Project Ideas/Suggestions

1. **(Recruitment Model)** Ricker's curve describes the relationship between the size of the parental stock of some fish and the number of recruits. If we denote the size of the parental stock by P and the number of recruits by R, then Ricker's curve is given by

$$R(P) = \alpha P e^{-\beta P} \text{ for } P \geq 0$$

where α and β are positive constants.[1]

We are interested in the size P of the parental stock that maximizes the number $R(P)$ of recruits. Since $R(P)$ is differentiable, we can use its first derivative test to solve this problem.

(a) Use the product rule to show that, for $P > 0$,

$$R'(P) = \alpha e^{-\beta P}(1 - \beta P) \qquad R''(P) = -\alpha\beta e^{-\beta P}(2 - \beta P)$$

(b) Show that $R'(1/\beta) = 0$ and $R''(1/\beta) < 0$. This shows that $R(P)$ has a local maximum at $P = 1/\beta$. Show that $R(1/\beta) = (\alpha/\beta)e^{-1} > 0$.

(c) To find the global maximum, you need to check $R(0)$ and $\lim_{P\to\infty} R(P)$. Show that

$$R(0) = 0 \text{ and } \lim_{P\to\infty} R(P) = 0$$

and that this implies that there is a global maximum at $P = 1/\beta$.

(d) Show that $R(P)$ has an inflection point at $= 2/\beta$.

(e) Sketch the graph of $R(P)$ for several choices of α and β (e.g. $\alpha = 2$ and $\beta = 1$).

[1] [1]Note that $R(0) = 0$; that is, without parents there are no offspring. Furthermore, $R(P) > 0$ when $P > 0$.