MA 123 Spring 2024 Exam 4

Elementary Calculus 05/01/24

Name: _____

Student ID #: 9_____ **Sec:** _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 2 short answer questions and 18 multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (a) is correct, you must shade

a **b** **c** **d** **e**

It is your responsibility to make it CLEAR which response has been chosen. **You will not get credit unless the correct answer has been clearly marked on this page.**

GOOD LUCK!

3. **a** **b** **c** **d** **e**

4. **a** **b** **c** **d** **e**

5. **a** **b** **c** **d** **e**

6. **a** **b** **c** **d** **e**

7. **a** **b** **c** **d** **e**

8. **a** **b** **c** **d** **e**

9. **a** **b** **c** **d** **e**

10. **a** **b** **c** **d** **e**

11. **a** **b** **c** **d** **e**

12. **a** **b** **c** **d** **e**

13. **a** **b** **c** **d** **e**

14. **a** **b** **c** **d** **e**

15. **a** **b** **c** **d** **e**

16. **a** **b** **c** **d** **e**

17. **a** **b** **c** **d** **e**

18. **a** **b** **c** **d** **e**

19. **a** **b** **c** **d** **e**

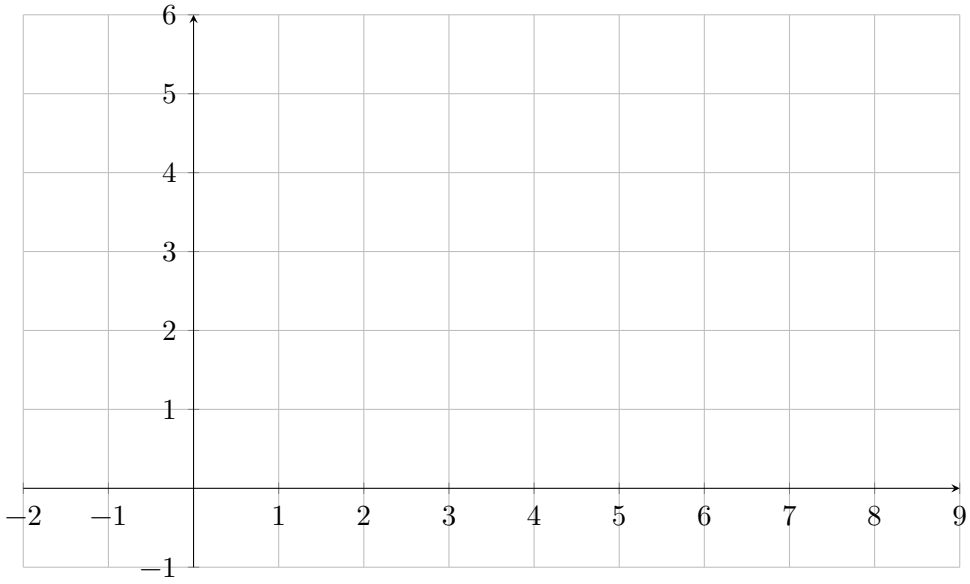
20. **a** **b** **c** **d** **e**

Short Answer Questions

Each question is an opportunity to earn 5 points. Points are earned on the clarity and correctness of your work, not merely on having a correct answer somewhere.

---

1. Sketch the graph of a function $y = f(x)$ which satisfies the following properties:
   $\lim_{x\to 3} f(x) = 1$, $\lim_{x\to 6^-} f(x) = 5$, $f(6) = 2$, $\lim_{x\to 6^+} f(x) = 3$ and $f(x)$ is continuous for all x except $x = 6$.



2. Evaluate $\displaystyle\int_2^3 \frac{6x^5}{x^6+9}\,dx$. **Answers that are not supported with work will receive no credit.**

Name: _____

Multiple Choice Questions

Clearly mark your answer on the cover page on this exam for credit.

3. Suppose $f(4) = 8$, $f'(4) = -7$, $f(10) = 3$ and $f'(10) = 9$. If $g(x) = f(x^2 - 6)$, determine $g'(4)$.

   Possibilities:

   (a) 90

   (b) -6

   (c) 24

   (d) 51

   (e) 72

4. Determine the minimum value of $f(x) = \dfrac{3x^2}{x-2}$ on the interval $[3, 7]$.

   Possibilities:

   (a) 27

   (b) $\dfrac{147}{5}$

   (c) 24

   (d) 0

   (e) 31

5. Evaluate $\lim_{t \to 3} \dfrac{t^2 - 9t + 18}{t^2 + 2t - 15}$.

Possibilities:

(a) The limit does not exist.

(b) $-\dfrac{9}{2}$

(c) 0

(d) 1

(e) $-\dfrac{3}{8}$

---

6. The graph of $y = f(x)$ is shown below. The function is continuous, except at which x values?

Possibilities:

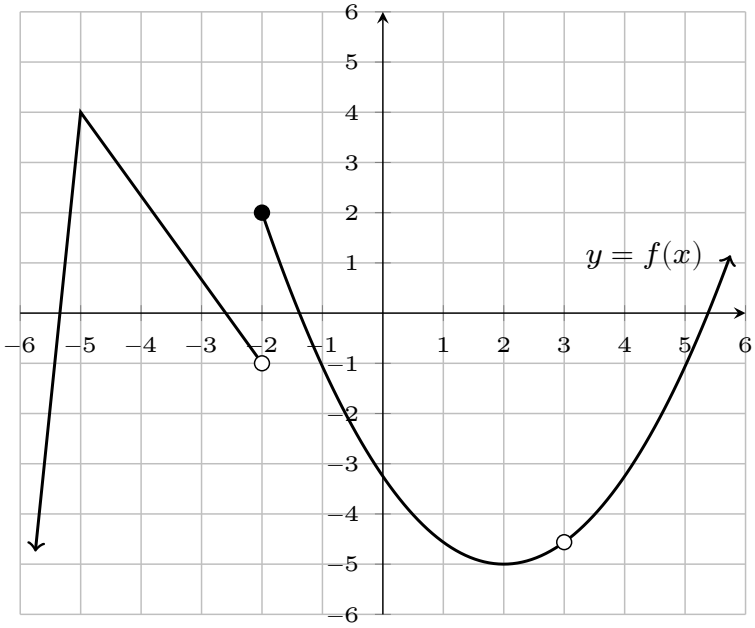
(a) -5, -2 and 3

(b) -2 and 3 only

(c) -2 only

(d) -5 and -2 only

(e) -5 only



7. A company that makes whatzits has a start up cost of $12000. It costs the company $2.81 to make each whatzit. The company charges $4.01 for each whatzit. Determine the minumum number of whatzits the company must produce and sell to make a profit.

Possibilities:

(a) 2993

(b) 10001

(c) 6214

(d) 1760

(e) 5212

8. A coal-burning electrical generating plant emits sulfur dioxide into the surrounding air. The concentration $C(x)$, in parts per million, is approximately given by the function

$$C(x) = \frac{1.7}{x^2},$$

where x is the distance away from the plant in miles. Determine the instantaneous rate of change of the sulfur dioxide concentration 9 miles from the plant.

Choose the numeric value that most closely approximates the answer.

Possibilities:

(a) -0.0047 parts per million per mile

(b) -30.6000 parts per million per mile

(c) -137.7000 parts per million per mile

(d) -0.3778 parts per million per mile

(e) 0.0210 parts per million per mile

9. Consider the point labeled A on the graph of the function $y = f(x)$. Use the graph to determine the signs of f' and f'' at A.

Possibilities:

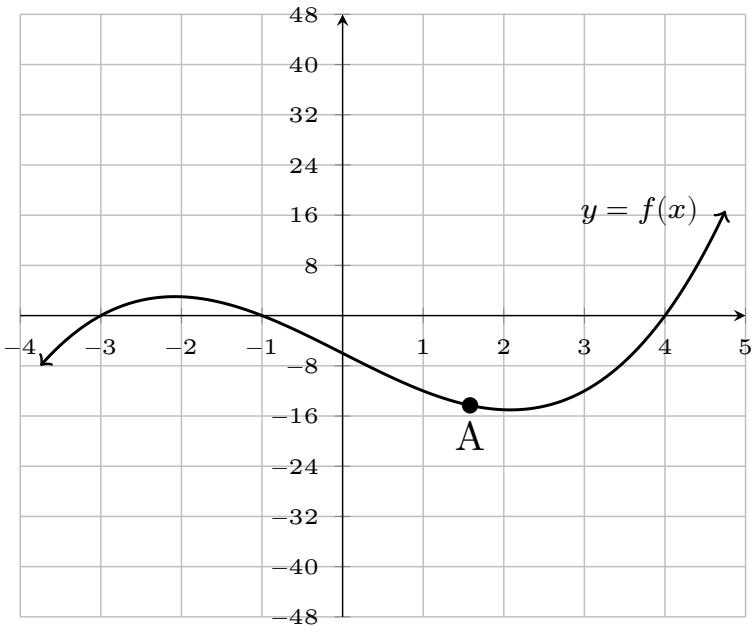
(a) $f' < 0$ and $f'' > 0$

(b) $f' = 0$ and $f'' = 0$

(c) $f' > 0$ and $f'' < 0$

(d) $f' > 0$ and $f'' > 0$

(e) $f' < 0$ and $f'' < 0$



10. Suppose the derivative of $g(t)$ is $g'(t) = t^2(t^2 + 4)(t + 6)$. Determine the value of t in the interval $[-60, 60]$ where $g(t)$ takes on its minimum value.

Possibilities:

(a) -6

(b) -2

(c) 60

(d) 0

(e) -60

11. Determine the slope of the tangent line to the graph of the function

$$f(x) = x^3 e^x$$

at $x = 2$.

Possibilities:

(a) $12e^2$

(b) $4e^2$

(c) $20e^2$

(d) $12e^2 - 4$

(e) $8e^2$

12. Let $f(x) = 3x^2 + 7x - 8$. Determine the slope-intercept form of the equation of the tangent line to the graph of $y = f(x)$ at $x = 4$.

Possibilities:

(a) $y = 23x - 24$

(b) $y = 68x + 31$

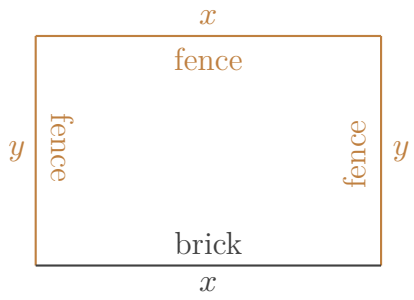
(c) $y = 31x + 68$

(d) $y = 31x - 56$

(e) $y = 68x - 241$

13. A landscape architect plans to enclose a rectangular garden on one side by a brick wall costing \$70 per foot and on the other three sides by a metal fence costing \$40 per foot. If the area of the garden must be 1320 square feet, determine the minimum cost to enclose the garden.

    Choose the numeric value that most closely approximates the answer.



Possibilities:

(a) \$7301.61

(b) \$6816.45

(c) \$6882.97

(d) \$6983.95

(e) \$6401.18

14. Let $f(x) = 3x^2$. Determine a value of x such that the average rate of change of $f(x)$ from 4 to x equals 36.

Possibilities:

(a) 16

(b) 8

(c) 9

(d) 4

(e) 12

15. Let $f(x) = x^4 + 8x^3 - 72x^2 + 84x + 108$. Determine all intervals on which $f(x)$ is concave up.

Possibilities:

(a) $(-\infty, -6) \cup (2, \infty)$

(b) $(-\infty, \infty)$

(c) $(-\infty, -2) \cup (6, \infty)$

(d) $(-2, 6)$

(e) $(-6, 2)$

16. Suppose that $\int_2^{18} f(x)\,dx = 11$. Determine the value of $\int_2^{18} (f(x) - 8)\,dx$.

Possibilities:

(a) 139

(b) 3

(c) -139

(d) -117

(e) 117

17. Suppose a rock is thrown from a Saturnian cliff. After t seconds, its speed in feet per second is $s(t) = 26t + 2$, at least until it hits the ground. If the rock hits the ground after 6 seconds, how high is the cliff?

Possibilities:

(a) 938 feet

(b) 948 feet

(c) 470 feet

(d) 158 feet

(e) 480 feet

18. Determine the indefinite integral $\int \left(\frac{1}{6x^5} + 3\sqrt{x}\right)\, dx$.

Possibilities:

(a) $\frac{1}{30x^4} - \frac{3}{2x^{3/2}} + C$

(b) $-\frac{5}{6x^6} + \frac{3}{2x^{1/2}} + C$

(c) $-\frac{1}{24x^4} + 2x^{3/2} + C$

(d) $-\frac{x^6}{36} + 2x^{3/2} + C$

(e) $-\frac{5}{6x^4} + \frac{3x^2}{4} + C$

19. Determine the indefinite integral $\int 21x^6 e^{x^7}\, dx$.

Possibilities:

(a) $3e^x + C$

(b) $\dfrac{1}{3}e^{x^7} + C$

(c) $147x^{12}e^{x^7} + 126x^5 e^{x^7} + C$

(d) $3e^{x^7} + C$

(e) $\dfrac{1}{3}e^x + C$

---

20. Determine the indefinite integral $\int \dfrac{(t+3)^2}{t}\, dt$.

Possibilities:

(a) $\dfrac{t^3 \ln|t|}{3} + 3t^2 \ln|t| + 9\ln|t| + C$

(b) $\dfrac{t^2}{2} + 9t + C$

(c) $\dfrac{t^2}{2} + 6t + 9\ln|t| + C$

(d) $\dfrac{t^3}{3} + 15t + C$

(e) $9t + C$

Formulas

Areas:

Circle: $A = \pi r^2$

Triangle: $A = \dfrac{bh}{2}$

Rectangle: $A = lw$

Trapezoid: $A = \dfrac{b_1 + b_2}{2}\, h$

Volumes:

Rectangular Solid: $V = lwh$