`Chapter 3: Practice/review problems`

The collection of problems listed below contains questions taken from previous MA123 exams.

## Limits and one-sided limits

**[1].** Suppose $H(t) = t^2 + 5t + 1$. Find the limit $\lim_{t\to 2} H(t)$.

**(a)** 15 **(b)** 1 **(c)** 9 **(d)** 6 **(e)** $2t + 5$

**[2].** Find the limit $\lim_{t\to 2} \dfrac{t^2 - 4}{t - 2}$.

**(a)** 2 **(b)** 4 **(c)** 6 **(d)** 8 **(e)** The limit does not exist

**[3].** Find the limit $\lim_{x\to 5} \dfrac{x - 5}{x^2 - 25}$.

**(a)** $-\dfrac{1}{10}$ **(b)** $-\dfrac{1}{5}$ **(c)** 0 **(d)** $\dfrac{1}{5}$ **(e)** $\dfrac{1}{10}$

**[4].** Compute $\lim_{x\to 3} \dfrac{x^2 - 7x + 12}{x - 3}$.

**(a)** 0 **(b)** 1 **(c)** $-1$ **(d)** 2 **(e)** The limit does not exist

**[5].** Find $\lim_{r\to 1} \dfrac{r^2 - 3r + 2}{r - 1}$.

**(a)** 1 **(b)** 0 **(c)** $-1$ **(d)** 2 **(e)** The limit does not exist

**[6].** Find the limit or state that it does not exist: $\lim_{x\to 4} \dfrac{x^2 + x - 20}{x - 4}$.

**(a)** 8 **(b)** $-20$ **(c)** $-15$ **(d)** 9 **(e)** Does Not Exist

**[7].** Compute $\lim_{x\to 0} \left( \dfrac{2x^2 - 3x + 4}{x} + \dfrac{5x - 4}{x} \right)$.

**(a)** 5 **(b)** 4 **(c)** 3 **(d)** 2 **(e)** 1

**[8].** Compute $\lim_{h\to 0} \dfrac{(h + 4)^2 - 16}{h}$.

**(a)** 4 **(b)** 5 **(c)** 6 **(d)** 7 **(e)** 8

**[9].** Find the limit $\lim_{t\to 0^+} \dfrac{\sqrt{t^3}}{\sqrt{t}}$.

**(a)** 0 **(b)** 1 **(c)** 2 **(d)** 3 **(e)** The limit does not exist

**[10].** Find the limit as $x$ tends to 0 from the left $\lim_{x\to 0^-} \frac{|x|}{2x}$.

**(a)** 1/3 **(b)** 1/2 **(c)** 0 **(d)** $-1/2$ **(e)** $-1/3$

**[11].** Find the limit $\lim_{h\to 0^-} \frac{|4h|}{h}$.

(**Hint:** Evaluate the quotient for some negative values of $h$ close to 0.)

**(a)** 0 **(b)** 2 **(c)** $-2$ **(d)** 4 **(e)** $-4$

**[12].** Compute $\lim_{x\to 3^-} \frac{|4x-12|}{x-3}$.

**(a)** 4 **(b)** $-4$ **(c)** 0 **(d)** Doesn't exist **(e)** Cannot be determined

**[13].** Find the limit of $f(x)$ as $x$ tends to 2 from the left if $f(x) = \begin{cases} 1+x^2 & \text{if } x<2 \\ x^3 & \text{if } x\geq 2 \end{cases}$

**(a)** 5 **(b)** 6 **(c)** 7 **(d)** 8 **(e)** 9

**[14].** Find the limit of $f(x)$ as $x$ tends to 2 from the left if $f(x) = \begin{cases} x^3-2 & \text{if } x\geq 2 \\ 1+x^2 & \text{if } x<2 \end{cases}$

**(a)** 5 **(b)** 6 **(c)** 7 **(d)** 8 **(e)** Does not exist

**[15].** For the function $f(x) = \begin{cases} 4x^2-1 & \text{if } x<1 \\ 3x+2 & \text{if } x\geq 1 \end{cases}$

Find $\lim_{x\to 1^+} f(x)$.

**(a)** 5 **(b)** 3 **(c)** 1 **(d)** 0 **(e)** The limit does not exist

**[16].** Let $f(x) = \begin{cases} x^2+8x+15 & \text{if } x\leq 2 \\ 4x+7 & \text{if } x>2. \end{cases}$

Find $\lim_{x\to 2^+} f(x)$.

**(a)** 15 **(b)** 20 **(c)** 30 **(d)** 35 **(e)** The limit does not exist

**[17].** Let $f(x) = \begin{cases} -5x+7 & \text{if } x<3 \\ x^2-16 & \text{if } x\geq 3. \end{cases}$

Find $\lim_{x\to 3^+} f(x)$.

**(a)** 6 **(b)** $-6$ **(c)** $-7$ **(d)** $-8$ **(e)** The limit does not exist

**[18].** Suppose $f(t) = \begin{cases} -t & \text{if } t<1 \\ t^2 & \text{if } t\geq 1 \end{cases}$

Find the limit $\lim_{t\to 1} f(t)$.

**(a)** $-1$ **(b)** 1 **(c)** 0 **(d)** 2 **(e)** The limit does not exist

**[19].** Suppose $f(t) = \begin{cases} (-t)^2 & \text{if } t < 1 \\ t^3 & \text{if } t \geq 1 \end{cases}$

Find the limit $\lim_{t \to 1} f(t)$.

**(a)** $-2$ **(b)** $-1$ **(c)** 1 **(d)** 2 **(e)** The limit does not exist

**[20].** Suppose the total cost, $C(q)$, of producing a quantity $q$ of a product equals a fixed cost of \$1000 plus \$3 times the quantity produced. So total cost in dollars is

$$C(q) = 1000 + 3q.$$

The average cost per unit quantity, $A(q)$, equals the total cost, $C(q)$, divided by the quantity produced, $q$. Find the limiting value of the average cost per unit as $q$ tends to 0 from the right. In other words find

$$\lim_{q \to 0^+} A(q)$$

**(a)** 0 **(b)** 3 **(c)** 1000 **(d)** 1003 **(e)** The limit does not exist

**Limits at infinity**

**[21].** Find the limit $\lim_{t \to \infty} \frac{3}{1 + t^2}$.

**(a)** 0 **(b)** 1 **(c)** 2 **(d)** 3 **(e)** The limit does not exist

**[22].** Find the limit $\lim_{x \to \infty} \frac{x^2 + x + 1}{(3x + 2)^2}$.

**(a)** 1 **(b)** 1/3 **(c)** 0 **(d)** 1/9 **(e)** The limit does not exist

**[23].** Find the limit $\lim_{s \to \infty} \frac{s^4 + s^2 + 13}{s^3 + 8s + 9}$.

**(a)** 0 **(b)** 1 **(c)** 2 **(d)** 3 **(e)** The limit does not exist

**[24].** Find the limit $\lim_{x \to \infty} \frac{2x^2}{(x + 2)^3}$.

**(a)** 0 **(b)** 1 **(c)** 2 **(d)** 3 **(e)** The limit does not exist

**[25].** Suppose the total cost, $C(q)$, of producing a quantity $q$ of a product is given by the equation

$$C(q) = 5000 + 5q.$$

The average cost per unit quantity, $A(q)$, equals the total cost, $C(q)$, divided by the quantity produced, $q$. Find the limiting value of the average cost per unit as $q$ tends to $\infty$. In other words find

$$\lim_{q \to \infty} A(q)$$

**(a)** 5 **(b)** 6 **(c)** 5000 **(d)** 5006 **(e)** The limit does not exist

## Continuity and differentiability

**[26].** Suppose $f(t) = \begin{cases} Bt & \text{if } t \leq 3 \\ 5 & \text{if } t > 3 \end{cases}$

Find a value of $B$ such that the function $f(t)$ is continuous for all $t$.

**(a)** 3/5 **(b)** 4/5 **(c)** 5/3 **(d)** 5/4 **(e)** 5/2

**[27].** Suppose that $f(x) = \begin{cases} A + x & \text{if } x < 2 \\ 1 + x^2 & \text{if } x \geq 2 \end{cases}$

Find a value of $A$ such that the function $f(x)$ is continuous at the point $x = 2$.

**(a)** $A = 8$ **(b)** $A = 1$ **(c)** $A = 2$ **(d)** $A = 3$ **(e)** $A = 0$

**[28].** Suppose $f(t) = \begin{cases} t & \text{if } t \leq 3 \\ A + \frac{t}{2} & \text{if } t > 3 \end{cases}$

Find a value of $A$ such that the function $f(t)$ is continuous for all $t$.

**(a)** 1/2 **(b)** 1 **(c)** 3/2 **(d)** 2 **(e)** 5/2

**[29].** Consider the function $f(x) = \begin{cases} 2x^2 + 3 & \text{if } x \leq 3 \\ 3x + B & \text{if } x > 3 \end{cases}$.

Find a value of $B$ such that $f(x)$ is continuous at $x = 3$.

**(a)** 6 **(b)** 9 **(c)** 12 **(d)** 15 **(e)** There is no such value of $B$.

**[30].** Find all values of $a$ such that the function $f(x) = \begin{cases} x^2 + 2x & \text{if } x < a \\ -1 & \text{if } x \geq a \end{cases}$ is continuous everywhere.

**(a)** $a = -1$ only **(b)** $a = -2$ only **(c)** $a = -1$ and $a = 1$

**(d)** $a = -2$ and $a = 2$ **(e)** all real numbers

**[31].** Which of the following is true for the function $f(x)$ given by

$$f(x) = \begin{cases} 2x - 1 & \text{if } x < -1 \\ x^2 + 1 & \text{if } -1 \leq x \leq 1 \\ x + 1 & \text{if } x > 1 \end{cases}$$

**(a)** $f$ is continuous everywhere

**(b)** $f$ is continuous everywhere except at $x = -1$ and $x = 1$

**(c)** $f$ is continuous everywhere except at $x = -1$

**(d)** $f$ is continuous everywhere except at $x = 1$

**(e)** None of the above

**[32].** Which of the following is true for the function $f(x) = |x - 1|$?

**(a)** $f$ is differentiable at $x = 1$ and $x = 2$.

**(b)** $f$ is differentiable at $x = 1$, but not at $x = 2$.

**(c)** $f$ is differentiable at $x = 2$, but not at $x = 1$.

**(d)** $f$ is not differentiable at either $x = 1$ or $x = 2$.

**(e)** None of the above.