

MA123, Chapter 9: Optimization Word Problems

Chapter Goals:

In this Chapter we learn a general strategy on how to approach optimization problems, one of the main types of word problems that one usually encounters in a first Calculus course.

Assignments:

Assignment 17

MAX-MIN PROBLEMS

All max-min problems ask you to find the largest or smallest value of a function on an interval. Usually, the hard part is reading the English and finding the formula for the function. Once you have found the function, then you can use the techniques from Chapter 6 to find the largest or smallest values.

► **Max-min guideline:** This guideline is found on pp. 131-133 of our textbook.

- (1.) Read the problem quickly.
- (2.) Read the problem carefully.
- (3.) Define your variables. If the problem is a geometry problem, draw a picture and label it.
- (4.) Determine whether you need to find the max or the min.
Determine exactly what needs to be maximized or minimized.
- (5.) Write the *general* formula for what you are trying to maximize or minimize. If this formula only involves *one* variable, then skip steps **6**, **7** and **8**.
- (6.) Find the relationship(s) (i.e., equation(s)) between the variables.
- (7.) Do the algebra to solve for one variable in the equation(s) as a function of the other(s).
- (8.) Use your formula from step **5** to rewrite the formula that you want to maximize or minimize as a function of one variable only.
- (9.) Write down the interval over which the above variable can vary, for the particular word problem you are solving.
- (10.) Take the derivative and find the critical points.
- (11.) Use the techniques from Chapter 6 to find the maximum or the minimum.

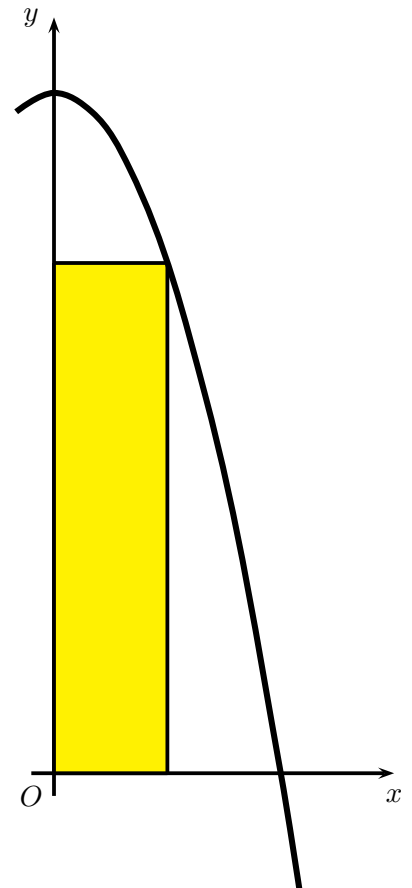
Example 1: What is the largest possible product you can form from two non-negative numbers whose sum is 30?

Example 2: Suppose the product of x and y is 26 and both x and y are positive. What is the minimum possible sum of x and y ?

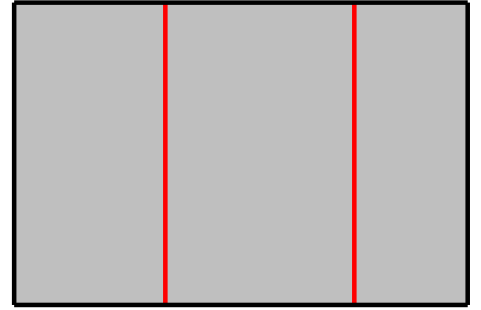
Note: An alternative wording for Example 2 above is:

“Suppose y is inversely proportional to x and the constant of proportionality equals 26. What is the minimum sum of x and y if x and y are both positive?”

Example 3: Find the area of the largest rectangle with one corner at the origin, the opposite corner in the first quadrant on the graph of the parabola $f(x) = 9 - x^2$, and sides parallel to the axes.



Example 4: A farmer builds a rectangular pen with three parallel partitions using 500 feet of fencing. What dimensions will maximize the total area of the pen?



Example 5: A landscape architect wishes to enclose a rectangular garden on one side by a brick wall costing \$20/ft and on the other three sides by a metal fence costing \$10/ft. If the area of the garden is 200 square feet, find the dimensions of the garden that minimize the cost.

Example 6: A box is constructed out of two different types of metal. The metal for the top and bottom, which are both square, costs \$5 per square foot and the metal for the sides costs \$10 per square foot. Find the dimensions that minimize cost if the box has a volume of 24 cubic feet.

Example 7: An open box is to be made out of a 12-inch by 20-inch piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides. Find the dimensions of the resulting box that has the largest volume.

Example 8: A car rental agency rents 180 cars per day at a rate of 30 dollars per day. For each 1 dollar increase in the daily rate, 5 fewer cars are rented. At what rate should the cars be rented to produce the maximum income, and what is the maximum income?