+1) dx$ as in 2(a) above.

4. Given that the area of the ellipse $30x^2 + y^2 = 30$ is $\sqrt{30}\pi$, evaluate the integral

$$\int_0^1 \sqrt{30-30x^2} dx.$$

5. Consider the limit $\lim_{n \rightarrow \infty} \frac{4}{n} \sum_{k=1}^n \sqrt{4^2 - \left(k \frac{4}{n}\right)^2}$.

- This limit is obtained by applying the definition of the definite integral to $\int_0^4 f(x) dx$ for what function $f(x)$?
- Use a graph of $f(x)$, and geometry, to evaluate this definite integral (and thus, the limit).

1. Find the indefinite integrals:

a. $\int (t^3 + 4t^2 - 8t + 3) dt$

b. $\int \frac{9}{\sqrt{t}} dt$

c. $\int \frac{9x^5 + 5x^3 - 4x + 1}{x^3} dx$

2. Use the fundamental theorem of calculus to find the derivative of $F(x)$ for

$$F(x) = \int_2^x \sqrt{t^3 + 5t - 8} dt.$$

3. Use the fundamental theorem of calculus to evaluate the definite integrals:

a. $\int_0^8 (4x - 7) dx$

b. $\int_1^2 (4x - 7) dx$

c. $\int_1^T \left(\frac{x+1}{x^4} \right) dx$

4. Find the value of x for which $F(x) = \int_{-8}^x (|t| + 200) dt$ takes its maximum on the interval $[-8, 40]$.

1. Find the definite integrals using the fundamental theorem of calculus. You may need to use a substitution.

a. $\int_0^x e^t dt$

b. $\int_0^x (t+3)^2 dt$

c. $\int_0^x \sqrt{t+9} dt$

d. $\int_0^x \frac{3}{(4t+5)} dt$

e. $\int_0^x 6e^{3t-2} dt$

f. $\int_0^x 3t^2 e^{t^3+2} dt$

2. Consider the function $F(x) = \int_{-2}^x \frac{1}{1+t^2} dt$.

Determine the intervals on which $F(x)$ is increasing.

3. Find the average value of $g(x) = e^{2x}$ on the interval $[1, 4]$.

4. A rock is dropped from a cliff. The velocity of the rock, measured in feet per second, after t seconds, is $v(t) = -32t$. The rock hits the ground 10 seconds later. How high is the cliff?