

## Chapter 7: Practice/review problems

The collection of problems listed below contains questions taken from previous MA123 exams.

## Max-min problems

[1]. A field has the shape of a rectangle with two semicircles attached at opposite sides. Find the radius of the semicircles if the field is to have maximum area, the perimeter of the field equals 100, and the width of the field (twice the radius of the semicircles) is at most 18. (**Caution:** Be sure your answer satisfies all conditions.) The radius equals

- (a) 6                      (b) 7                      (c) 8                      (d) 9                      (e) 10

[2]. Find the area of the largest rectangle with one corner at the origin and the opposite corner in the first quadrant and on the line  $y = 10 - 2x$ . Assume the sides of the rectangle are parallel with the axes.

- (a)  $73/2$                       (b)  $67/2$                       (c)  $55/2$                       (d)  $49/2$                       (e)  $25/2$

[3]. If you sell an item at price  $p$ , your revenue will equal the price  $p$  times the number sold,  $n$ . Suppose price is linearly related to the number sold by the equation

$$n = 100 - 10(p - 10)$$

How should you set the price to maximize revenue? The price should equal

- (a) 10                      (b) 15                      (c) 20                      (d) 25                      (e) 30

[4]. A rectangle in the first quadrant has one corner at  $(0, 0)$  and the opposite corner on the curve  $y = 2 - x^2$ . What is the largest possible area of this rectangle?

- (a)  $\frac{2}{3}\sqrt{\frac{7}{3}}$                       (b)  $\frac{8}{3}\sqrt{\frac{2}{3}}$                       (c)  $\frac{8}{3}\sqrt{\frac{4}{3}}$                       (d)  $\frac{4}{3}\sqrt{\frac{2}{3}}$                       (e)  $\frac{2}{3}\sqrt{\frac{4}{3}}$

[5]. Find the length of the shortest line segment that connects the point  $(4, 0)$  in the  $(x, y)$  plane to the line  $y = 2x$ .

- (a)  $\frac{8}{5}\sqrt{5}$                       (b)  $\frac{10}{7}\sqrt{7}$                       (c)  $\frac{16}{17}\sqrt{17}$                       (d)  $\frac{12}{15}\sqrt{15}$                       (e)  $\frac{18}{19}\sqrt{19}$

[6]. Find the area of the triangle of minimum area with base equal to the unit interval  $0 \leq x \leq 1$  on the  $x$  axis and with opposite vertex lying on the curve  $y = 8x + \frac{4}{x^2}$  with  $x > 0$ .

- (a) 1                      (b) 2                      (c) 3                      (d) 4                      (e) 6

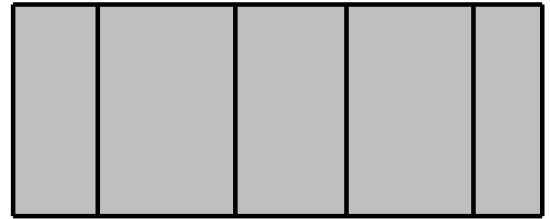
[7]. Find the area of the largest rectangle with one corner at the origin, the opposite corner in the first quadrant on the graph of the line  $f(x) = 6 - 3x$ , and sides parallel to the axes.

- (a) 1                      (b) 2                      (c) 3                      (d) 4                      (e) 5

[8]. What is the maximum area of the rectangle with sides parallel to the coordinate axes, one corner at the origin, and the opposite corner in the first quadrant on the ellipse given by the equation  $2x^2 + y^2 = r^2$ ?

- (a)  $r^2$                       (b)  $\frac{r^2}{\sqrt{2}}$                       (c)  $\frac{r^2}{2}$                       (d)  $\frac{r^2}{2\sqrt{2}}$                       (e)  $\frac{r^2}{4}$

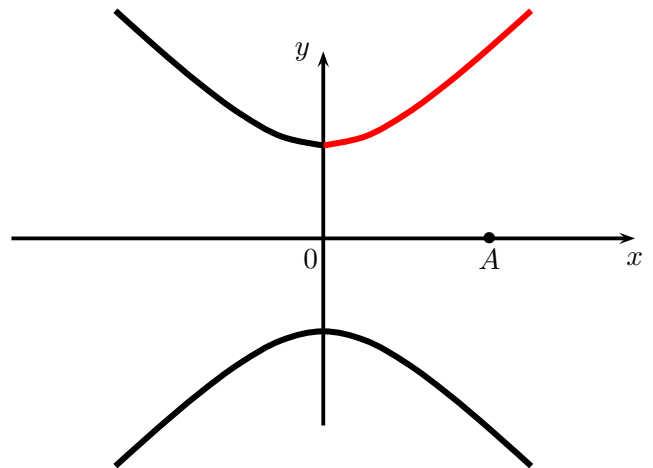
[9]. A rectangular field as shown below is constructed using 2400 feet of fencing. (There are six parallel fences in the vertical direction.) What is the maximum possible area in square feet of the rectangular field?



- (a) 100,000                      (b) 110,000                      (c) 120,000                      (d) 130,000                      (e) None of the above

[10]. Find the point  $(x_0, y_0)$  in the first quadrant that lies on the hyperbola  $y^2 - x^2 = 5$  and is closest to the point  $A(4, 0)$ . Then  $(x_0, y_0)$  is

- (a)  $(1, \sqrt{6})$   
 (b)  $(2, 3)$   
 (c)  $(2.5, \sqrt{11.25})$   
 (d)  $(3, \sqrt{14})$   
 (e)  $(4, \sqrt{21})$



[11]. Suppose you want to find the shortest distance between the point  $(1, 0)$  on the  $x$ -axis and a point on the ellipse  $x^2 + 4y^2 = 16$ . Which problem do you need to solve?

- (a) Minimize  $D = \sqrt{(x-1)^2 + \left(\sqrt{\frac{16-x^2}{4}}\right)^2}$  where  $-4 \leq x \leq 4$ .  
 (b) Minimize  $D = \sqrt{(x)^2 + \left(\sqrt{\frac{16-x^2}{4}} - 1\right)^2}$  where  $-4 \leq x \leq 4$ .  
 (c) Minimize  $D = \sqrt{(x)^2 + \left(\sqrt{16-4x^2} - 1\right)^2}$  where  $-2 \leq x \leq 2$ .  
 (d) Minimize  $D = \sqrt{(x-1)^2 + \left(\sqrt{16-4x^2}\right)^2}$  where  $-2 \leq x \leq 2$ .  
 (e) None of the above.

- [12]. Suppose  $y = \frac{32}{x^2}$ . What is the minimum sum of  $x$  and  $y$  if  $x$  and  $y$  are both positive?
- (a) 6                      (b) 9                      (c) 3                      (d) 2                      (e) 4
- [13]. Suppose that the sum of  $x$  and  $y$  is 12,  $x$  and  $y$  both positive. What is the value of  $x$  that gives the largest possible value of  $x^2y$ ?
- (a) 6                      (b)  $\sqrt{6}$                       (c) 8                      (d)  $\sqrt{8}$                       (e) 4
- [14]. Suppose the product of  $x$  and  $y$  is 64 and both  $x$  and  $y$  are positive. What is the minimum possible sum of  $x$  and  $y$ ?
- (a) 9                      (b) 12                      (c) 15                      (d) 16                      (e) 20
- [15]. Find the area of the rectangle of maximum area with one vertex (corner) at  $(0,0)$  and opposite corner on the ellipse  $x^2 + 4y^2 = 4$ .
- (a)  $3/4$                       (b)  $\sqrt{5}/4$                       (c)  $\sqrt{7}/4$                       (d) 1                      (e)  $\sqrt{11}/4$
- [16]. Let  $T$  be the triangle enclosed by the  $x$ -axis, the  $y$ -axis, and the line  $y = 4 - 2x$ . Find the area of the largest rectangle with sides parallel to the coordinate axes that can be inscribed in  $T$ .
- (a) 2 square units                      (b) 8 square units                      (c) 4 square units  
(d) 6 square units                      (e) 3 square units
- [17]. Let  $(a,b)$  be the point on the line  $y = 4 - 2x$  that is closest to the origin  $(0,0)$ . What is the distance from  $(a,b)$  to  $(0,0)$ ? (**Hint:** Draw a picture.)
- (a)  $2\sqrt{5}/5$                       (b)  $3\sqrt{5}/5$                       (c)  $4\sqrt{5}/5$                       (d)  $5\sqrt{5}/5$                       (e)  $6\sqrt{5}/5$

**Related rate problems**

- [18]. At 12:00 noon a ship sailing due East at 20 miles per hour passes directly North of a lighthouse located on the coast exactly one mile South of the ship. How fast is the distance between the ship and the lighthouse increasing at 1:00 pm?
- (a)  $\frac{100}{\sqrt{101}}$                       (b)  $\frac{200}{\sqrt{201}}$                       (c)  $\frac{300}{\sqrt{301}}$                       (d)  $\frac{400}{\sqrt{401}}$                       (e)  $\frac{500}{\sqrt{501}}$
- [19]. Water is evaporating at a rate of .5 cubic feet per day from a cylindrical tank. The circular base of the tank (parallel to the ground) has a radius of 4 feet. How fast is the depth of the water decreasing when the tank is half full (measured in feet per day)?
- (a)  $\frac{1}{64\pi}$                       (b)  $\frac{1}{32\pi}$                       (c)  $\frac{1}{16\pi}$                       (d)  $\frac{1}{8\pi}$                       (e)  $\frac{1}{4\pi}$

- [20]. A triangle has a base of length 5 on the  $x$  axis. The altitude of the triangle is increasing at a rate of 3 units per second. How fast is the area of the triangle increasing when the area of the triangle equals 14 square units?
- (a)  $\frac{45}{2}$       (b)  $\frac{40}{2}$       (c)  $\frac{35}{2}$       (d)  $\frac{25}{2}$       (e)  $\frac{15}{2}$
- [21]. A train travels along a straight track at a constant speed of 50 miles per hour. A straight road intersects the track at right angles and a truck is parked on the road one mile from the track. How fast is the train traveling away from the truck when the train is 3 miles past the intersection?
- (a)  $10\sqrt{10}$       (b)  $15\sqrt{10}$       (c)  $20\sqrt{10}$       (d)  $5\sqrt{10}$       (e)  $20\sqrt{5}$
- [22]. Water is evaporating from a pool at a constant rate. The area of the pool is 5000 square feet. Assume the sides of the pool drop straight down (perpendicular) from the edge. The water in the pool drops .5 feet in one day. How fast is the water evaporating in cubic feet per day?
- (a) 2000      (b) 2500      (c) 3000      (d) 3500      (e) 4000
- [23]. A point moves along the line  $y = 4 + 3x$  so that the  $y$  coordinate of the point increases at a constant rate of 2 units per second. How fast is the  $x$  coordinate of the point increasing?
- (a)  $2/3$       (b) 1      (c)  $3/2$       (d) 2      (e) 3
- [24]. Two trains leave a station at the same time. One travels north on a track at 30 miles per hour. The second travels east on a track at 40 miles per hour. How fast are the trains travelling away from each other in miles per hour when the northbound train is 60 miles from the station?
- (a) 60 miles per hour      (b) 40 miles per hour      (c) 50 miles per hour  
(d) 130 miles per hour      (e)  $50\sqrt{5}$  miles per hour
- [25]. A sandbox with square base is being filled with sand at the rate of 9 cubic feet per minute. The sandbox is 9 feet long and 9 feet wide. How fast is the level of sand in the sandbox rising?
- (a)  $3/5$  feet per minute      (b)  $2/9$  feet per minute      (c)  $2/5$  feet per minute  
(d)  $1/9$  feet per minute      (e)  $1/5$  feet per minute
- [26]. The length of the horizontal side of a rectangle is increasing at a rate of 3 inches per minute. Suppose the instantaneous rate of change of the area of the rectangle equals zero at the instant that the length of the horizontal side is 2 inches and the length of the vertical side is 5 inches. How fast is the length of the vertical side decreasing at this instant?
- (a)  $13/2$       (b)  $15/2$       (c)  $17/2$       (d)  $19/2$       (e)  $21/2$

[27]. At 12:00 noon a boat is 15 miles due north of a lighthouse. The boat is moving east at 20 miles per hour. How fast is the distance from the boat to the lighthouse increasing one hour later?

- (a) 20 miles per hour                      (b) 13 miles per hour                      (c) 12.8 miles per hour  
(d) 25 miles per hour                      (e) 16 miles per hour

[28]. The relationship between degrees Celsius,  $C$ , and degrees Fahrenheit,  $F$  is

$$F = 32 + \frac{9}{5}C$$

Suppose you heat water at a constant rate of 9 degrees  $F$  per minute. How fast are you heating the water measured in degrees  $C$  per minute?

- (a) 1                      (b) 5                      (c) 9                      (d)  $5/9$                       (e)  $9/5$

[29]. A rectangular pool of dimensions 10 ft.  $\times$  20 ft.  $\times$  6 ft. (length  $\times$  width  $\times$  height) is filled with water at a rate of  $1.5 \text{ ft}^3/\text{min}$ . How fast is the level of the water rising when the pool is half full?

- (a)  $0.0075 \text{ ft}/\text{min}$ .                      (b)  $0.0085 \text{ ft}/\text{min}$ .                      (c)  $0.006 \text{ ft}/\text{min}$ .  
(d)  $0.005 \text{ ft}/\text{min}$ .                      (e)  $0.009 \text{ ft}/\text{min}$ .

[30]. Two trains leave a station at the same time. One travels north on a track at 50 mph. The second travels east on a track at 120 miles per hour. How fast are they traveling away from one another in miles per hour when the northbound train is 100 miles from the station?

- (a) 125                      (b) 130                      (c) 132.4                      (d) 135                      (e) 138.6

[31]. Two trains leave a station at the same time. One train travels east at a speed of 15 miles per hour. The other train travels north at a speed of 20 miles per hour. How fast (in miles per hour) are the trains traveling away from each other when the eastbound train is 30 miles from the station?

- (a) 25                      (b) 35                      (c) 40                      (d) 50                      (e) 100

[32]. A stone is dropped into a pond and causes a circular ripple. If the radius of the circle increases at a rate of  $0.25 \text{ ft}/\text{sec}$ ., how fast does the area increase (in  $\text{ft}^2/\text{sec}$ .) when the radius equals 0.4 ft.?

- (a)  $0.2\pi$                       (b)  $0.4\pi$                       (c)  $0.8\pi$                       (d)  $0.16\pi$                       (e)  $0.64\pi$

[33]. An expanding rectangle has its length always equal to three times its width. The area is increasing at a rate of 42 square feet per minute. At what rate (in feet per minute) is the width increasing when the width is 4 feet?

- (a) 1.50                      (b) 1.75                      (c) 2.00                      (d) 2.25                      (e) 2.75

[34]. An expanding rectangle has its length always equal to twice its width. The area is increasing at a rate of 40 square feet per minute. At what rate is the width increasing when the width is 2 feet?

- (a) 10                      (b) 8                      (c) 6                      (d) 5                      (e) 4