MA 137 — Calculus 1 with Life Science Applications
The Definite Integral
(Section 6.1)

Department of Mathematics
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Some Properties of Definite Integrals

1. $$\int_a^a f(x)\,dx = 0$$

2. $$\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$$

3. $$\int_a^b (f(x) \pm g(x))\,dx = \left(\int_a^b f(x)\,dx\right) \pm \left(\int_a^b g(x)\,dx\right)$$

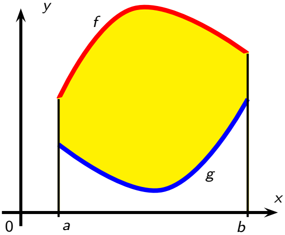
4. $$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

5. $$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

6. If $m \le f(x) \le M$ on $[a, b]$ then
$$m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$$

Geometric Illustration of Some of the Properties

Property **3.** says that if f and g are two positive valued functions with



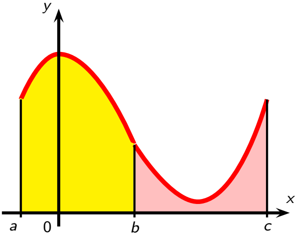
f greater than g, then

$$\int_a^b (f(x) - g(x))\, dx$$

gives the area between the graphs of f and g

$$\int_a^b f(x)\, dx - \int_a^b g(x)\, dx$$

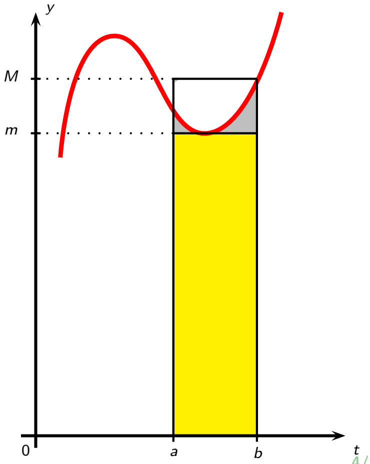
Property **4.** says that if $f(x)$ is a positive valued function then the area underneath the graph of $f(x)$ between a and b plus the area underneath the graph of $f(x)$ between b and c equals the area underneath the graph of $f(x)$ between a and c.



Property **5.** follows from Properties **4.** and **1.** by letting $c = a$.

$$0 = \int_a^a f(x)\,dx \; = \; \int_a^b f(x)\,dx + \int_b^a f(x)\,dx.$$

Property **6.** is illustrated in the picture below.



Example 1: (Online Homework, HW23, # 8)

The sum

$$\int_{-2}^{2} f(x)\,dx + \int_{2}^{5} f(x)\,dx - \int_{-2}^{-1} f(x)\,dx$$

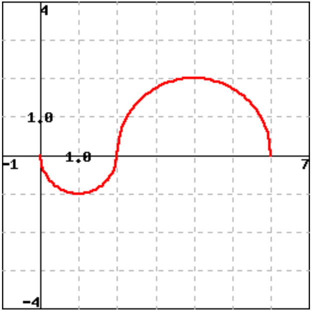
can be written as a single definite integral of the form

$$\int_{a}^{b} f(x)\,dx$$

for appropriate a and b. Determine these values.

Example 2: (Online Homework, HW23, # 5)

Evaluate the integrals for $f(x)$ shown in the figure below. The two parts of the graph are semicircles.



$$\int_0^2 f(x)\,dx \qquad \int_0^6 f(x)\,dx \qquad \int_1^4 f(x)\,dx \qquad \int_1^6 |f(x)|\,dx.$$

Example 3: (Neuhauser, Problem # 37, p. 320)

Use an area formula from geometry to find the value of the integral below

$$\int_{-2}^{3} |x|\, dx$$

by interpreting it as the (signed) area under the graph of an appropriately chosen function.

Example 4: (Neuhauser, Problem # 41, p. 320)

Use an area formula from geometry to find the value of the integral below

$$\int_{-2}^{2} \left(\sqrt{4-x^2} - 2\right) dx$$

by interpreting it as the (signed) area under the graph of an appropriately chosen function.

Example 5: (Neuhauser, Problem # 50(c),(f), p. 320)

Given that

$$\int_0^a x^2\,dx = \frac{1}{3}a^3$$

evaluate the following

$$\int_{-1}^{3} \frac{1}{3}x^2\,dx \qquad\qquad \int_2^4 (x-2)^2\,dx.$$