`Chapter 8: Practice/review problems`

The collection of problems listed below comprises questions taken from previous MA123 exams.

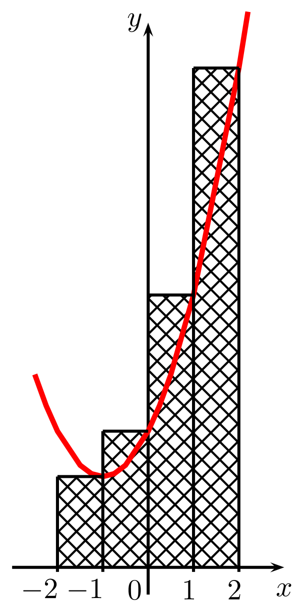
---

[1]. Estimate the area under the graph of $f(x) = x^2 + 2$ on the interval $[0, 2]$ by dividing the interval into four equal parts. Use the right endpoint of each interval as a sample point.

(a) 11.75 square units **(b)** 5.71875 square units **(c)** 9.5 square units

(d) 5.75 square units **[(e)]** 7.75 square units

[2]. Estimate the area under the graph of $y = x^2 + 2x + 3$ for x between -2 and 2. Use a partition that consists of 4 equal subintervals of $[-2, 2]$ and use the right endpoint of each subinterval as a sample point.



[(a)] 22

(b) 23

(c) 24

(d) 25

(e) 26

[3]. A train starts from rest (velocity equal to 0 miles per hour) at 12:00 noon. The velocity increases at a constant rate until 12:15 when the velocity equals 64 miles per hour. How far does the train travel from 12:00 to 12:15?

(a) 7 **[(b)]** 8 **(c)** 9 **(d)** 10 **(e)** 11

[4]. Use a calculator to estimate the integral $\displaystyle\int_{.1}^{.25} 2^x\,dx$

Use three (3) subintervals and the left endpoint of each subinterval to determine the height of the rectangles used in the approximation. The approximate value of the integral is

[(a)] .166 **(b)** .168 **(c)** .172 **(d)** .174 **(e)** .178

[5]. Use a calculator to estimate the integral $\displaystyle\int_{2}^{2.25} \log(x)\,dx$

Use five (5) subintervals and the left endpoint of each subinterval to determine the height of the rectangles used in the approximation. The approximate value of the integral is

(a) .131 **(b)** .128 **(c)** .113 **(d)** .104 **[(e)]** .08

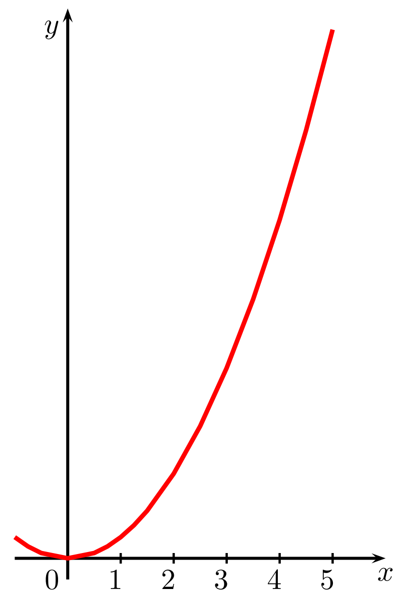
[6]. Use a calculator to estimate the integral

$$\int_1^2 \ln(x)\,dx.$$

Use four subintervals and the right endpoint of each subinterval to determine the height of the rectangles used in the approximation. The approximate value of the integral is

(a) 0.218 **(b)** 0.297 **(c)** 0.352 **[(d)]** 0.470 **(e)** 0.521

[7]. Estimate the area under the graph of $y = 2x^2$ for x between 1 and 5. Use a partition that consists of 4 equal subintervals of $[1, 5]$ and use the right endpoint of each subinterval as a sample point.



(a) 92

(b) 94

(c) 96

(d) 102

[(e)] 108

[8]. Suppose you estimate the integral

$$\int_{10}^{20} (1+x)^2\,dx$$

by the sum of the areas of 10 rectangles of equal base length. Use the right endpoint of each base to determine the height. What is the area of the first (left most) rectangle?

[(a)] 144 **(b)** 244 **(c)** 341 **(d)** 441 **(e)** 541

[9]. Suppose you estimate the area under the graph of $f(x) = x^3$ from $x = 5$ to $x = 25$ by adding the areas of rectangles as follows: partition the interval into 20 equal subintervals and use the right endpoint of each interval to determine the height of the rectangle. What is the area of the 11th rectangle?

(a) 1000 **(b)** 1331 **(c)** 2744 **(d)** 3375 **[(e)]** 4096

[10]. You want to estimate the integral $\displaystyle\int_{10}^{30} \frac{1}{x}dx$ as the sum of areas of rectangles. You break the interval $[10, 30]$ into 20 subintervals of equal length. If you use the left endpoint of each subinterval to determine the height of each rectangle, which estimate is correct?

(**Hint:** Draw a picture!)

(a) $\displaystyle\int_{10}^{30} \frac{1}{x}dx \geq \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \cdots + \frac{1}{29}$

[(b)] $\displaystyle\int_{10}^{30} \frac{1}{x}dx \leq \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \cdots + \frac{1}{29}$

(c) $\displaystyle\int_{10}^{30} \frac{1}{x}dx \leq \frac{1}{11} + \frac{1}{12} + \cdots + \frac{1}{29} + \frac{1}{30}$

(d) $\displaystyle\int_{10}^{30} \frac{1}{x}dx \geq \frac{1}{11} + \frac{1}{12} + \cdots + \frac{1}{29} + \frac{1}{30}$

(e) $\displaystyle\int_{10}^{30} \frac{1}{x}dx \leq \frac{1}{10} + \frac{1}{12} + \frac{1}{14} + \cdots + \frac{1}{28} + \frac{1}{30}$

[11]. Suppose you estimate the integral $\int_{10}^{20} x^2 dx$ by the sum of the areas of 50 rectangles of equal base length. Use the left endpoint of each base to determine the height. What is the area of the first (leftmost) rectangle?

(a) 20 **(b)** 30 **(c)** 40 **(d)** 50 **(e)** 60

[12]. Evaluate the sum $\sum_{k=4}^{10}(1+k)$

(a) 56 **(b)** 60 **(c)** 63 **(d)** 73 **(e)** 74

[13]. Write the sum $7+9+11+13+15+17+19+21$ in summation notation as

$$\sum_{k=2}^{N}(A+2k)$$

What are the values of A and N?

(a) $A=1,\ N=10$ **(b)** $A=2,\ N=10$ **(c)** $A=1,\ N=9$ **(d)** $A=2,\ N=9$ **(e)** $A=3,\ N=9$

[14]. Evaluate the sum $\sum_{k=2}^{4}(5k+1)$

(a) 112 **(b)** 66 **(c)** 29 **(d)** 64 **(e)** 48

[15]. Evaluate the sum $\sum_{k=3}^{6}(k^2-1)$

(a) 35 **(b)** 60 **(c)** 82 **(d)** 98 **(e)** 122

[16]. Evaluate the sum $\sum_{k=2}^{6}(k^2-k)$.

(a) 60 **(b)** 63 **(c)** 67 **(d)** 70 **(e)** 72

[17]. Suppose you estimate the integral

$$\int_{2}^{6} f(x)\,dx$$

by evaluating the sum

$$\sum_{k=1}^{n}(\Delta x)f(2+k\Delta x).$$

If you use $\Delta x = .2$, what value should you use for n?

(a) 25 **(b)** 10 **(c)** 30 **(d)** 20 **(e)** 15

[18]. You make two estimates using rectangles for the integral

$$\int_0^1 (1 - x^2)dx$$

The first estimate uses 50 equal length subintervals and the left endpoint of each subinterval. The second estimate uses 50 equal length subintervals and the right endpoint of each subinterval. What is the difference between the two estimates (first minus second)?

(a) $\frac{8}{50}$ **(b)** $\frac{6}{50}$ **(c)** $\frac{4}{50}$ **(d)** $\frac{2}{50}$ **(e)** $\frac{1}{50}$

[19]. The integral

$$\int_1^6 x^3\,dx$$

is computed as

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{A}{n}\left(1 + k \cdot \frac{A}{n}\right)^3$$

What is the value of A?

(a) 8 **(b)** 5 **(c)** 7 **(d)** 6 **(e)** 4

[20]. Suppose you estimate the integral

$$\int_2^6 f(x)\,dx$$

by adding the areas of n rectangles of equal base length, and you use the right end point of each subinterval to determine the height of each rectangle. If the sum you evaluate is written as

$$\sum_{k=1}^{n} \frac{A}{n} \cdot f\left(B + \frac{A}{n}k\right),$$

what are A and B?

(a) $A = 2, B = 4$ **(b)** $A = 4, B = 4$ **(c)** $A = 4, B = 2$ **(d)** $A = 2, B = 2$ **(e)** None of the above

[21]. Suppose that you estimate the integral

$$\int_2^8 f(x)\,dx$$

by evaluating a sum

$$\sum_{k=1}^{n} \Delta x \cdot f(2 + k \cdot \Delta x).$$

If you use 12 intervals of equal length, what value should you use for Δx?

(a) 0.1 **(b)** 0.2 **(c)** 0.3 **(d)** 0.4 **(e)** 0.5

[22]. Suppose that the integral $\int_1^{11} f(x)\,dx$ is estimated by the sum $\sum_{k=1}^{N} f(a+k\Delta x)\cdot\Delta x$. The terms in the sum equal areas of rectangles obtained by using right endpoints of the subintervals of length Δx as sample points. If $N = 20$, then what is Δx?

(a) .05 **(b)** .1 **[(c)]** .5 **(d)** 1 **(e)** Cannot be determined

[23]. Suppose that the integral $\int_2^{52} f(x)\,dx$ is estimated by the sum $\sum_{k=1}^{N} f(a+k\Delta x)\cdot\Delta x$. The terms in the sum equal areas of rectangles obtained by using right endpoints of the subintervals of length Δx as sample points. If $f(x) = \frac{1}{x^2}$ and $N = 50$, then find the area of the second rectangle.

[(a)] 1/16 **(b)** 1/9 **(c)** 1/8 **(d)** 1/4 **(e)** 1/2

[24]. Suppose that the integral $\int_6^{12} \sqrt{x}\,dx$ is estimated by the sum $\sum_{k=1}^{N} \sqrt{(a+k\Delta x)}\cdot\Delta x$, where $\Delta x = .2$ and $N = 30$. The terms in the sum equal areas of rectangles obtained by using right endpoints of the subintervals of length Δx as sample points. What is a?

(a) 2 **(b)** 3 **(c)** 4 **(d)** 5 **[(e)]** 6