SageMathCell

$$\frac{dy}{dx} = y+2$$

Type some Sage code below and press Evaluate.

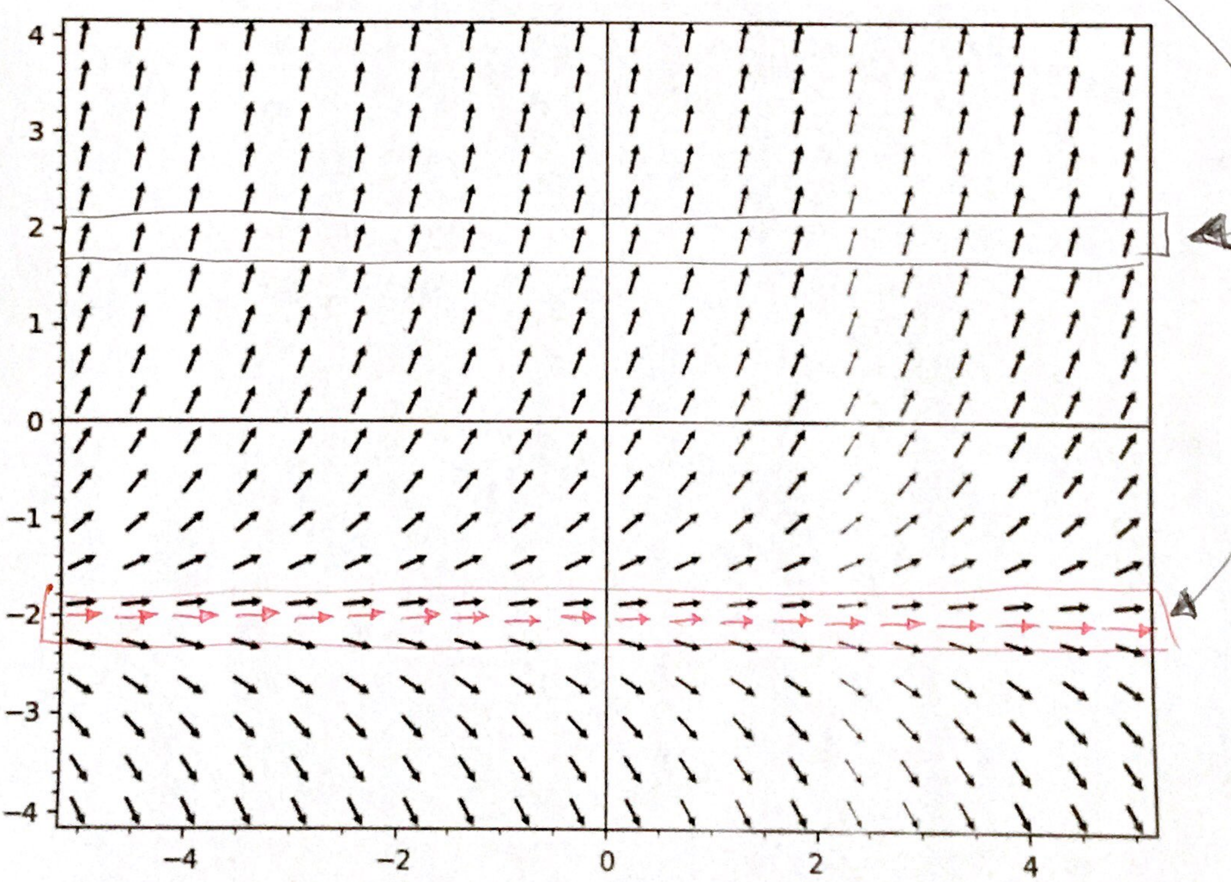
```
x,y=var('x,y')
plot_slope_field(y+2,(x,-5,5),(y,-4,4), headaxislength=3, headlength=3)
```

notice that this is an autonomous D.E.

For a fixed value y the arrows are the same on that row

For $y=-2$ $\frac{dy}{dx}=0$ hence horizontal arrows

Evaluate



SageMathCell

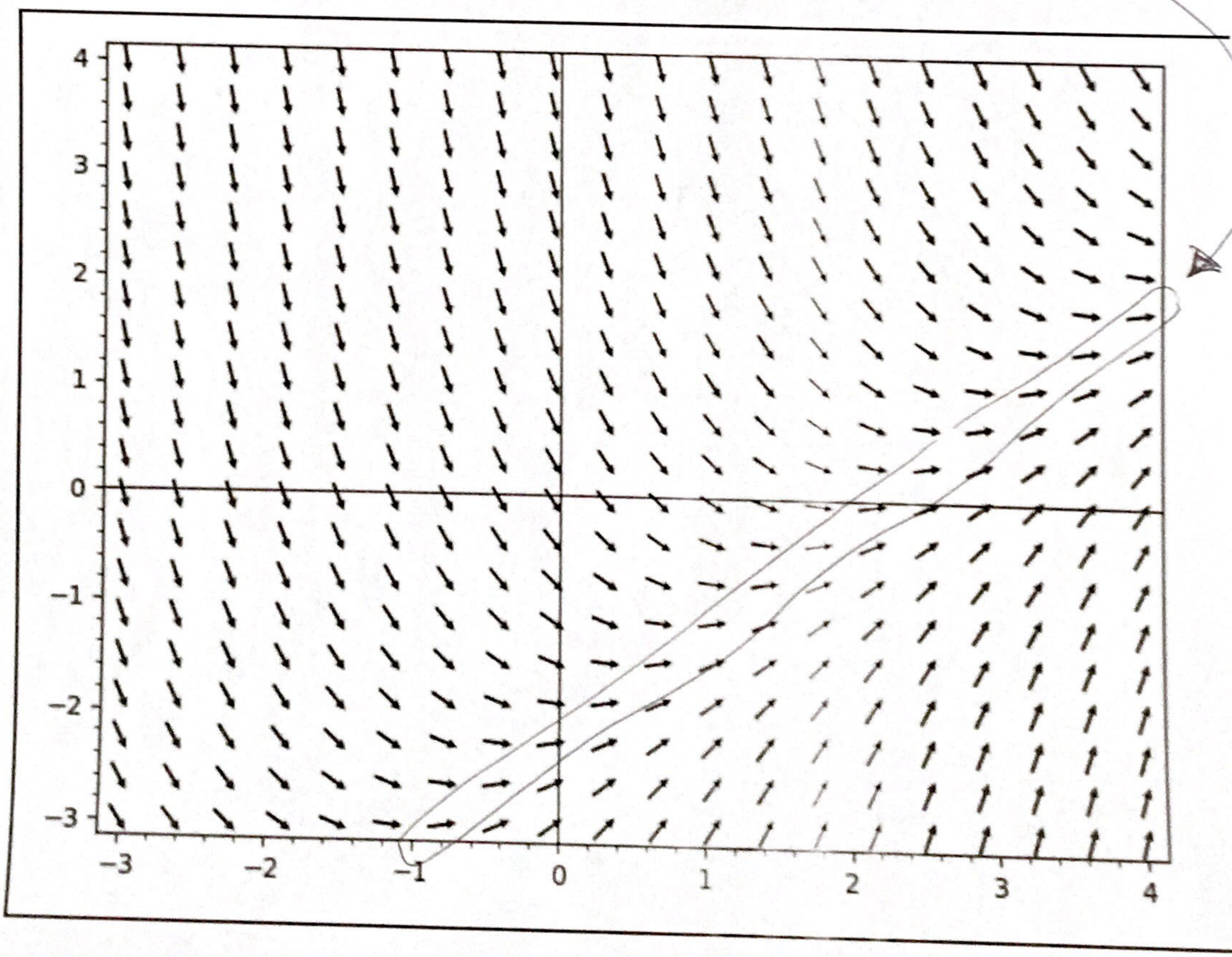
$$\frac{dy}{dx} = -2 + x - y$$

Type some Sage code below and press Evaluate.

```
x,y=var('x,y')
plot_slope_field(-2+x-y,(x,-3,4),(y,-3,4), headaxislength=3, headlength=3)
```

note that $\frac{dy}{dx} = 0$ corresponds to the line $y = x - 2$. There are "almost" horizontal arrows

Evaluate



SageMathCell

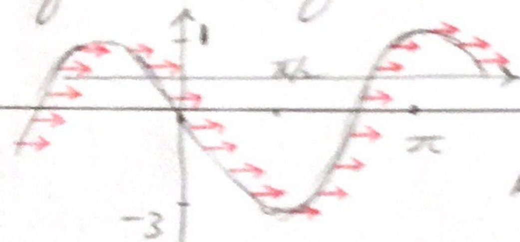
$\frac{dy}{dx} = 2\sin(x) + 1 + y$

Type some Sage code below and press Evaluate.

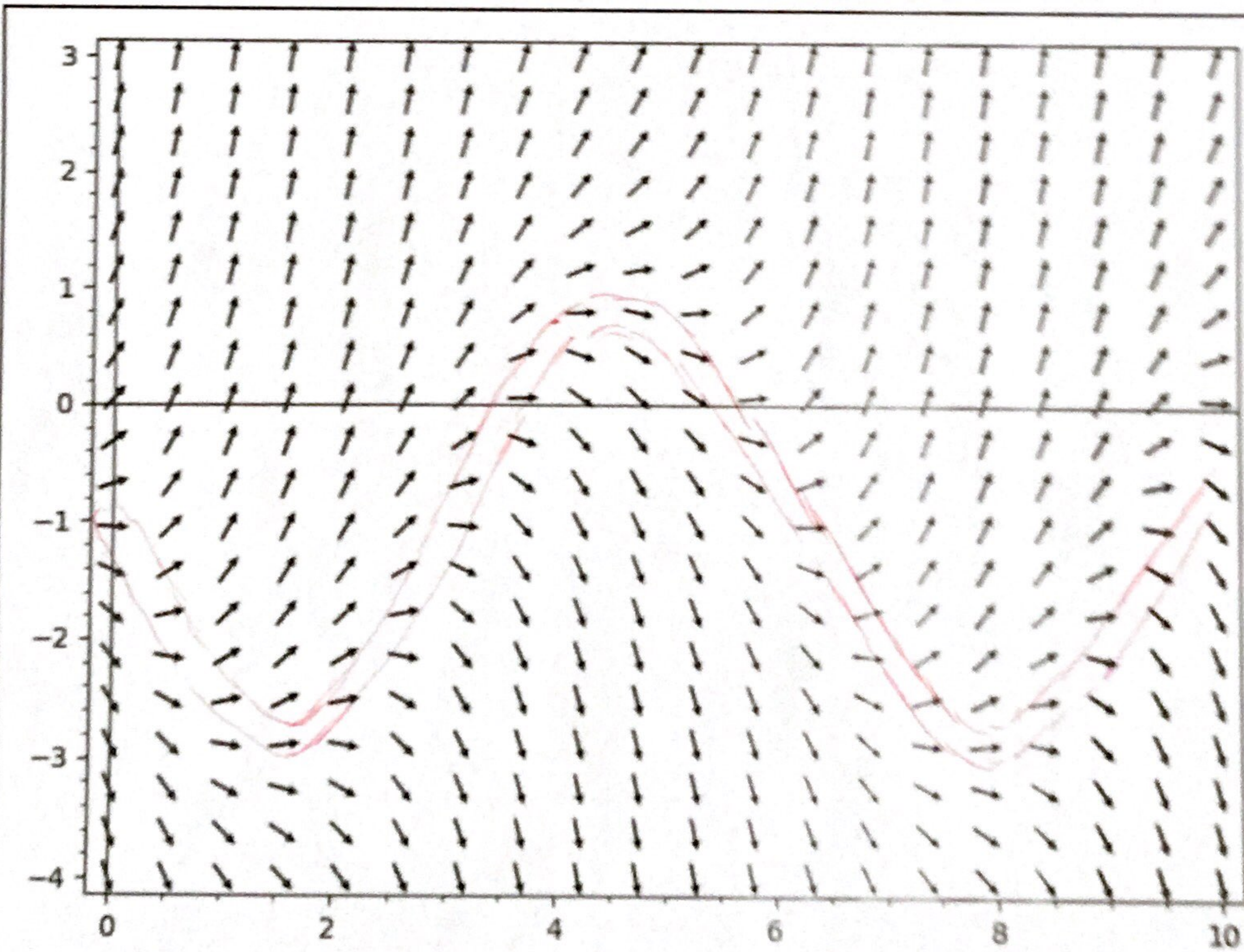
```
x,y=var('x,y')
plot_slope_field(2*sin(x)+1+y,(x,0,10),(y,-4,3), headaxislength=3, headlength=3)
```

notice that $\frac{dy}{dx} = 0$ hence there are horizontal arrows in the direction field for

$y = -1 - 2\sin(x)$



Evaluate



$\boxed{\#2}$

$$\frac{dy}{dx} = (4-y)(5-y) = g(y)$$

The equilibria for this D.E. are obtained by $g(y)=0$. This gives $\boxed{\hat{y} = 4 \text{ or } 5}$

$$g(y) = (4-y)(5-y) = y^2 - 9y + 20$$

$$g'(y) = 2y - 9$$

so the "eigenvalues" are

$$g'(4) = 2 \cdot 4 - 9 = -1 < 0$$

$$g'(5) = 2 \cdot 5 - 9 = 1 > 0$$

hence $\hat{y} = 4$ is <u>locally stable</u>

$\hat{y} = 5$ is <u>unstable</u>

SageMathCell

Type some Sage code below and press Evaluate.

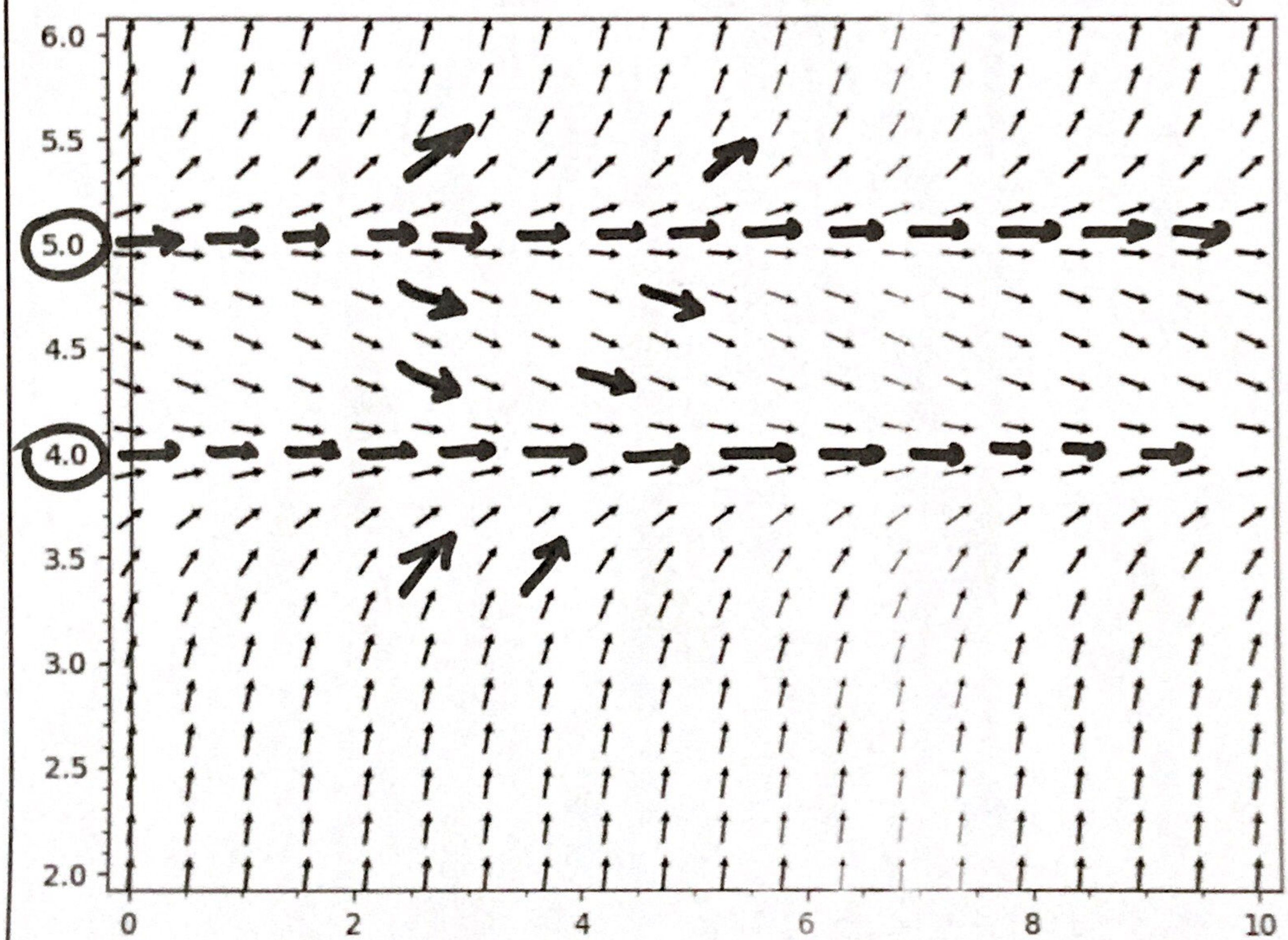
```
x,y=var('x,y')
plot_slope_field((4-y)*(5-y),(x,0,10),(y,2,6), headaxislength=3, headlength=3)
```

$\hat{y} = 4$ is locally stable

$\hat{y} = 5$ is unstable

Evaluate

arrows point toward to the line $y = 4$; the point away from $y = 5$



$$\frac{dy}{dx} = (4-y)(5-y) \qquad \text{separate variables}$$

$$\int \frac{1}{(4-y)(5-y)}\, dy = \int dx$$

need partial fractions

$$\frac{1}{(4-y)(5-y)} = \frac{A}{4-y} + \frac{B}{5-y} = \frac{A(5-y)+B(4-y)}{(4-y)(5-y)}$$

requires $\quad 1 = A(5-y) + B(4-y)$

set $y=4 \rightsquigarrow 1 = A\cdot(5-4) + B\cdot 0 \qquad \boxed{A=1}$

set $y=5 \rightsquigarrow 1 = A(0) + B(4-5) \qquad \boxed{B=-1}$

Hence $\quad \displaystyle\int \left(\frac{1}{4-y} - \frac{1}{5-y}\right) dy = \int dx$

$$\int \left(\frac{1}{y-5} - \frac{1}{y-4}\right) dy = x + C$$

$$\ln|y-5| - \ln|y-4| = x + C$$

$$\ln\left|\frac{y-5}{y-4}\right| = x + C$$

Take exp and get rid of $|\cdot|$

$$\frac{y-5}{y-4} = \underbrace{\pm e^{c}}_{A} \cdot e^{x}$$

A new constant dependent on the initial condition

$$\frac{y-5}{y-4} = Ae^{x} \iff y-5 = (y-4)Ae^{x}$$

$$y-5 = yAe^{x} - 4Ae^{x}$$

$$y - yAe^{x} = 5 - 4Ae^{x}$$

$$y(1-Ae^{x}) = 5-4Ae^{x}$$

$$\boxed{y = \frac{5-4Ae^{x}}{1-Ae^{x}}} \quad \text{OR} \quad \boxed{y = \frac{5e^{-x}-4A}{e^{-x}-A}}$$

multiply top and bottom by e^{-x}

notice $\lim_{x\to\infty} y(x) = \lim_{x\to\infty} \frac{5e^{-x} - 4A}{e^{-x} - A} = \frac{-4A}{-A} = 4$

the locally stable equilibrium