MA 123 Fall 2024 — Exam 1

Elementary Calculus — 09/26/24

Name: _____

Student ID #: 9 _____ **Sec:** _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 2 short answer questions and 18 multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (a) is correct, you must shade

(a) (b) (c) (d) (e)

It is your responsibility to make it CLEAR which response has been chosen. **You will not get credit unless the correct answer has been clearly marked on this page.**

GOOD LUCK!

3. (a) (b) (c) (d) (e)   12. (a) (b) (c) (d) (e)

4. (a) (b) (c) (d) (e)   13. (a) (b) (c) (d) (e)

5. (a) (b) (c) (d) (e)   14. (a) (b) (c) (d) (e)

6. (a) (b) (c) (d) (e)   15. (a) (b) (c) (d) (e)

7. (a) (b) (c) (d) (e)   16. (a) (b) (c) (d) (e)

8. (a) (b) (c) (d) (e)   17. (a) (b) (c) (d) (e)

9. (a) (b) (c) (d) (e)   18. (a) (b) (c) (d) (e)

10. (a) (b) (c) (d) (e)   19. (a) (b) (c) (d) (e)
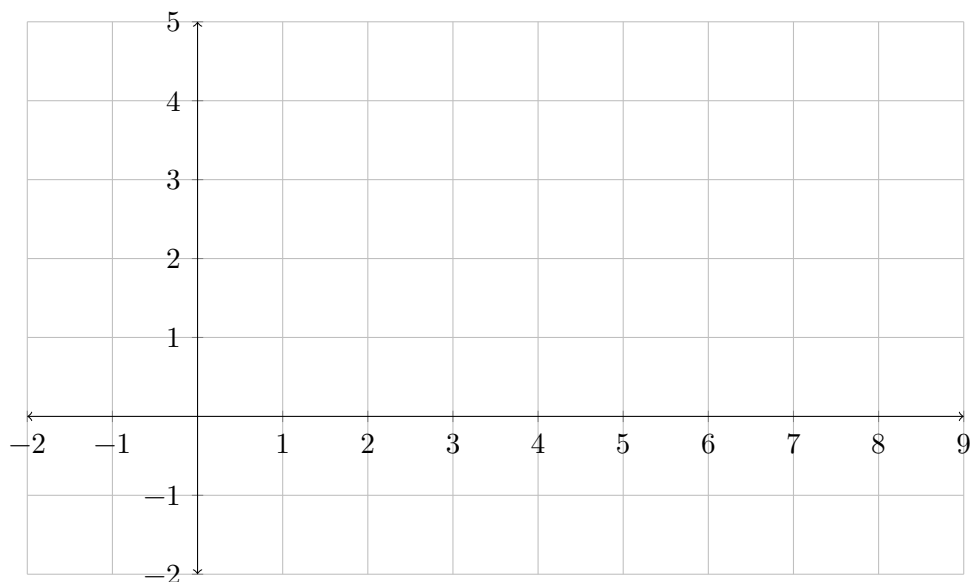
11. (a) (b) (c) (d) (e)   20. (a) (b) (c) (d) (e)

Short Answer Questions

Each question is an opportunity to earn 5 points. Points are earned on the clarity and correctness of your work, not merely on having a correct answer somewhere.

1. Sketch the graph of a function $y = f(x)$ which satisfies the following properties:
   $\lim_{x \to 5} f(x) = 2$, $\lim_{x \to 6^-} f(x) = 1$, $f(6) = 4$, $\lim_{x \to 6^+} f(x) = 3$, and $f(x)$ is continuous for all x except $x = 6$.

2. If $h(t) = -16t^2 + 54t + 28$ represents the height of an object in feet above ground level at time t seconds, determine the height of the object at the time when its speed is zero (prior to hitting the ground).

Multiple Choice Questions

Clearly mark your answer on the cover page on this exam for credit.

3. Determine the average rate of change of $f(x) = x^3 - 15x + 8$ from $x = 5$ to $x = 9$.

   Possibilities:

   (a) $\dfrac{1175}{2}$

   (b) 136

   (c) -136

   (d) $-\dfrac{1}{136}$

   (e) $\dfrac{5315}{9}$

4. Evaluate $\displaystyle\lim_{x\to\infty} \dfrac{13x + 19x^6 + 8}{5x^6 + x^2 + 11}$.

   Possibilities:

   (a) $\dfrac{19}{5}$

   (b) $\dfrac{8}{11}$

   (c) $\dfrac{13}{5}$

   (d) 0

   (e) The limit does not exist.

5. Let $f(x) = x^2 + 8x - 18$. Determine a value of x such that the average rate of change of $f(x)$ from 0 to x equals 30.

   Possibilities:

   (a) 10

   (b) 38

   (c) 30

   (d) 50

   (e) 22

6. A train traveled from city A to city B then traveled from city B to city C. The train left city A at 9:00am and arrived at city B at 3:30pm. The train left city B at 5:30pm and arrived at city C at 10:30pm. The average speed from city A to city B was 54 miles per hour. The distance from city B to city C is 175 miles. What was the average speed of the train from city A to city C, including the 2 hour wait at city B?

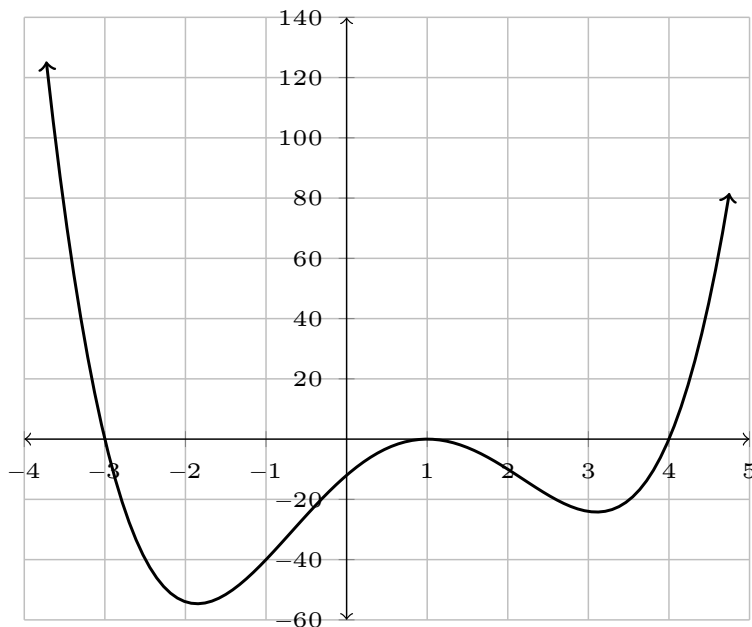   Choose the numeric value that most closely approximates the answer.

   Possibilities:

   (a) 55.13 miles per hour

   (b) 45.74 miles per hour

   (c) 46.96 miles per hour

   (d) 39.50 miles per hour

   (e) 38.96 miles per hour

7. The graph of $y = x^4 - 3x^3 - 9x^2 + 23x - 12$ is shown below. Determine all values of x for which $f(x) = \left| x^4 - 3x^3 - 9x^2 + 23x - 12 \right|$ is not differentiable.

Possibilities:

(a) -3 and 4

(b) $-3, 1$ and 4

(c) 1 and 4

(d) $1, 3$ and 4

(e) 1

8. Let $f(x) = 8x^2 + 3x - 5$. The tangent line to the graph of $y = f(x)$ at $x = 6$ passes through the point $(7, y_1)$. Determine the value of y_1.

Possibilities:

(a) 408

(b) 400

(c) 301

(d) 994

(e) -293

9. Evaluate $\lim\limits_{t \to 7} \dfrac{t^2 - 9t + 14}{t^2 - 49}$.

Possibilities:

(a) $\dfrac{9}{14}$

(b) $\dfrac{5}{14}$

(c) 0

(d) 1

(e) The limit does not exist.

10. Given

$$f(x) = \begin{cases} \sqrt{x^2 - 4} & \text{if } x \leq -2, \\ -4 + 4x & \text{if } -2 < x \leq 5, \\ 5x^2 + 3x + 9 & \text{if } x > 5, \end{cases}$$

evaluate $\lim\limits_{x \to 5^+} f(x)$.

Possibilities:

(a) 0

(b) 16

(c) $\sqrt{21}$

(d) 149

(e) -12

11. Let $f(x) = 3x^2 - 6x + 10$. At what value of x does the tangent line to the graph of $y = f(x)$ have slope 7?

Possibilities:

(a) 36

(b) $\dfrac{1}{2}$

(c) $-\dfrac{3}{2}$

(d) $\dfrac{13}{6}$

(e) $\dfrac{1}{6}$

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12. The graph of $y = f(x)$ is shown below. Compute the average rate of change of $f(x)$ from $x = -5$ to $x = 3$.
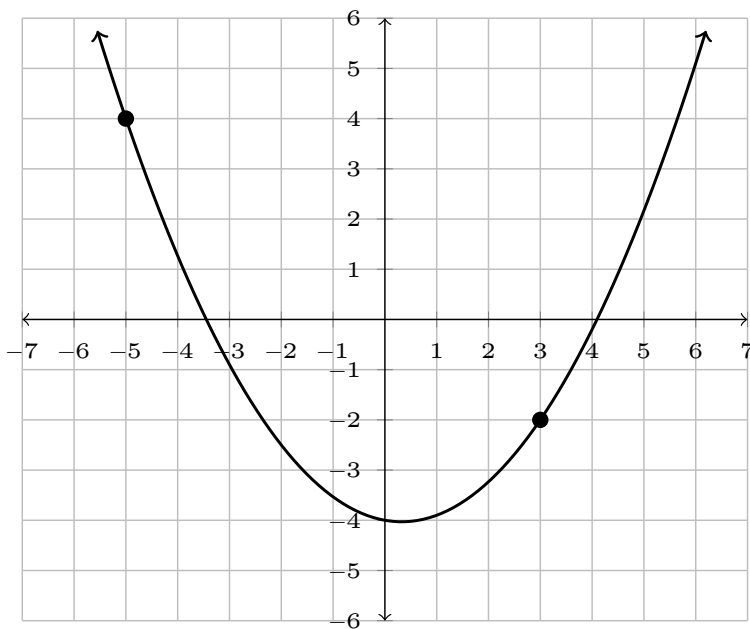
Possibilities:

(a) $\dfrac{3}{4}$
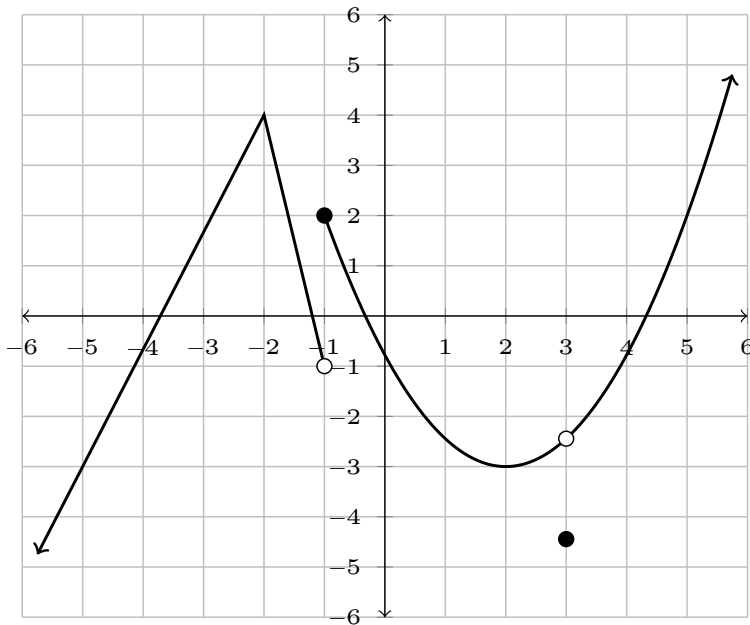
(b) $-\dfrac{4}{3}$

(c) $-\dfrac{3}{4}$

(d) 6

(e) -1

13. The graph of $y = f(x)$ is shown below. The function is continuous except at which x values?

Possibilities:

(a) -1 and 3 only

(b) -2 and -1 only

(c) -2, -1 and 3

(d) -1 only

(e) -2 only

14. Consider the function $f(x) = \begin{cases} Ax^2 & \text{if } x < 2, \\ 11 - Ax & \text{if } x \geq 2. \end{cases}$ Determine a value of A such that $f(x)$ is continuous at $x = 2$.

Possibilities:

(a) $\dfrac{11}{2}$

(b) $\dfrac{11}{6}$

(c) $-\dfrac{1}{2} + \dfrac{1}{2}\sqrt{23}$

(d) $\dfrac{1}{2} - \dfrac{1}{2}\sqrt{23}$

(e) 0

15. Let $f(x) = x^2 + 3x - 8$. Determine a value of c between $x = 1$ and $x = 14$ such that the average rate of change of $f(x)$ from $x = 1$ to $x = 14$ is equal to the instantaneous rate of change at $x = c$.
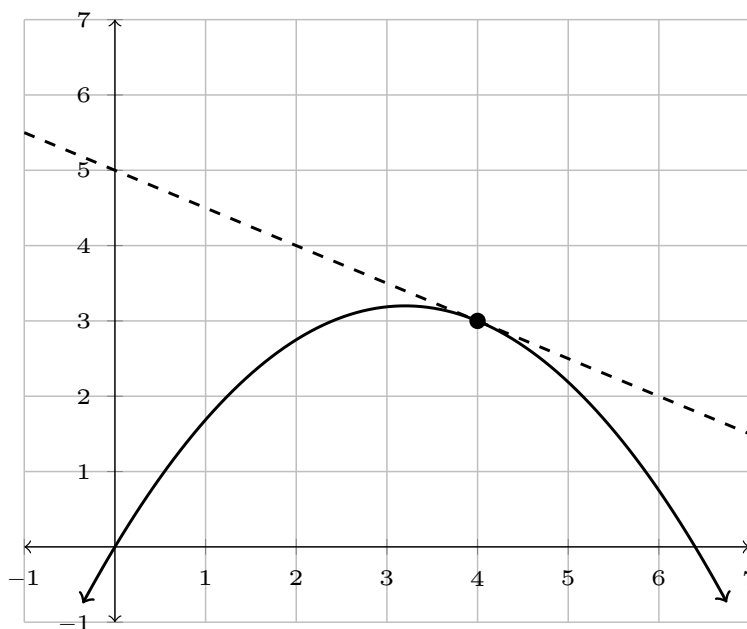
Possibilities:

(a) 7

(b) $\dfrac{2}{15}$

(c) 18

(d) 39

(e) $\dfrac{15}{2}$

16. The graph of $y = f(x)$ and its tangent line at $x = 4$ is shown below. What is the value of $f'(4)$?
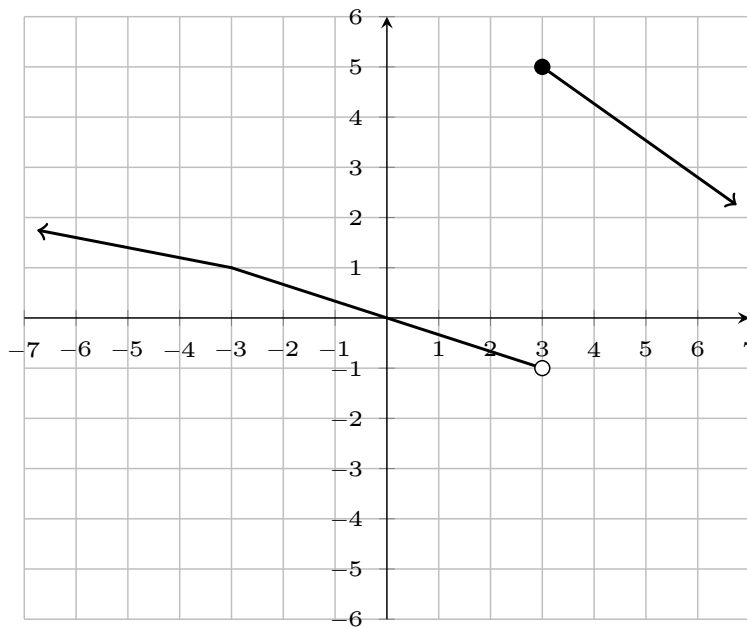
Possibilities:

(a) 2

(b) 3

(c) -2

(d) -1

(e) $-\dfrac{1}{2}$

17. The graph of $y = f(x)$ is shown below. Evaluate $\lim\limits_{x \to 3} f(x)$.

Possibilities:

(a) The limit does not exist.

(b) 1

(c) -1

(d) -5

(e) 5

18. Let $f(x) = 4x^2 + 3x + 9$. Choose the simplified form of $\dfrac{f(x+h) - f(x)}{h}$.

Possibilities:

(a) $6x + 18$

(b) $8h + 3$

(c) $14x + 8h + 21$

(d) $8x + 4h + 3$

(e) 1

19. Suppose $\lim\limits_{x \to 7} f(x) = 2$, $\lim\limits_{x \to 7} g(x) = 9$, $f(7) = 4$ and $g(7) = -5$.

Evaluate $\lim\limits_{x \to 7} \left((6x + 3)f(x) - \dfrac{72}{g(x)} \right)$.

Possibilities:

(a) $\dfrac{522}{5}$

(b) $\dfrac{972}{5}$

(c) 82

(d) 172

(e) $\dfrac{1251}{7}$

20. Let $f(x) = 8x^2 - 3x + 5$. Determine the instantaneous rate of change of $f(x)$ at $x = 6$.

Possibilities:

(a) 96

(b) 98

(c) 93

(d) $\dfrac{9}{16}$

(e) 9