4. (Hematopoietic cell replication and control)

Periodic hematopoiesis is a disease characterized by large oscillations in the red blood cell count. Red blood cells are generated by a feedback mechanism that approximately obeys the difference equation

$$x_{t+1} = \frac{\tau}{1+\gamma\tau} F(x_t) + \frac{1}{1+\gamma\tau} x_t .$$

The function $F(x_t)$ describes production as a function of number of cells and takes the form

$$F(x) = F_0 \frac{\theta^n}{\theta^n + x^n} .$$

The terms here are:

Parameter	Meaning	Normal Value
x_t	Number of cells at time t (in billions)	About 330
τ	Time for cell development	5.7 days
γ	Fraction of cells that die each day	0.0231
F_0	Maximum production of cells	76.2 billion
θ	Value where cell production is halved	247 billion
n	Shape parameter	7.6

a) Graph and explain $F(x)$ with these parameter values. Does this sort of feedback system make sense?

b) Use a computer to find the equilibrium points with the normal parameters.

c) Graph and cobweb the difference equation. If you start near the equilibrium point, do values remain nearby? What might a solution look like?

d) Certain autoimmune diseases increase γ, the death rate of cells. Study what happens to the equilibrium and the solution as γ increases. Can you explain your results in biological terms?

e) Explore what would happen if the value of n were decreased. Does the equilibrium become more sensitive to small changes in γ?