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① $f(x) = \dfrac{x^2-9}{x^2+9}$ First find critical values

$$f'(x) = \frac{2x(x^2+9) - (2x(x^2-9))}{(x^2+9)^2} = 0$$

$x^2+9 \neq 0$ for any x

$$2x(x^2+9) - 2x(x^2-9) = 0$$

Factor a $2x$

$$2x(x^2+9-x^2+9) = 0$$

$2x = 0$ $\quad x^2 - x^2 + 9 + 9 = 0$

$x = 0$ $\quad 18 = 0$ ⊗ cant happen

So $x=0$ is our only critical value

We need to test

-9, 0, and 9 into $f(x)$

notice $f(9) = f(-9) = \dfrac{(-9)^2-9}{(-9)^2+9} = \dfrac{72}{90} = \dfrac{4}{5} = .8$

$f(0) = \dfrac{-9}{9} = -1$

Max at $(-9, .8)$ and $(9, .8)$

Min at $(0, -1)$

2. $f(x) = 2x - 3\ln(x)$ on $[1,5]$

$$f'(x) = 2 - \frac{3}{x} = 0$$

$$\boxed{x = \frac{3}{2}}$$

So we need to test

$(1, \frac{3}{2}, 5)$

$f(1) = 2$

$f(3/2) \approx 1.784$

$f(5) \approx 5.172$

Max at $(5, 5.172)$

Min at $(\frac{3}{2}, 1.784)$

3. $f(x)$ is cont. on $[0,4]$ ✓

$f(x)$ is differentiable on $(0,4)$ ✓

So there is a c in $[0,4]$ such that

$$f'(c) = \frac{f(4) - f(0)}{4-0} = \frac{\frac{2}{3}(4)^3 - 6(4) - \frac{2}{3}(0)^3 + 6(0)}{4}$$

$$= \frac{32}{3} - 6 = \frac{14}{3}$$

So

$$f'(c) = \frac{14}{3}$$

$$f'(c) = 2c^2 - 6 = \frac{14}{3}$$

$$2c^2 = \frac{14}{3} + 6$$

$$2c^2 = \frac{14}{3} + \frac{18}{3} = \frac{32}{3}$$

So

$$c = \pm\sqrt{\frac{16}{3}}$$

$$c = \frac{4}{\sqrt{3}}$$

Notice $\frac{-4}{\sqrt{3}}$ is not in $[0,4]$

4, $f'(x) = \frac{2}{3} x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$ which is

undefined at $x = 0$

So f is <u>not</u> differentiable at zero

(It has a corner point) so it does <u>not</u>

satisfy MVT