`Chapter 6: Practice/review problems`

The collection of problems listed below contains questions taken from previous MA123 exams.

## Extreme values problems on a closed interval

**[1].** Suppose $f(t) = \begin{cases} \sqrt{4-t} & \text{if} \quad t < 4 \\ \sqrt{t-4} & \text{if} \quad t \geq 4. \end{cases}$

Find the minimum of $f(t)$ on the interval $[0, 6]$.

**(a)** 0 **(b)** 2 **(c)** 4 **(d)** 6 **(e)** 8

**[2].** Let $g(s) = \dfrac{s-1}{s+1}$. Find the maximum of $g(s)$ on the interval $[0, 2]$.

**(a)** $-1/3$ **(b)** 0 **(c)** $1/3$ **(d)** $2/3$

**(e)** Neither the maximum nor the minimum exists on the given interval.

**[3].** Suppose $f(t) = \begin{cases} t^2 - 2t + 2 & \text{if} \quad t < 1 \\ t^3 & \text{if} \quad t \geq 1. \end{cases}$

Find the minimum of $f(t)$ on the interval $[0, 2]$.

**(a)** $-1$ **(b)** 0 **(c)** 1 **(d)** 2 **(e)** 8

**[4].** Let $f(x) = 3x^2 + 6x + 4$. Find the maximum value of $f(x)$ on the interval $[-2, 1]$.

**(a)** 5 **(b)** 7 **(c)** 9 **(d)** 13 **(e)** $-1$

**[5].** Let $G(x) = \begin{cases} (x-3) + 6 & \text{if } x \geq 3 \\ -(x-3) + 6 & \text{if } x < 3. \end{cases}$

Find the minimum of $G(x)$ on the interval $[-10, 10]$.

**(a)** 3 **(b)** 1 **(c)** $-6$ **(d)** 19 **(e)** 6

**[6].** Let $g(s) = \dfrac{1}{s+1}$. Find the maximum of $g(s)$ on the interval $[0, 2]$.

**(a)** $-1$ **(b)** 0 **(c)** 1 **(d)** 2

**(e)** Neither the maximum nor the minimum exists on the given interval.

**[7].** Find the minimum value of $f(x) = x^3 - 3x + 3$ on the interval $[-2, 4]$.

**(a)** 2 **(b)** 1 **(c)** 0 **(d)** $-1$ **(e)** $-2$

**[8].** Find the maximum of $g(t) = |t + 4| + 10$ on the interval $[-12, 12]$.

**(a)** 19 **(b)** 20 **(c)** 24 **(d)** 26 **(e)** 28

**[9].** Find the minimum value of $f(x) = \sqrt{x^2 - 2x + 16}$ on the interval $[0, 5]$.

**(a)** 1 **(b)** 2 **(c)** $\sqrt{15}$ **(d)** $\sqrt{12}$ **(e)** 0

**[10].** Let $f(x) = |x^2 - 1| + 2$. Find the minimum of $f(x)$ on the interval $[-3, 3]$.

**(a)** 3 **(b)** 0 **(c)** 1 **(d)** 2 **(e)** $-1$

**[11].** Suppose $f(t) = 2t^3 - 9t^2 + 12t + 31$. Find the value of $t$ in the interval $[0, 3]$ where $f(t)$ takes on its minimum.

**(a)** 0 **(b)** 1 **(c)** 2 **(d)** 3

**(e)** Neither the maximum nor the minimum exists on the given interval.

**[12].** Let $Q(t) = t^2$. Find a value $A$ such that the average rate of change of $Q(t)$ from 1 to $A$ equals the instantaneous rate of change of $Q(t)$ at $t = 2A$

**(a)** 1 **(b)** $\frac{1}{3}$ **(c)** $\frac{1}{4}$ **(d)** $\frac{1}{5}$ **(e)** Does not exist

## Mean Value Theorem problems

**[13].** Find the value of $A$ such that the average rate of change of the function $g(s) = s^3$ on the interval $[0, A]$ is equal to the instantaneous rate of change of the function at $s = 1$.

**(a)** $\sqrt{2}$ **(b)** $\sqrt{3}$ **(c)** $\sqrt{5}$ **(d)** $\sqrt{6}$ **(e)** $\sqrt{12}$

**[14].** Suppose $k(s) = s^2 + 3s + 1$. Find a value $c$ in the interval $[1, 3]$ such that $k'(c)$ equals the average rate of change of $k(s)$ on the interval $[1, 3]$.

**(a)** $-1$ **(b)** 0 **(c)** 1 **(d)** 2 **(e)** 3

**[15].** Let $k(x) = x^3 + 2x$. Find a value of $c$ between 1 and 3 such that the average rate of change of $k(x)$ from $x = 1$ to $x = 3$ is equal to the instantaneous rate of change of $k(x)$ at $x = c$.

**(a)** 30 **(b)** 15 **(c)** $\sqrt{\frac{28}{3}}$ **(d)** $\sqrt{\frac{13}{3}}$ **(e)** 60

## Increasing/decreasing problems

**[16].** Which function is always increasing on $(0, 2)$

**(a)** $\sqrt{x} + x^2$ **(b)** $x + (1/x)$ **(c)** $x^3 - 3x$

**(d)** $7 - |x|$ **(e)** $(x - 1)^4$

**[17].** Suppose that a function $f(x)$ has derivative $f'(x) = x^2 + 1$. Which of the following statements is true about the graph of $y = f(x)$?

**(a)** The function is increasing on $(-\infty, \infty)$

**(b)** The function is decreasing on $(-\infty, \infty)$

**(c)** The function is increasing on $(-\infty, -1)$ and $(1, \infty)$, and the function is decreasing on $(-1, 1)$.

**(d)** The function is increasing on $(-\infty, 0)$, and the function is decreasing on $(0, \infty)$.

**(e)** The function is decreasing on $(-\infty, 0)$, and the function is increasing on $(0, \infty)$.

**[18].** Find the largest value of $A$ such that the function $g(s) = s^3 - 3s^2 - 24s + 1$ is increasing on the interval $(-5, A)$.

**(a)** $-4$ **(b)** $-2$ **(c)** $0$ **(d)** $2$ **(e)** $4$

**[19].** Let $f(x) = e^{-x^2}$. Find the intervals where $f(x)$ is decreasing.

**(a)** $(-\infty, 0)$ **(b)** $(0, \infty)$ **(c)** $(-\infty, -1)$ **(d)** $(1, \infty)$ **(e)** $(-1, 1)$

**[20].** Let $f(x) = x \ln x$. Find the intervals where $f(x)$ is increasing.

**(a)** $(0, \infty)$ **(b)** $(1, \infty)$ **(c)** $(e, \infty)$ **(d)** $(1/e, \infty)$ **(e)** $(1/e, e)$

**[21].** Suppose the cost, $C(q)$, of stocking a quantity $q$ of a product equals $C(q) = \dfrac{100}{q} + q$. The rate of change of the cost with respect to $q$ is called the marginal cost. When is the marginal cost positive?

**(a)** $q > 10$ **(b)** $q > 15$ **(c)** $q < 20$ **(d)** $q < 25$ **(e)** $q = 30$

**[22].** For which values of $t$ is the function $t^3 - 2t + 1$ increasing?

**(a)** $t > \sqrt{2/3}$ or $t < -\sqrt{2/3}$ **(b)** $-\sqrt{2/3} < t < \sqrt{2/3}$ **(c)** $0 < t < \sqrt{4/3}$ **(d)** $-\sqrt{4/3} < t < 0$ **(e)** Never

**[23].** Suppose that $g'(x) = x^2 - x - 6$. Find the interval(s) where $g(x)$ is increasing.

**(a)** $(-1, 2)$ **(b)** $(-\infty, -2)$ and $(3, \infty)$ **(c)** $(-\infty, -1)$ and $(2, \infty)$

**(d)** $(-2, 3)$ **(e)** It cannot be determined from the information given

**[24].** Let $f(x) = xe^{2x}$. Then $f$ is decreasing on the following interval.

**(a)** $(-\infty, -1/2)$ **(b)** $(-1/2, \infty)$ **(c)** $(-\infty, 1/2)$ **(d)** $(1/2, \infty)$ **(e)** $(-\infty, 0)$

**[25].** Find the interval(s) where $f(x) = -x^3 + 18x^2 - 105x + 4$ is increasing.
(Note that the coefficient of $x^3$ is $-1$, so compute carefully.)

**(a)** $(-\infty, 5)$ and $(7, \infty)$ **(b)** $(5, 7)$ **(c)** $(-\infty, -5)$ and $(7, \infty)$
**(d)** $(-5, 7)$ **(e)** $(-7, 5)$

**[26].** Suppose that $f(x) = xg(x)$, and for all positive values of $x$ the function $g(x)$ is negative (i.e., $g(x) < 0$) and decreasing. Which of the following is true for the function $f(x)$?

**(a)** $f(x)$ is negative and decreasing for all positive values of $x$.

**(b)** $f(x)$ is positive and increasing for all positive values of $x$.

**(c)** $f(x)$ is negative and increasing for all positive values of $x$.

**(d)** $f(x)$ is positive and decreasing for all positive values of $x$.

**(e)** None of the above.

**[27].** Suppose the derivative of a function $g(x)$ is given by $g'(x) = x^2 - 1$. Find all intervals on which $g(x)$ is increasing.

**(a)** $(-\infty, \infty)$ **(b)** $(-1, 1)$ **(c)** $(-\infty, -1)$ and $(1, \infty)$
**(d)** $(0, \infty)$ **(e)** $(-\infty, 0)$

**Extreme values problems using the first derivative**

**[28].** Suppose the derivative of the function $h(x)$ is given by $h'(x) = 1 - |x|$. Find the value of $x$ in the interval $[-1, 1]$ where $h(x)$ takes on its minimum value.

**(a)** $-1/2$ **(b)** $-1$ **(c)** $0$ **(d)** $1/2$ **(e)** $1$

**[29].** Suppose the total cost, $C(q)$, of producing a quantity $q$ of a product equals

$$C(q) = 1000 + q + \frac{1}{10}q^2.$$

The average cost, $A(q)$, equals the total cost divided by the quantity produced. What is the minimum average cost? (Assume $q > 0$)

**(a)** 20 **(b)** 21 **(c)** 26 **(d)** 30 **(e)** 31

**[30].** Suppose that a function $h(x)$ has derivative $h'(x) = x^2 + 4$. Find the $x$ value in the interval $[-1, 3]$ where $h(x)$ takes its minimum.

**(a)** $-1$ **(b)** 3 **(c)** 5 **(d)** 13 **(e)** 29

**[31].** Suppose the cost, $C(q)$, of stocking a quantity $q$ of a product equals $C(q) = \frac{100}{q} + q$. Which positive value of $q$ gives the minimum cost?

**(a)** 10 **(b)** 15 **(c)** 20 **(d)** 25 **(e)** 30

**[32].** Find a local extreme point of $f(x) = \frac{\ln x}{x}$.

**(a)** $(1, 0)$ is a local maximum point.
**(b)** $(1, 0)$ is a local minimum point.
**(c)** $(e, 1/e)$ is a local minimum point.
**(d)** $(e, 1/e)$ is a local maximum point.
**(e)** $f(x)$ has no local extreme points.

**[33].** Suppose the derivative of $G(q)$ is given by $G'(q) = q^2(q + 1)^2(q + 2)^2$. Find the value of $q$ in the interval $[-5, 5]$ where $G(q)$ takes on its maximum.

**(a)** $-5$ **(b)** $-2$ **(c)** $-1$ **(d)** $0$ **(e)** $5$

**[34].** Suppose the derivative of $H(s)$ is given by $H'(s) = s^2(s+1)$. Find the value of $s$ in the interval $[-100, 100]$ where $H(s)$ takes on its minimum.

**(a)** $-100$ **(b)** $-1$ **(c)** $0$ **(d)** $1$ **(e)** $100$

## Concavity problems

**[35].** Find the intervals where $f(x) = x^4 - 12x^3 + 48x^2 + 10x - 8$ is concave downward.

**(a)** $(-\infty, \infty)$
**(b)** $(1, \infty)$
**(c)** $(-\infty, -4)$ and $(-2, \infty)$
**(d)** $(-\infty, 2)$ and $(4, \infty)$
**(e)** $(2, 4)$

**[36].** Let $f(x) = e^{-x^2}$. Find the intervals where $f(x)$ is concave upward.

**(a)** $(1, \infty)$
**(b)** $(-e, e)$
**(c)** $(-\infty, -\sqrt{1/2})$ and $(\sqrt{1/2}, \infty)$
**(d)** $(-\sqrt{1/2}, \sqrt{1/2})$
**(e)** $(-\infty, -e)$ and $(e, \infty)$

**[37].** Let $f(x) = x \ln x$. Find the intervals where $f(x)$ is concave downward.

**(a)** $(0, 1)$
**(b)** $(0, \infty)$
**(c)** $(0, 1/e)$
**(d)** $(1/e, \infty)$
**(e)** $f(x)$ is not concave downward anywhere

**[38].** Suppose that the derivative of $f(x)$ is given by $f'(x) = x^2 - 5x + 6$. Then the graph of $f(x)$ is concave downward on the following intervals(s).

**(a)** $(-\infty, 2)$ and $(3, \infty)$
**(b)** $(2, 3)$
**(c)** $(-\infty, 2.5)$
**(d)** $(2.5, \infty)$
**(e)** $f(x)$ in not concave downward on any interval

**[39].** Find the interval(s) where the graph of $f(x) = x^4 + 18x^3 + 120x^2 + 10x + 50$ is concave downward.

**(a)** $(-5, 4)$
**(b)** $(4, 5)$
**(c)** $(-\infty, 4)$ and $(5, \infty)$
**(d)** $(-5, -4)$
**(e)** $(-\infty, -5)$ and $(-4, \infty)$