MA 123 Fall 2024
Elementary Calculus

Exam 3
11/21/24

Name: _____

Student ID #: 9_____ **Sec:** _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 2 short answer questions and 18 multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (a) is correct, you must shade

(a) (b) (c) (d) (e)

It is your responsibility to make it CLEAR which response has been chosen. **You will not get credit unless the correct answer has been clearly marked on this page.**

GOOD LUCK!

3. (a) (b) (c) (d) (e)

4. (a) (b) (c) (d) (e)

5. (a) (b) (c) (d) (e)

6. (a) (b) (c) (d) (e)

7. (a) (b) (c) (d) (e)

8. (a) (b) (c) (d) (e)

9. (a) (b) (c) (d) (e)

10. (a) (b) (c) (d) (e)

11. (a) (b) (c) (d) (e)

12. (a) (b) (c) (d) (e)

13. (a) (b) (c) (d) (e)

14. (a) (b) (c) (d) (e)

15. (a) (b) (c) (d) (e)

16. (a) (b) (c) (d) (e)

17. (a) (b) (c) (d) (e)

18. (a) (b) (c) (d) (e)

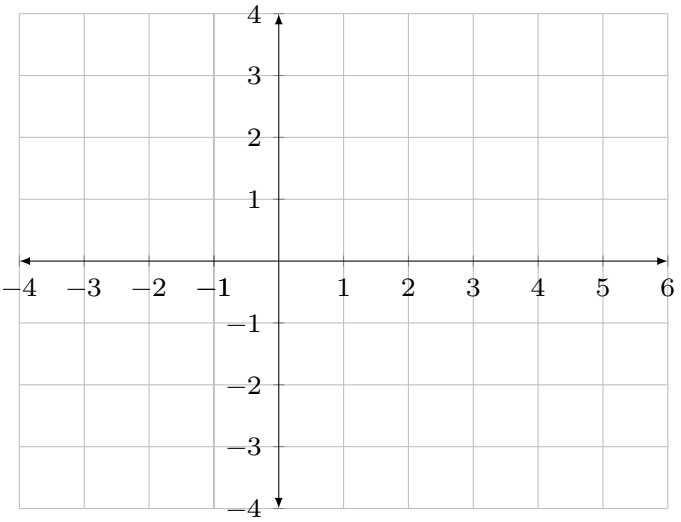
19. (a) (b) (c) (d) (e)

20. (a) (b) (c) (d) (e)

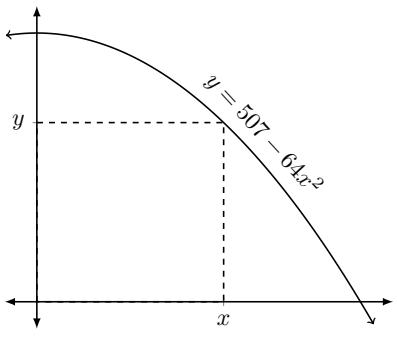
Short Answer Questions

Each question is an opportunity to earn 5 points. Points are earned on the clarity and correctness of your work, not merely on having a correct answer somewhere.

1. Sketch the graph of a continuous function $y = f(x)$ which satisfies the following properties: $f(3) = 2$, $f'(x) > 0$ on $(-\infty, 1)$, $f'(x) < 0$ on $(1, \infty)$, $f''(x) < 0$ on $(-\infty, 1)$ and $f''(x) > 0$ on $(1, \infty)$.



2. Determine the area of the largest rectangle with one corner at the origin, the opposite corner in the first quadrant on the graph of $y = 507 - 64x^2$, and sides parallel to the axes. **You must show all steps of an optimization problem to earn full credit.**

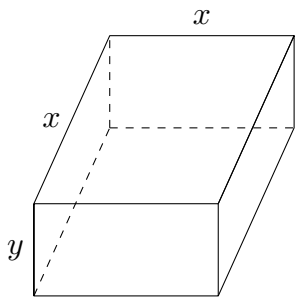


Name: _____

Multiple Choice Questions

Clearly mark your answer on the cover page on this exam for credit.

3. A person is constructing a box out of two different types of metal. The metal for the top and bottom, which are both square, costs \$7 per square foot and the metal for the sides costs \$27 per square foot. The person wants to minimize the cost to construct the box, which must have a volume of 40 cubic feet. What is the objective function for this optimization problem? In the answer choices below, C denotes cost and V denotes volume.



Possibilities:

(a) $V = x^2y$

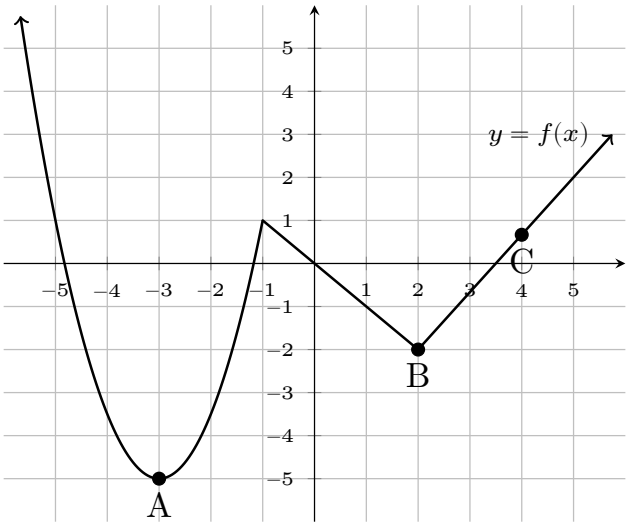
(b) $C = 122x + 108y$

(c) $2x + y = 40$

(d) $C = 14x^2 + 108xy$

(e) $x^2y = 40$

4. The graph of $y = f(x)$ is shown below. At which of the labeled points does $f(x)$ attain a local minimum?



Possibilities:

(a) A and B only

(b) B and C only

(c) B only

(d) A, B and C

(e) A only

5. Let $f(x) = x^4 - 4x^3 - 210x^2 + 24x + 48$. Determine all intervals on which $f(x)$ is concave up.

Possibilities:

(a) $(-\infty, -7) \cup (5, \infty)$

(b) $(-\infty, -5) \cup (7, \infty)$

(c) $(-5, 7)$

(d) $(-7, 5)$

(e) $(-\infty, \infty)$

6. At which value of x does $f(x) = xe^{18x}$ attain its minimum value?

Possibilities:

(a) $f(x)$ does not attain a minimum value

(b) $-\dfrac{1}{9}$

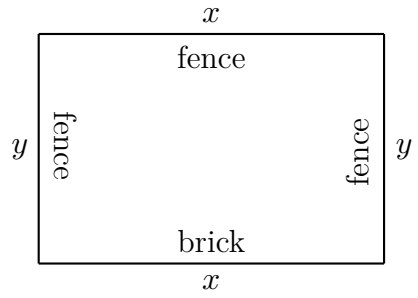
(c) $-\dfrac{1}{18}$

(d) 18

(e) 0

7. A landscape architect plans to enclose a 935 square foot rectangular garden on one side with a brick wall costing \$80 per foot and on the other three sides with a metal fence costing \$90 per foot. Determine the minimum cost to enclose the garden.

   Choose the numeric value that most closely approximates the answer.



   Possibilities:

   (a) \$10403.18

   (b) \$10519.13

   (c) \$11115.44

   (d) \$10697.85

   (e) \$10589.99

8. Suppose the derivative of $g(t)$ is $g'(t) = -t^2(t^2+9)(t-5)$. Determine the value of t in the interval $[-20, 20]$ where $g(t)$ takes on its maximum value.

   Possibilities:

   (a) 20

   (b) -20

   (c) 0

   (d) -3

   (e) 5

9. Suppose the derivative of $g(t)$ is $g'(t) = -t^4 + 12t^3 + 26$. Determine all values of t where $g(t)$ has an inflection point. You may assume that $g(t)$ is defined for all t.

Possibilities:

(a) 9

(b) 6

(c) 0 and 9

(d) 0 and 6

(e) 0

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10. Determine all critical values of the function $f(x) = -x^4 + 24x^3 - 162x^2$.

Possibilities:

(a) 3 and 9

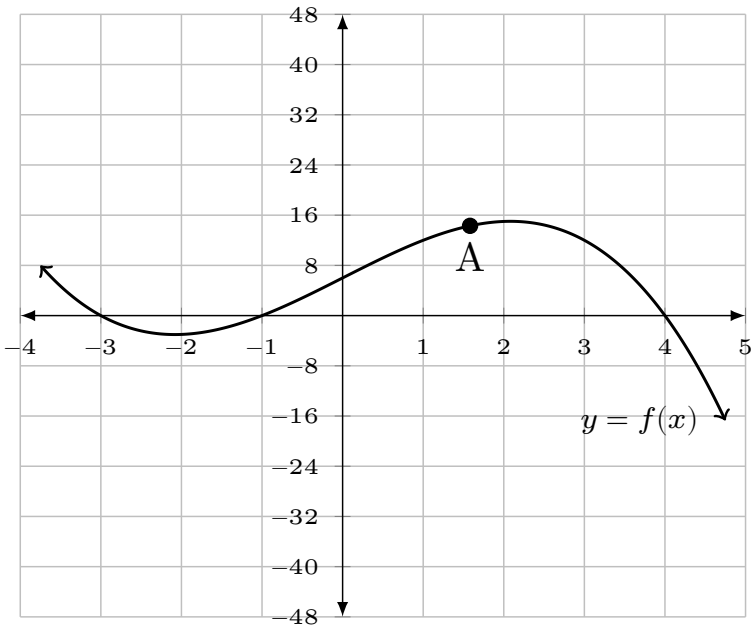
(b) $-9, 0$ and 9

(c) -9 and 9

(d) -9 and 0

(e) 0 and 9

11. Consider the point labeled A on the graph of the function $y = f(x)$. Use the graph to determine the signs of f' and f'' at A.



Possibilities:

(a) $f' = 0$ and $f'' = 0$

(b) $f' > 0$ and $f'' < 0$

(c) $f' < 0$ and $f'' < 0$

(d) $f' < 0$ and $f'' > 0$

(e) $f' > 0$ and $f'' > 0$

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12. Let $f(x) = (x + 15) \cdot \ln(x - 10)$ for $x > 10$. Determine all intervals on which $f(x)$ is concave down.

Possibilities:

(a) $(10, 35)$

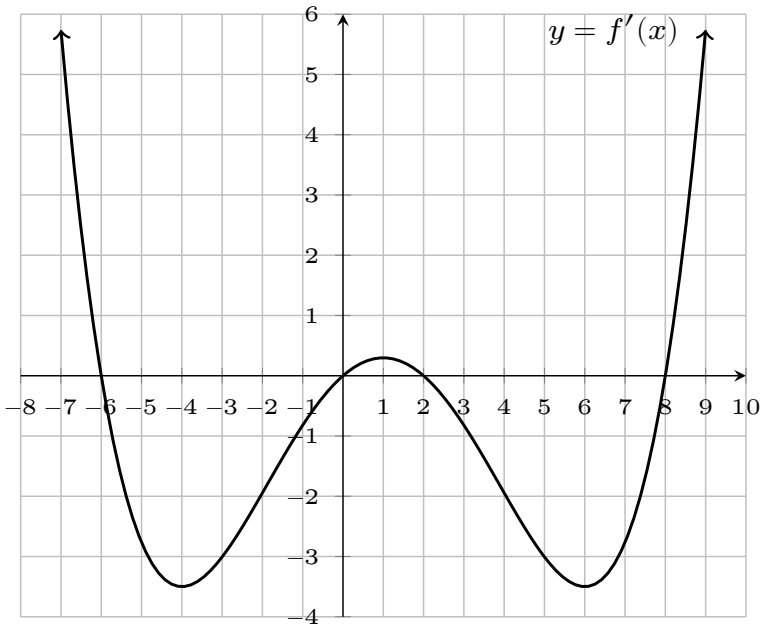
(b) $(10, 15)$

(c) $(15, 35)$

(d) $(35, \infty)$

(e) $(15, \infty)$

13. Below is the **graph of the derivative**, $f'(x)$, of a function $f(x)$. Determine all intervals on which the original function $f(x)$ is concave down.



Possibilities:

(a) $(-\infty, \infty)$

(b) $(-6, 0) \cup (2, 8)$

(c) $(-\infty, -4) \cup (1, 6)$

(d) $(-4, 1) \cup (6, \infty)$

(e) $(-\infty, -6) \cup (0, 2) \cup (8, \infty)$

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14. Determine all intervals on which the function $f(x) = \dfrac{6}{(x-8)^2}$ is increasing.

Possibilities:

(a) $(6, 8)$

(b) $(-\infty, 8)$

(c) $(-\infty, 8) \cup (8, \infty)$

(d) $f(x)$ is never increasing

(e) $(8, \infty)$

15. Suppose that $\int_7^{15} f(x)\,dx = 11$. Determine the value of $\int_7^{15} (f(x) - 9)\,dx$.

Possibilities:

(a) -61

(b) 83

(c) -83

(d) 2

(e) 61

16. The graph of $y = f(x)$ is given below. The numbers shown represent the geometric area of each region. Evaluate the definite integral

$$\int_2^0 f(x)\,dx.$$

Possibilities:

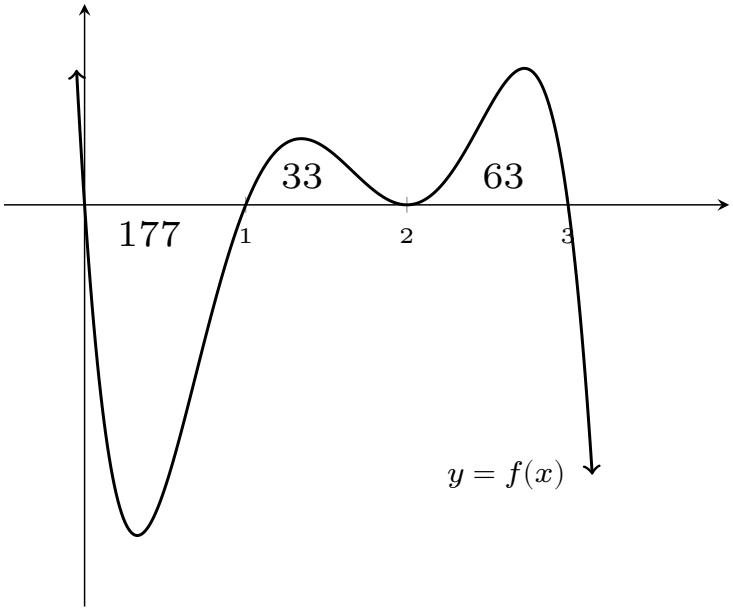
(a) 81

(b) -144

(c) -273

(d) -81

(e) 144



17. If $\int_1^{17} f(x)\,dx = 22$ and $\int_3^{17} f(x)\,dx = 20$, then determine $\int_1^3 f(x)\,dx$.

Possibilities:

(a) -42

(b) 2

(c) 42

(d) -2

(e) 36

18. The graph of $y = f(x)$ shown below consists of straight lines. Evaluate the definite integral $\int_{-5}^{5} f(x)\,dx$.

Possibilities:

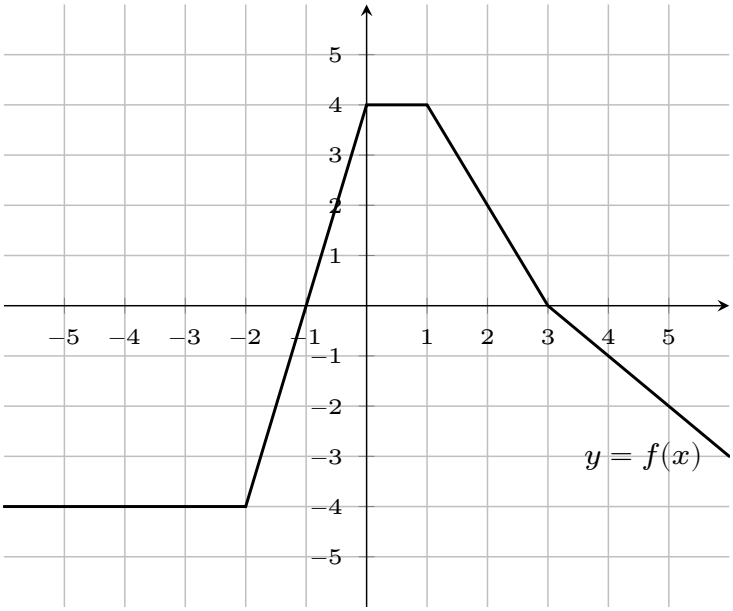
(a) 26

(b) -2

(c) -26

(d) -6

(e) 22



19. Given the function $f(x) = \begin{cases} 5 & \text{if } x < 3, \\ x + 2 & \text{if } x \geq 3, \end{cases}$ evaluate the definite integral $\int_0^9 f(x)\,dx$.

Possibilities:

(a) 147

(b) 111

(c) 207

(d) 63

(e) 48

20. Suppose the average value of a function $f(x)$ on the interval $[7, 12]$ is 20. Determine the value of $\int_7^{12} f(x)\,dx$.

Possibilities:

(a) 70

(b) 15

(c) 100

(d) $\frac{1}{4}$

(e) 4

Formulas

Areas:

Circle: $A = \pi r^2$

Triangle: $A = \dfrac{bh}{2}$

Rectangle: $A = lw$

Trapezoid: $A = \dfrac{b_1 + b_2}{2}\, h$

Volumes:

Rectangular Solid: $V = lwh$